ESSAYS ON FISCAL POLICY, PUBLIC INFRASTRUCTURE, AND
PUBLIC R&D AND HETEROGENEOUS LABOR

By

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This dissertation includes three essays on the dynamic effects of fiscal policy, public infrastructure, and public R&D and heterogeneous labor. In the first essay, we develop and estimate a DSGE model in which congested public infrastructure and public R&D are the crucial elements of a neoclassical production function in order to investigate the dynamic effects of increasing in government investment in R&D and government investment in infrastructure. For the second essay, we estimate a small scale DSGE model for the Thai economy. The endogenous growth model is developed to incorporate congestion due to production inputs and the population density. The estimates suggest that all factors have congested public infrastructure, and that density may be the dominant influence. The third essay analyzes the effects of public R&D on differentiated labor for the US economy compared to the Thailand economy, including male skilled labor, female skilled labor, male unskilled labor, and female unskilled labor. Two sectors of R&D and final goods productions are analyzed in which the final goods firms produce homogenous final output to the economy, and intermediate goods firms invest in R&D knowledge and utilize public R&D to produce blueprints and then produce variety of capital. We find that for the R&D sector, the elasticities of substitution between the two inputs of the two nested levels for the Thai economy and for the United States are the same in terms of complementarity and substitution. For the final goods sector, all the inputs are substitutes for the Thai economy but the for the United States, unskilled female and variety capital are complements.
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CHAPTER ONE

DYNAMIC EFFECTS OF FICAL POLICY IN AN ENDOGENOUS GROWTH MODEL WITH
PUBLIC CAPITAL AND PUBLIC R&D

1.1 Introduction

One of the most important roles of government is implementing fiscal policy so that the country’s resources are optimally allocated to sustain long-run economic growth and increase per capita income. This study examines the dynamic effects of government activities on the key macroeconomic variables by incorporating endogenous growth model and fiscal rules in a dynamic stochastic general equilibrium (DSGE) model, and estimating the model by Bayesian techniques. The technology for this economy is an applied endogenous growth model constructed to include congested public infrastructure and public research and development (R&D) for the purpose of analyzing the impact of public inputs when fiscal rule is defined to respond to output and the state of debt endogenously and dynamically.

Public R&D and public infrastructure have significant effects on output, private inputs, and other components of the economy. A number of studies emphasize that both investments are productive but a consensus has not emerged since the specific effects rely on the specified technology in terms of the category and definition of public inputs and the specific functional form chosen for production.

Public R&D, or public-funded R&D, increases the available stock of knowledge. This causes knowledge spillovers, and may elicit additional private capital accumulation. Whether such investments are productive or not is one of the key criteria for evaluating the economic development and competitiveness of a nation. It affects economic growth through multiple channels such as innovation, capital accumulation, and human resource development, all of which gradually lead to the overall development of the economy. This has become a major concern for policy-makers as they compare the economic costs of such investments with the economic benefits.
The empirical results have been thus far inconclusive and the model specifications only include public R&D as a public input for the private production, while ignoring public infrastructure which is also a productive government activity affecting the economy. The varieties in private capital developed by public R&D knowledge spillovers are analyzed without specifying how capital formation evolves and how this evolution is affected by the simultaneous decisions of agents in both the private and public sectors. Also, the studies are mostly theoretical or only provide calibration results. There are not result that rely on estimation of the impact on the overall economy in the R&D literature.

Government investment in infrastructure is also a crucial productive element of government activities. Several studies (Aschauer (1989a), Munnell (1990), and Finn (1993)) emphasize that public capital is productive, increasing output, households’ income, and hours worked. However, a substantial part of public capital such as roads and highways, airports, harbors, etc. are clearly not pure public goods (Stiglitz (1988)). Roads and highways, for example, are subject to congestion and individuals can be excluded at some cost from using a given road. Both households and firms not only receive the benefit of a positive externality, but also face congestion costs from using public infrastructure. Taking congestion effects into account in considering the overall effects of public infrastructure may lead to greater investment, rather than underinvestment, and allows us to develop a more realistic picture of the value of such investments.

There are several studies that incorporate congestion effects of infrastructure on economic growth. However, these studies examine the effects of public capital on economic growth mostly in theoretical models, or partial equilibrium frameworks, but do not include a stochastic technology, or allow the parameters under the control of the government to respond to the state of the economy via a fiscal rule. That is the results obtained from econometric models have fallen short of investigating the factors contributing to economic growth, to structural change, and to policy changes. In contrast, the DSGE models emphasize the dependence of current choices on expected future outcomes. A key advantage of DSGE models is that they share core assumptions on behavior of households and firms, which make them easily scalable to include details that are relevant to address the questions at hand.
This study employs Bayesian techniques for estimating the DSGE model. Bayesian estimation is becoming increasingly popular in the field of macroeconomics. It fits the complete, solved DSGE model, as opposed to GMM estimation which is based on particular equilibrium relationships such as the Euler equation in consumption. Bayesian techniques allow the consideration of priors which work as weights in the estimation process so that the posterior distribution avoids peaking at strange points where the likelihood peaks. Also, Bayesian estimation explicitly addresses model misspecification by including shocks, which can be interpreted as observation errors, in the structural equations. Recent papers have attracted significant attention; some of these include Smets and Wouters (2003), An and Schorfheide (2007), Negro Schorfheide (2010).

The main objectives of this study include 1) constructing and estimating a DSGE model in which congested public infrastructure and public R&D are considered as public inputs of production, 2) examining the dynamics of changes in fiscal shocks on congestion effects, and on other key macroeconomic variables, and 3) analyzing the net effects/benefits of public infrastructure and public R&D on private outputs and overall economy in order to propose fiscal policy whether the government should decide to invest in infrastructure or invest in R&D or both in short run and long run.

Evaluating the dynamic effects of a complete set of public inputs and fiscal rules on the overall economy is a complex task, and there is a need for a comprehensive macroeconomic input–output- fiscal policy–stochastic framework or a DSGE system model as well as the powerful method for model estimation. Although there are vast numbers of DSGE and Bayesian estimation studies in the literature, the literature that incorporates R&D investment and congested infrastructure is not exist, to the best of my knowledge.

Therefore, this paper contributes to the literature in several ways. First, this study estimates the dynamic effects of fiscal policy on the level and the degree of congestion of public infrastructure and other key macroeconomic variables. We construct a model that can distinguish the effects of public infrastructure from those of public R&D in a DSGE framework, i.e. the DSGE model is extended to incorporate both
congestion effects and public R&D effects simultaneously and dynamically. Second, the detailed specifications of both congestible infrastructure and public R&D knowledge spillovers modeled as public inputs are different from the existing literature.

The remainder of this paper is structured as follows. Section 2 reviews related literature. Section 3 describes the details of the DSGE model, the preferences of the representative household, the technology, the fiscal policy of the government, and the stochastic shocks that affect the economy. Section 4 presents the estimation procedure and model solution. Section 5 explains how Bayesian methods work for estimating a DSGE model, and describes the data construction. Section 6 reports the estimation results and discussion. Section 7 contains concluding remarks.

1.2 Literature Review

This study employs Bayesian methods to estimates a DSGE model with endogenous growth emphasized on public-sector contributions to productivity, and on dynamic effects of fiscal shocks. Public R&D and congested public infrastructure are considered as public inputs for output production. The related literature follows.

1.2.1 Public Capital and an Endogenous Growth Model

The effects of public investment on output and private capital formation is a crucial public policy issue. Empirical research following this question was stimulated by Aschauer (1989). He uses annual data for the U.S. from 1949 to 1985 and OLS, instrumental variables (IV), and nonlinear least square (NLLS) to estimate the elasticities of output with respect to public capital and core infrastructure. The results show that the coefficient of the aggregate nonmilitary public capital is 0.39, while that of core infrastructure is 0.24. Munnell (1990) extends Aschauer’s insight and uses a similar approach to explore whether changes in the amount of public capital, combined with the growth of private capital and labor can explain most of the slowdown in productivity growth. This study finds that the shortfall in public investment reduces labor
productivity growth by 0.1 to 0.2 percentage points for the period 1949 – 1987.

However, these results are considered relatively large due to the problems of model misspecification, omitting time trend, nonstationarity of the time series, cointegration of model variables, and the direction of causality between public capital and private output. Finn (1993) examines components of government capital that play a role in production, and estimates the magnitudes of the associated productivity coefficients, controlling for possible reverse causation. By using GMM to estimate the model, the result shows that highway capital is significantly productive with its productivity coefficient of 0.16, which is much smaller than the original estimates of Aschauer. Also, Tatom (1991) corrects for these problems by adding a time trend and the energy price, and estimating the production function in first difference form in order to correct for stationarity problems. His estimates show that the public capital effect on private sector output is not statistically different from zero. However, Munnell (1992), Evans and Karras (1994), and Hulten and Schwab (1991b) argue that the use of first differences is contested on both conceptual and empirical grounds in that it leads to implausible coefficients for private input and for public capital.

1.2.2 Public R&D

Government supports R&D activities by its direct investment in R&D, funding to private institutions for investing in public R&D projects, and subsidizing investments by private firms in R&D. This funding is calculated as public R&D investment. A numbers of studies conclude that public R&D expenditure or government-funded industrial research and development activity has positive effects on private outputs in many ways: public funded R&D can stimulate private sector expenditure on R&D, it can generate knowledge spillovers and other technology that may raise the productivity of inputs into private R&D, and generate greater variety and quality of private inputs. We will add to this the idea that public R&D increases the variety of private capital used in producing consumption. Greater capital variety induced by public R&D knowledge spillovers are crucial in increasing productivity and growth both directly and indirectly.
Robson (1993) and Diamond (1998) conclude that federal R&D expenditures and private investment in R&D have a positive relationship. Robson (1993) uses U.S. data for the period of 1956 – 1988 to regress the change in private basic research investment in industry level on the change in federal basic research expenditures, the level of federal applied R&D, the change in private applied R & D investment, and the government and non-government ratio of industry sales. He finds that both the change in federal basic and the level of federal applied R&D have positive and significant coefficients in this regression equation. Diamond (1998) similarly uses NSF data for the earlier portion of the same period of 1953 – 1969 to examine the impact of changes in federal basic research expenditures on changes in basic research spending by industry. He finds a positive and significant coefficient on the federal spending variable. However, these studies do not conclude how and through which components an increasing in public capital can stimulate private R&D expenditure.

Archibald and Pereira (2003) analyze the effects of public and private research and development (R&D) on private-sector output, employment, and investment using a VAR system with partial analysis only for the production sector. They study aggregate data for the US economy during the period of 1956 to 1988. They find that public R&D does not affect employment in the long term, but it does induce greater private investment and private R&D. That is, public R&D crowds in private investment with a long-term elasticity of 0.328. As a consequence, the long-term effects of public R&D on output are positive and substantial.

Gómez and Sequeira (2014) present a model of R&D, human capital, and physical capital in order to examine externalities of R&D spillovers and propose a social planner’s optimal solution as the equilibrium. The model is calibrated to the U.S. economy and intertemporally budget-neutral policies are compared. They show that subsidies to R&D are most welfare improving when the main target is to keep the intertemporal budget balanced.

Voyvoda and Yeldan (2015) investigate alternative public policy intervention schemes within the context of an applied endogenous growth model with human capital and varieties of private capital. Final output is
produced using skilled labor, unskilled labor, and differentiated capital varieties as inputs. The varieties of capital depend on new research conducted which is generated by skilled labor (human capital). The varieties of capital are produced by monopolists as intermediate goods for final production and the capital used is assumed to be non depreciable. Government sector plays a role in subsidizations for (1) some of costs of this intermediate production and (2) for education to build-up of human capital. Using data from Canada economy and a calibration strategy, the results reveal long-lasting effects and non-conventional trade-offs both across instruments and also across time periods. The subsidy to the cost of R&D creates a more direct effect in terms of the allocation of resources to the marketed activities increasing the varieties of capital and the average growth rate.

1.2.3 Congestion

Several studies introduce congestion effects into the endogenous growth models mostly in order to examine the effects of congested public capital on economic growth. Barro and Sala-i-Martin (1995) argue that virtually all public services are characterized by some degree of congestion. The incorporation of congestion is an important consideration in assessing the relationship between public and private capital formation. Glomm and Ravikumar (1997) examine the effects of congested public capital on economic growth. Public capital is congested by aggregate private stocks and aggregate labor input. In their model, private and public capital fully depreciates each period. In particular, it does not feature transitional dynamics. The results show that long-run growth depends on the degree of nonrivalry of public capital and the ratio of public capital and total input of the economy.

Fisher and Turnovsky (1998) analyze the impact of public investment on the dynamics of private capital formation in an intertemporal optimizing market clearing framework. The key feature of this study is that the public good is treated as a durable capital good, subject to congestion. Labor supply is inelastic and households only choose level of consumption. Public capital is congested by relative private capital. Both lump-sum and distortionary tax financing are considered. They show that in the presence of congestion the
effect of government investment on private capital formation involves a tradeoff between the degree of substitution between private and public capital in production and the degree of congestion.

Piras (2001) investigates theoretically how changes in policy variables and in the degree of congestion of public goods influence the growth rate of the economy and the welfare of the representative agent. He employs an endogenous growth model in which government expenditure is divided into public consumption and investment in public capital, and where both components suffer from some degree of congestion. The public good is congested by aggregate private capital. The results reveal that the socially optimal growth rate is negatively related to the degree of congestion, an increase in congestion leads to a decrease in government spending on infrastructure, and that the optimal share of government spending on output can increase or decrease with congestion.

Gómez (2008) analyzes the effect of public investment in an endogenous growth model with private and public physical capital, and human capital which allows for relative and absolute congestion. The method used in analyzing theoretically growth effects by introduces a lump-sum tax and tax on income in the equilibrium dynamics of the market economy. He allows for public capital to be subject to both relative and absolute congestion. He shows that an increase in absolute congestion reduces the long-run growth rate of output. In contrast, relative congestion does not affect long-run growth. In the absence of congestion, it is optimal to use lump-sum taxation, and with congestion it is optimal to also tax income as a Pigovian externality tax.

---

1 The services, $K_s$, derived by the agent from the stock of public capital, $K_g$, is defined as $K_s = K_g (k/K)^{\theta_R} K^{-\theta_A}$ where $k$ is the agent’s capital stock, $K$ is the aggregate private capital stock. If $\theta_R > 0$ and $\theta_A = 0$, it is referred to pure relative congestion, in which congestion increases relative to individual usage. If $\theta_R = 0$ and $\theta_A > 0$, it corresponds to pure absolute congestion, in which congestion is directly proportional to the aggregate level of private capital.
1.2.4 Bayesian Analysis of DSGE Models with Fiscal Policy

The Bayesian technique is now a standard tool for the estimation of dynamic stochastic general equilibrium (DSGE) models and in practical applications, the Bayesian approach delivers very strong performance. DSGE models capture microeconomic foundations and emphasize agents’ intertemporal choice. The dependence of current choices on future uncertain outcomes makes the models dynamic and assigns a central role to agents’ expectations in the determination of current macroeconomic outcomes. In addition, they capture the interaction between policy actions and agents’ behavior.

An and Schorfheide (2007) review and evaluate Bayesian estimation of DSGE models. The model economy consists of a final goods producing firm, a continuum of intermediate goods producing firms, a representative household, and a monetary as well as a fiscal authority. The evaluation of models based on Bayesian model checking, posterior odds comparisons, and comparisons to vector autoregressions, as well as the non-linear estimation based on a second-order accurate model solution, is an advantage of this technique. These methods are applied to data generated from correctly specified and misspecified linearized DSGE models and a DSGE model that was solved with a second-order perturbation method.

Leeper et al. (2009) construct a variety of specifications of fiscal policy rules to evaluate which fiscal rule best matches the U.S. data by using the Bayesian method to estimate the models. They employ a DSGE model that includes policy rules for government spending, lump-sum transfers, and distortionary taxation on labor and capital income and on consumption expenditures. They find that the best fitting model is a rich set in which all fiscal instruments are allowed to adjust to output, public debt, and allowed to respond to changes in other fiscal instruments.

Forni et al. (2009) estimate a medium scale DSGE model that takes into account distortionary taxation on labor and capital income and on consumption, while expenditures are broken down into purchases of goods and services, compensation of public employees and transfers to households. The Bayesian technique is
used to estimate the parameters of the model. They find that government purchases of goods and services and compensations for public employees have small and short-lived expansionary effects on private consumption, while innovations in transfers to households show a slightly more sizeable and lasting effect. The effects are more significant on the revenue side: decreases in labor income and consumption tax rates have sizeable effects on consumption and output, while a reduction in capital income tax favors investment and output in the medium run.

Leeper et al. (2010) employ Bayesian methods to estimate the effects of government investment in a neoclassical growth model which allows for implementation delays and distorting fiscal adjustments. The model incorporates several real frictions – habit formation in consumption, investment adjustment costs, and variable capital utilization. U.S. quarterly data from 1960Q1 to 2008Q1 are used for estimation. They find that implementation delays can produce small or even negative labor and output responses to increases in government investment in the short run. Anticipated fiscal adjustments matter both quantitatively and qualitatively for long-run growth effects. When public capital is insufficiently productive, distorting financing can make government investment contractionary at longer horizons.

1.3 A Dynamic Stochastic General Equilibrium (DSGE) Model

The construction of a DSGE model in this paper starts with the specification of the characteristics of economy including households’ preferences, technology, government, constraints, and a set of structural shocks to the economy. The model economy is an endogenous growth model focusing on the dynamic effects of productive and congested public infrastructure and public research and development (R&D) in a dynamic stochastic general equilibrium framework.

1.3.1 Households

We assume that the economy consists of a set of identical households with measure normalized to 1. The representative household owns all firms in the economy. There is no population growth and the population
is normalized to 1. Each household supplies some labor in two sectors each period, the consumption goods sector and the R&D sector. Households own the inputs and assets of the economy, including ownership rights in firms, choose the fractions of their income to consume and save, determine how much to work, pay income and consumption taxes, and receive transfer payments from government.

A representative household has preferences over sequences of consumption \( c_t \) and labor \( l_t \) and chooses these sequences to maximize expected discounted lifetime utility,

\[
\max_{\{c_t,l_t,k_t,b_t\}_{t=0}^{\infty}} U = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{u_l}{1+\frac{1}{\beta}} \right\},
\]

(1)

where the labor supply shock, \( u_l^t = \rho_t u_{l,t-1} + \sigma_t \epsilon_t^l \), follows an AR(1) process, \( \rho_t \in [0,1] \), \( \sigma_t \) is the standard deviation, and \( \epsilon_t^l \sim iid \mathcal{N}(0,1) \). Future consumption and labor are discounted using the subjective discount factor, \( \beta \in (0,1) \), and \( v \) is the aggregate labor supply elasticity. Since returns to labor in both sectors are the same, a representative household derives utility at time \( t \) from the amount of final private consumption good it consumes and disutility from total amount of hours worked.

The representative household maximizes expected discounted lifetime utility (1) subject to flow budget constraint:

\[
(1 + \tau_c^c) c_t + i_t + b_t = (1 - \tau_l^l) w_t l_t + (1 - \tau_k^k) r_t^k k_{t-1} + R_{t-1}^b b_{t-1} + tr_t,
\]

(2)

where \( \tau_c^c, \tau_l^l, \) and \( \tau_k^k \) are the tax rates on consumption, labor income, and capital income respectively. \( r_t^k \) is the rate of return on capital. \( w_t \) is the wage rate which is the same for two sectors. \( R_{t-1}^b \) is the gross return on government bond holding, \( b_{t-1} \). A household’s total wealth consists of labor income net of tax, capital income net of tax, lump sum transfer, \( tr_t \), and interest bearing government bonds. This wealth can be used for consumption, investment, \( i_t \), and government bonds holding.
The household owns the stock of capital, whose value at the beginning of time \( t \) denoted by \( k_t \) and evolves according to

\[
k_t = (1 - \delta)k_{t-1} + i_t,
\]

(3)

where \( i_t \) is the investment in capital, and \( \delta \) is the depreciation rate.

The transversality conditions are given by the following two equations,

\[
\lim_{t \to \infty} E_t [\beta^t u(c_t)k_t] = 0,
\]

(4)

\[
\lim_{t \to \infty} E_t [\beta^t u(c_t)b_t] = 0.
\]

(5)

The representative household chooses levels of consumption, total hours worked for both sectors, government bond holding and investment in a dynamically efficient manner by equating the marginal utility of consumption to its opportunity cost, the marginal utility of leisure to the value of forgone earnings, and the opportunity cost of investment to its future returns, subject to the transversality conditions. We assume that the household acts competitively, treating the tax rates, wage rate, interest rates, and transfer payments as exogenous to the choices of consumption, bond holding, and investment.

1.3.2 Technology

This study aims at analyzing how government investment in infrastructure and research and development (R&D) which are determined endogenously following fiscal rule affects productivity. The technology developed in this paper therefore is an endogenous growth model with congestion and endogenous technological changes due to public R&D. We apply a Neoclassical growth model augmented to include public infrastructure and public R&D as the inputs of private production. These public factors should induce endogenous growth when public infrastructure is productive and also when public R&D contributes to the production and leads to crucial effects on macroeconomic aggregates.
The economy admits a representative firm: all firms in the economy have access to the same production function for the final good and the same input markets. The firm has access to a technology, hires inputs, capital and labor, and takes public R&D and public infrastructure as given, and uses them to produce goods that they sell to households or other firms.

1.3.2.1 Public R&D

Another important productive input supplied by the government is investment in public R&D. The characteristic of R&D investment, which distinguishes it from other types of investment is that its output may have the property of pure public goods (Arrow (1962), and Romer (1990)). Public R&D (and privately funded R&D) produces ideas and information about new materials or compounds, about new ways of arranging or using them, and about new ways of designing new goods or services for the satisfaction of potential wants of consumers and producers. Often the idea or compound is embodied in a new product or range of products.

This paper investigates how much public R&D contributes to productivity when public R&D expenditure can expand a variety of private input or machines used in production. We apply the study by Acemoglu (2009) examining the lab-equipment model of growth with input varieties derived from private R&D spending. In a steady of using privately funded R&D, in our study, firms take research and knowledge from public R&D spending, which reflects the flow of public R&D expenditure in each period as given to produce a variety of capital for their production. The innovation possibilities frontier takes the function

$$\Delta N_{t} = f(G_{t}^{RD}, I_{t}^{RD}),$$

(6)

where $f_{G} > 0, f_{RD} > 0$, $N_{t}$ denotes the different number of varieties of machines $k_{t}(v)$ available to be used in the production process, $G_{t}^{RD}$ is the expenditure on R&D. The economy starts with some initial technology stock $N_{0} > 0$. Equation (6) implies that the number of varieties of inputs depends on the amount of public R&D available in that period and hours worked for R&D.
Therefore, public R&D produces a variety of private machines which are invested in each period by firms. The representative firm takes this knowledge advancement derived from public R&D expenditure as given (with no costs) and uses labor to increase various kinds of machines. Finally, we obtain, following the Dixit-Stiglitz model, the bundle or index of variety of private capital or effective capital \( \bar{k}_t \) for private production.

\[
\bar{k}_t \equiv \left( \int_0^{N_t} (k_t(v))^{\frac{\varepsilon_a}{\varepsilon_a - 1}} d\nu \right)^{\frac{\varepsilon_a - 1}{\varepsilon_a}} = F(k_t, N_t) = F(k_t, G_{RD}^{t}, l_{RD}^{t}),
\]

where \( \varepsilon_a \) is the elasticity of substitution between machines, \( k_t(v) \) is the total amount of variety \( v \) used at time \( t \). The greater varieties of machines \( N_t \) induce more effective investment, and therefore more effective private capital, \( \bar{k}_t \).

Following equation (7), we can also define the effective private capital in term of a functional form as

\[
\bar{k}_{t-1} = k_{t-1} \eta G_{RD}^{t} l_{RD}^{t}
\]

where \( \eta > 0 \) is a scale parameter.

### 1.3.2.2 Congestion

Congestible public goods are those for which crowding or congestion reduces the benefits to existing consumers when more consumers are accommodated. That is, there is depletion in consumption. For public infrastructure that serves as an input to private production, we develop Glomm and Ravikumar (1997)\(^2\) congestion model for elastic-labor supply. In our study, public infrastructure, \( K_{RD}^{t} \), is congested by effective inputs consisting of aggregate effective capital, \( \bar{k}_{t-1} \), and aggregate employment in the output sector, \( L_{Y}^{t} \). This specification differs from that of the literature, since we have two different labor inputs, \( L_{RD}^{t} \) and \( L_{Y}^{t} \) causing different kinds of congestion. \( L_{Y}^{t} \) causes congestion when commuting to work and

\(^2\) Public capital is congested by aggregate private stocks and aggregate labor input where private and public capital fully depreciates each period.
not only causes commuting congestion but also causes a second kind of congestion when used for increasing private capital. Specifically, public infrastructure is congested as

\[
K^\text{cong}_{g,t-1} = \frac{K^S_{g,t-1}}{(K^S_{g,t-1})^\psi (L^Y_t)^\phi} = \frac{K^S_{g,t-1}}{(K^S_{g,t-1})^\psi (L^Y_t)^\phi} \tag{9}
\]

where \( K^\text{cong}_{g,t-1} \) is the congested public infrastructure services available to an individual firm in period \( t \).

Equation (9) implies that public R&D also causes congestion by increasing the amount of capital in the economy. Moreover, labor supplies in both sectors have different degrees of nonrivalry, which could be thought of as different labor supplies causing different degrees of congestion.

The parameters \( \psi \) and \( \phi \) are indicators of the degree of rivalry, \( \psi, \phi \geq 0 \). The infrastructure is congested by the total private inputs and the degree of rivalry. A greater rivalry means there is less public infrastructure available to a single user. When \( \psi = \phi = 0 \), the infrastructure is completely nonrival and is a pure public good. In this case, there are no congestion externalities and public investments in infrastructure increase linearly with population size. Thus, the services flowing from infrastructure to each firm are increasing with population size since there are no congestion externalities.

However, congestion externalities have a negative effect on the service flow from infrastructure if are positive or either one of them is positive. Also, an increase in private inputs for given \( K^S_{g,t-1} \) lowers the public services available to other firms and therefore reduces output. This formulation is assumed that \( K^S_{g,t-1} \) has to rise in relation to \( (K^S_{g,t-1})^\psi (L^Y_t)^\phi \) in order to expand the public services available to each user.

The representative firm uses hours worked for output production, \( L^Y_t \), effective private capital \( k^\text{eff}_{t-1} \), and congested public infrastructure \( K^\text{cong}_{g,t-1} \) available at the beginning of period \( t \) as its inputs to produce output, \( y_t \) given exogenous technology, \( A_t \).
After simplifying, the production function takes the following form,

\[ y_t = (A_t)^{1-\theta-2(\alpha_k+\psi\theta)} (k_{t-1}\eta G_t^{RD} l_t^{RD})^{-\alpha_k} (l_t^{-\phi})^{\phi (k_{t-1}^{\eta \phi})^\phi}. \]  

(10)

Thus, in order for his production function to exhibit constant returns to scale across private inputs, we impose \( 2\alpha_k + \alpha_l = 1 \). However, it may also exhibit increasing returns to scale across all inputs depending on degree of congestion. The degree of technology is defined so that the common trend can be eliminated.

Because the population size is normalized to 1 and there is no population growth in this economy,

\[ y = Y, k = K, l^Y = L^Y, l^{RD} = L^{RD} \] in every period. The aggregate production function is

\[ Y_t = (A_t)^{1-\theta-2(\alpha_k+\psi\theta)} (K_{t-1}\eta G_t^{RD} L_t^{RD})^{-\alpha_k} (L_t^{-\phi})^{\phi (K_{t,t-1}^{\eta \phi})^\phi}, \]  

(11)

where \( 0 < \alpha_k < 1, 0 < \alpha_l < 1 \), and \( \psi, \phi, \theta \geq 0 \).

This production function exhibits long-run growth if \( \alpha_k - \psi \theta + \theta = \alpha_k + (1-\psi)\theta = 1 \). We also need to impose assumptions of positive and diminishing marginal products on the aggregate production function to ensure the existence of an interior equilibrium that is globally asymptotically stable. Marginal product of capital is positive and diminishing if

\[ MP_K = (\alpha_k - \psi \theta) \frac{Y_t}{K_{t-1}} > 0, \]  

(12)

\[ \frac{\partial MP_K}{\partial K} = (\alpha_k - \psi \theta)(\alpha_k - \psi \theta - 1) \frac{Y_t}{(K_{t-1})^2} < 0. \]  

(13)

Equation (12) implies \( \alpha_k - \psi \theta > 0 \), and (13) implies \( \alpha_k - \psi \theta - 1 < 0 \implies \alpha_k - \psi \theta < 1 \). Therefore, it has to be \( 0 < \alpha_k - \psi \theta < 1 \).

Similarly, marginal products of labor for the R&D sector and for the output sector are positive and diminishing if

\[ MP_{l^{RD}} = (\alpha_k - \psi \theta) \frac{Y_t}{L_t} > 0, \]  

(14)
\[
\frac{\partial MP_{k}^{RD}}{\partial L^{RD}} = (\alpha_k - \psi \theta)(\alpha_k - \psi \theta - 1) \frac{Y_t}{(L^{RD})^2} < 0.
\]

(15)

\[
MP_{L'Y} = (\alpha_t - \phi \theta) \frac{Y_t}{L_t} > 0,
\]

(16)

\[
\frac{\partial MP_{L'Y}}{\partial L'Y} = (\alpha_t - \phi \theta)(\alpha_t - \phi \theta - 1) \frac{Y_t}{(L_t')^2} < 0.
\]

(17)

Equations (16) and (17) imply that \( \alpha_t - \phi \theta > 0 \) and \( \alpha_t - \phi \theta < 1 \), thus we have that \( 0 < \alpha_t - \phi \theta < 1 \).

This production function follows the Neoclassical model so that in the equilibrium, firms make no profits.

In each period, for a given levels of public R&D, stock of public infrastructure, aggregate private capital and labor, given factor prices \( r_1 \) and \( w_1 \), and a given technology level \( A_t \), the representative firm rents private capital and labor for both sectors, output and R&D sectors, from the household to maximize its profits.

\[
\pi_t = (A_t)^{1-\theta - 2(\alpha_k + \psi \theta)} (k_{t-1} \eta G_t^{RD} L_t^{RD})^{\alpha_t} (\frac{K_{t-1}^{RD}}{(k_{t-1})^{\psi (L_t')^{\phi}}} )^\theta - w_t (l^{RD} + l_t') - r_t^k k_{t-1}.
\]

(18)

The first order conditions of the representative firm problem are given by

\[
r_t^k = \alpha_k \frac{y_t}{k_{t-1}}
\]

(19)

\[
w_t = \alpha_k \frac{y_t}{L_t^{RD}} = \alpha_t \frac{y_t}{L_t'}.
\]

(20)

A representative household chooses the level of total hours worked (labor supply) for given the same wage for both sectors, while a representative firm will choose the hours worked (labor demand) for each sector, \( L' \) and \( L^{RD} \), so that in equilibrium labor demand is equal to labor supply and this will determine wages. The workers in both sectors receive the same wage rate, but different hours worked depending on value of \( \alpha_k, \alpha_t \). Because these parameters are constant over time, the ratio of workers in the two sectors is also constant. Households will spend more hours working in output sector if \( \alpha_t > \alpha_k \). The representative firm makes no profit regardless of contributions of R&D spillovers and productive public infrastructure, since all these contributions are transferred to the households.
1.3.3 Fiscal Policy

In establishing the basis for including the endogenous fiscal rules in the macroeconomic model, first the government budget constraint must be satisfied: government purchases of goods and services, investment in infrastructure and R&D, and transfer payments must be less than or equal to its taxes received net of government debt. The flow of the government budget constraint is

$$G_t^c + G_t^s + G_t^{RD} + TR_t + R_t^b B_{t-1} = \tau_t^k r_t^k K_{t-1} + \tau_t^l w_t L_t + \tau_t^c C_t + B_t,$$  (21)

where $G_t^c, G_t^s, G_t^{RD}, TR_t, R_t^b B_{t-1}, \tau_t^k r_t^k K_{t-1}, \tau_t^l w_t L_t, \tau_t^c C_t$ and $B_t$ are the aggregate levels of government consumption, investment in infrastructure, investment in R&D, transfer payments, debt payments, capital tax revenues, labor income tax revenues, consumption tax revenues, and debt.

Public capital evolves according to

$$K_t^S = (1 - \delta_G) K_{t-1}^S + G_t^S$$  (22)

For simplicity, we rewrite equation (21) in terms of total revenues, total spending, transfers, and debt, as the following

$$B_t = R^-_{t-1} B_{t-1} + G_t + TR_t - T_t,$$  (23)

where $G_t = G_t^c + G_t^s + G_t^{RD}$, and $T_t = \tau_t^k r_t^k K_{t-1} + \tau_t^l w_t L_t + \tau_t^c C_t$.

Solving equation (23) forward, we can express debt as a function of its determinants,

$$B_t = \sum_{i=0}^{\infty} \Pi_{j=0}^i (R_{t+i}^b)^{-1} (T_{t+i+1} - G_{t+i+1} - TR_{t+i+1}) + \lim_{i \to \infty} \Pi_{j=0}^i (R_{t+i}^b)^{-1} B_{t+i+1}.$$  (24)

For the government to be solvent, we impose the transversality condition,

$$\lim_{i \to \infty} \Pi_{j=0}^i (R_{t+i}^b)^{-1} B_{t+i+1}.$$  (25)
In the long run agents believe that tax revenue is sufficient to cover expenditures. This is the usual no-Ponzi game condition.

The remaining terms in equation (24) imply that for the current level of debt to be consistent with the current status of the economy, any deviation from tax or spending plans at any point in the future due to any shocks affecting revenue, spending, and interest rates has to be backed by changes in policy instruments via the fiscal rules. In the long run agents believe tax revenues are sufficient to cover spending.

Fiscal rule

This study defines the endogenous fiscal rules to be consistent with the optimizing behavior already inherent in the model. The rules must be used to generate solvency for the fiscal sector, guaranteeing that the intertemporal government’s budget constraint is satisfied, and generating model closure. That is the possibility of an unstable or explosive path for the government debt ratio is ruled out, and therefore households in the model are willing to hold public debt. Also, the rules embody some behavioral elements regarding the intertemporal behavior of the government. That is the time path of adjustments in fiscal and other variables in the model are influenced by the formulation of the fiscal rule.

Fiscal instruments include government consumption, government investment in infrastructure, government investment in R&D, capital tax, labor tax, consumption tax, and public transfers. Fiscal policy here follows rules that embed two features. First, we allow for automatic stabilizers components. All fiscal instruments except the consumption tax are allowed to contemporaneously respond to deviations of output from steady state. This design offsets fluctuations in economic activity without direct intervention by policymakers. Second, all instruments are permitted to respond to the state of government debt.

The fiscal rules in terms of the level variables are presented in Appendix A1. In terms of deviations from steady state levels, the rules follow.

$$\dot{g}^s_t = -\eta_{gs}\bar{y}_t - \gamma_{gs}\bar{b}_{t-1} + u^{gs}_t,$$

(26)
\[ g^{RD}_t = -\eta_{RD}\dot{y}_t - \gamma_{RD}\dot{b}_t + u^{RD}_t, \]  
\[ g^c_t = -\eta_{gc}\dot{y}_t - \gamma_{gc}\dot{b}_t + u^g_c, \]  
\[ \dot{\tau}^k_t = \eta_{tk}\dot{y}_t + \gamma_{tk}\dot{b}_t + u^r_k, \]  
\[ \dot{\tau}^l_t = \eta_{tl}\dot{y}_t + \gamma_{tl}\dot{b}_t + u^r_l, \]  
\[ \dot{\tau}^c_t = \gamma_{tc}\dot{b}_t + u^r_c, \]  
\[ \dot{\bar{r}}_t = -\eta_{tr}\dot{y}_t - \gamma_{tr}\dot{b}_t + u^r_{tr}. \]  

(27)  
(28)  
(29)  
(30)  
(31)  
(32)

The fiscal instruments are allowed to adjust to the cyclical position of the economy (\( \eta_i > 0 \) for \( i = \{gs, gc, RD, tk, tl, tr\} \)) and to changes in the level of government debt (\( \gamma_i > 0 \) for \( i = \{gs, gc, RD, tk, tl, tr\} \)). Consumption taxes do not respond to the deviation of output from steady state because most of U.S. federal consumption tax revenues are collected from specific goods and used mainly for special funds. However, it does respond to the state of government debt, like all other instruments in order to prevent explosive deviations of debt from its normal steady-state value.

As described previously, the fiscal rules are set in order to stabilize the economy. The parameters of the fiscal rules or fiscal reaction functions are set at levels that produce a stable outcome. The government budget constraint (23) is an unstable difference equation in debt if the other variables are stationary but the coefficient for debt is greater than one. We impose the fiscal rules so that they can be thought of as specifying each fiscal instrument as a function of past debt. When solving for equation (21) and the fiscal rules (26)-(32), we will have a stable result if the coefficients attached to past debt are less than one.

It is difficult to derive the conditions on the parameters for the endogenous fiscal policy rule to be stationary in equations (26)-(32). This is because each fiscal instrument is also a reaction function of output, which is determined endogenously. So we need to use numerical methods to find such parameter values that produce a stable outcome.
1.3.4 Shocks

Following Negro and Schorfheide (2010), log technology evolves according to

\[
\ln A_t = \ln A_0 + (\ln \gamma) t + \ln \tilde{A}_t, \quad \text{and} \quad \ln \tilde{A}_t = \rho_A \ln \tilde{A}_{t-1} + \sigma_A \varepsilon^A_t
\]

where \( \rho_A \in [0,1] \). If \( 0 \leq \rho_A < 1 \), the technology process is stationary. If \( \rho_A = 1 \), then \( A_t \) is a random walk process with drift.

The technology shock, exogenous labor supply shift, government consumption, investment in infrastructure, investment in R&D, capital income tax, labor income tax, consumption tax, and transfer shifts are assumed to follow stationary AR(1) processes,

\[
\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \sigma_A \varepsilon^\tilde{A}_t, \quad (34)
\]

\[
u^l_t = \rho_l u^l_{t-1} + \sigma_l \varepsilon^l_t, \quad (35)
\]

\[
u^{gc}_t = \rho_{gc} u^{gc}_{t-1} + \sigma_{gc} \varepsilon^{gc}_t, \quad (36)
\]

\[
u^{gs}_t = \rho_{gs} u^{gs}_{t-1} + \sigma_{gs} \varepsilon^{gs}_t, \quad (37)
\]

\[
u^{RD}_t = \rho_{RD} u^{RD}_{t-1} + \sigma_{RD} \varepsilon^{RD}_t, \quad (38)
\]

\[
u^{tk}_t = \rho_{tk} u^{tk}_{t-1} + \sigma_{tk} \varepsilon^{tk}_t, \quad (39)
\]

\[
u^{tl}_t = \rho_{tl} u^{tl}_{t-1} + \sigma_{tl} \varepsilon^{tl}_t, \quad (40)
\]

\[
u^{tc}_t = \rho_{tc} u^{tc}_{t-1} + \sigma_{tc} \varepsilon^{tc}_t, \quad (41)
\]

\[
u^{tr}_t = \rho_{tr} u^{tr}_{t-1} + \sigma_{tr} \varepsilon^{tr}_t, \quad (42)
\]

where \( \varepsilon^x_t \sim iidN(0,1) \) and \( x = \{A, l, gc, gs, RD, tk, tl, tc, tr\} \).
1.4 Model Solution

1.4.1 Equilibrium Conditions

The procedure for solving a model using linearized Euler equations and matrix decomposition starts with solving the problems of the optimizing agents. This yields a set of first order conditions and the agents’ decision rules. The equations obtained from this process are then subject to aggregation procedures, to obtain a set of equations describing the behavior of economic agents as a whole. The equilibrium conditions are shown in Appendix A1. In addition, markets must clear and the budget constraint of the government will hold with equality.

1.4.2 Market Clearing Conditions

The final good market is in equilibrium if aggregate production equals aggregate demand for consumption, aggregate investment, and government expenditure,

\[ Y_t = C_t + I_t + G^C_t + G^S_t + G^{RD}_t \]  \hspace{1cm} (43)

The labor market is in equilibrium when the quantity of labor supplied equals the quantity of labor demanded and the wage will adjust for the equilibrium to be satisfied. The capital rental market is in equilibrium if demand for capital equals supply for capital and the rental rate will adjust the demand and supply to get the equilibrium. In addition, any equilibrium must satisfy the transversality conditions for debt and capital accumulation.

Combining the agents’ behavioral equations, policy rules and constraints with the market clearing conditions and the shock processes, we obtain a set of equations expressing the endogenous variables of the model in each period as a function of its past, present, and expected future path, of the set of parameters qualifying the relations between variables, and of the structural exogenous innovations hitting the economy in each period.
1.4.3 Correcting for Stationarity and Misspecification

1.4.3.1 Correcting for Stationarity

Before solving the model by constructing the local approximation of its dynamics near a steady state, all variables have to be stationary and cointegration relationship problems must be solved. In this real business cycle model, there is the technology process \( \ln A_1 \) which induces a common trend in output, consumption, private and public investments, private and public capitals, public R&D, tax revenues, and transfers, with the exception of \( L_t \), which is stationary as there is no population growth. All these non-stationary variables have to be scaled by the level of technology.

From the definition of technology process in equation (33), the process \( a_t \) follows

\[
a_t = \ln \frac{A_t}{A_{t-1}} = \ln y + (\rho_A - 1) \ln A_{t-1} + \sigma_a \varepsilon_t^A
\]

(44)

Since \( \rho_A \in [0,1] \), this process is always stationary. This is because if \( \rho_A = 1 \), the \( \ln A_{t-1} \) drops out. Hence, in a stochastic environment, the detrended variables follow a stationary law of motion, even if the underlying technology shock is nonstationary.

Moreover, when \( \rho_A = 1 \), the model generates a number of cointegration relationships. We correct for this problem by taking pairwise differences of log of the model variables. By doing this we obtain the differences of log of detrended variables, plus a constant \( \ln y \) which will be estimated. The equilibrium conditions in terms of the detrended variables are shown in Appendix A2.

1.4.3.2 Correcting for Model Misspecification

Model misspecification can occur especially for any DSGE model that generates a rank-deficient covariance matrix (singularity) for the observables. This model induces estimation results that may be at odds with the data. This singularity of the forecast error covariance matrix is an obstacle to likelihood
estimation. Hence in this paper, we will modify the model specification to remove the singularity by adding so-called measurement errors, or additional shocks as in Leeper and Sims (1995), and Smets and Wouters (2003).

Our model contains nine observable variables. Thus, we add nine exogenous shocks to the model so that the number of structural shocks equals the number of observables to which the model is fitted. This is because fluctuations are generated by nine exogenous disturbances, the likelihood function for more than nine variables is degenerate.

Our measurement equation takes the following form

\[ s_t = \Psi_0(\theta) + \Psi_1(\theta) t + \Psi_2(\theta) y_t, \]  

where \( s_t \) is a vector of observables, \( s_t = \{ GDP_t, L_t, G_t^C, G_t^Z, G_t^{RD}, R^K, T^L, T^C, T_R \} \), and \( y_t \) is a vector of model variables corresponding to \( s_t \).

Specifically, the percentage changes from the steady states of detrended variables are

\[ \ln s_t = \ln s_0 + (\ln \gamma) t + \tilde{s}_t + \tilde{A}_t, \tilde{s}_t = \tilde{s}_t. \]  

For hours worked, we define it differently since it does not fluctuate by a common trend. That is

\[ \ln L_t = \ln L + \tilde{L}_t, \text{ where } L \text{ is the steady state of hours worked.} \]

Following this procedure, we have detrended variables \( \tilde{s}_t \) and the trended generated by technology added \((\ln \gamma) t + \tilde{A}_t\) in the measurement equation. Thus we can learn about the technology growth rate \( \gamma \) and its persistence \( \rho_A \) from the available information about the level of output.

### 1.4.4 Solving the System of Equations

A DSGE model is a collection of first order and equilibrium conditions that take the general form,

\[ E_t \{ f(Y_{t+1}, y_t, y_{t-1}, \epsilon_t) \} = 0, \]  

\[ (47) \]
where \( y \) is a vector of endogenous variables, and \( \varepsilon \) is a vector that stacks the innovations for the structural shocks.

The solution of the rational expectations system which is called the policy function takes the form

\[
y_t = g(y_{t-1}, \varepsilon_t; \theta) \tag{48}
\]

This solution is a set of equations relating variables in the current period to the past state of the system and current shocks, that satisfy the original system. \( y_t \) can be viewed as a (partially latent) state vector in a non-linear state space model and (48) is the state transition equation.

We will employ linear approximation methods to solve this rational expectations system. These methods are very popular in the context of likelihood-based DSGE model estimation because they lead to a state-space representation of the DSGE model that can be analyzed with the Kalman filter.

**Log-linearization**

Most DSGE models, as well as the model of this study, are characterized by a set of non-linear equations, including Euler equations. This study proceeds under the assumption that the DSGE model’s equilibrium law of motion is approximated by log-linearization techniques.

First, we linearize this model (47) around a steady state defined as

\[
f(\bar{y}, \bar{y}, \bar{y}, 0) = 0, \tag{49}
\]

having the property that

\[
\bar{y} = g(\bar{y}, 0). \tag{50}
\]
The variables are assumed to be constant over time in the steady state, and the shock stays at its unconditional mean, which is zero. The model becomes deterministic. This steady state is a function of the model parameters. With the steady state conditions, we can find the steady state values of the model variables as shown in Appendix A3.

Then, we will obtain a new set of equations defining an approximate linear model whose endogenous variables correspond to the percentage deviations of the original variables from their steady-state. Using these equations, (47) and (48) become

\[
E_t\{f_{yt+1} \hat{y}_{t+1} + f_{yt} \hat{y}_t + f_{yt-1} \hat{y}_{t-1} + f_{t} \varepsilon_t\} = 0, \quad \text{(51)}
\]

\[
\hat{y}_t = g_{y} \hat{y}_{t-1} + g_{\varepsilon} \varepsilon_t, \quad \text{(52)}
\]

where \(g_{y}\) and \(g_{\varepsilon}\) are functions of structural parameters \(\theta\).

The solution takes the form

\[
y_t = g_{y}(\theta)y_{t-1} + g_{\varepsilon}(\theta)\varepsilon_t. \quad \text{(53)}
\]

Equation (53) constitutes the general form of the approximate linear model expressing the corresponding endogenous forward-looking variables as a function of the endogenous state variables and the innovations. It is characterized by an approximate linear policy function \(g_{y}\) dependent on matrices and \(g_{\varepsilon}\) which are a function of the structural and policy parameters of the model. The log-linearized equilibrium conditions of the DSGE model which are the system of linear expectational difference equations are presented in Appendix A4. This rational expectations system has to be solved before the DSGE model can be estimated.

Once we have obtained \(g_{y}\) and \(g_{\varepsilon}\), we can recover the values of the original endogenous variables by this expression of a first order linearization of the function \(g\),

\[
y_t = \bar{y} + g_{y} \hat{y} + g_{\varepsilon} \varepsilon. \quad \text{(54)}
\]
Solving the Approximate Linear Model

To find \( g_y \) and \( g_e \), a series of complex algebraic procedures are needed. Depending on the parameterization of the DSGE model, there are three possibilities for the solution, no stable rational expectations solution exists, the stable solution is unique (determinacy), or there are multiple stable solutions (indeterminacy). We will focus on the case of determinacy and will present here in only rough terms\(^3\) to show how to obtain a unique stable solution to the model.

Having obtained the system of linear expectational difference equations, they can be put into the following matrix representation,

\[
AE_t[x_{t+1}] = Bx_t + C\epsilon_t, \tag{55}
\]

where \( x \) is a vector of model variables, \( A, B \) and \( C \) are the coefficient matrices that contain the deep parameters of the original model. There are multiple solutions to this system but we will focus on solutions that guarantee a nonexplosive law of motion for the endogenous variables, with the justification that any explosive solution would violate the transversality conditions associated with the underlying dynamic optimization problems. The stability condition will replace the transversality condition. Following Blanchard and Kahn (1980), this occurs if the system’s number of eigenvalues\(^4\) larger than one, called explosive, is equal to the number of non-predetermined variables. If the Blanchard and Kahn condition is satisfied, we can proceed in finding the model solution which also needs a rank condition to be satisfied in order to obtain a unique stable solution to (55).

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3 To obtain a detailed exposition of this subject, refer to Klein (2000), and Sims (2001).

4 To find the eigenvalues of the system, one can apply a generalized Schur decomposition to matrices \( A \) and \( B \). See Klein (2000) for the derivations and proof.
1.5 Estimation

1.5.1 Bayesian Estimation of DSGE Models

This subsection describes how Bayesian inference can be used to estimate DSGE models and how to deal with estimation problems. The main references in these respects are An and Schorfheide (2007), and Negro and Schorfheide (2010). The Bayesian analysis is system-based and fits the solved DSGE model to a vector of aggregate time series. The estimation is based on the likelihood function generated by the DSGE model rather than, for instance, the discrepancy between DSGE model responses and VAR impulse responses. Prior distributions can be used to incorporate additional information into the parameter estimation.

In a Bayesian framework, the likelihood function is reweighted by a prior density in order to give more importance to certain areas of the parameter subspace. The prior can bring to bear information that is not contained in the estimation sample $Y$. The priors can also be obtained from Micro-level estimations. The posterior represents the probabilities assigned to different values of the parameters after observing the data and the corresponding priors. It basically constitutes an update of the probabilities given by the prior, based on the additional information provided by the variables in our sample. More technically, these two building blocks, priors and likelihood functions, are tied together by Bayes’ rule to estimate the posterior distribution.

To express this formally, we apply Bayes theorem to the two random events $\theta$ and $Y_T$, which produces

$$p(\theta|Y_T) = \frac{p(\theta)p(Y_T|\theta)}{p(Y_T)} ,$$  \hspace{1cm} (56)

where $p(\theta|Y_T)$ is the density of the parameters conditional on the data (the posterior), $p(Y_T|\theta)$ is the density of the data conditional on the parameters (the likelihood), $p(\theta)$ is the unconditional density of the parameters (the prior), $p(Y_T)$ is the marginal density of the data $Y_T$, and are the observations until period $T$. Because $p(Y_T)$ does not depend on the parameters, we can write the posterior distribution (56) as
\[ p(\theta|Y_T) \propto p(\theta)p(Y_T|\theta) \equiv K(\theta|Y_T), \tag{57} \]

where \( K(\theta|Y_T) \) is the posterior kernel (or un-normalized posterior density) which is proportional to the posterior by the factor \( p(Y_T) \).

To find the likelihood function that describes the density of the observed data \( L(\theta|Y_T) \), we first take logs,

\[
\ln K(\theta|Y_T) = \ln p(Y_T|\theta) + \ln p(\theta) = L(Y_T|\theta) + \ln p(\theta). \tag{58}
\]

Assuming that the priors are independently distributed, this equation may be calculated as

\[
\ln K(\theta|Y_T) = L(Y_T|\theta) + \sum_{n=1}^{N} \ln p(\theta_n), \tag{59}
\]

where \( N \) is the number of parameters being estimated. This is the equation that we will estimate the posteriors. However, it is analytically intractable since it is a complicated nonlinear function of the parameters contained in \( \theta \). Therefore, the analysis has to be performed with numerical methods.

The joint posterior distribution of all estimated parameters is obtained in two steps. First, the posterior mode and an approximate covariance matrix, based on the inverse Hessian matrix evaluated at the mode, is obtained by numerical optimization on the log posterior density. Second, the posterior distribution is subsequently explored by generating draws using the Metropolis – Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode.

We start by maximizing (59) with respect to \( \theta \) to obtain an estimate for the mode of the posterior distribution, \( \hat{\theta} \), and for the Hessian matrix evaluated at the mode, \( H(\hat{\theta}) \) (note that the maximum of \( p(\theta|Y_T) \) will be the same as the maximum of \( K(\theta|Y_T) \)). This is carried out using an optimization routine (csmiwel) developed by Christopher Sims. We then use the Random-Walk Metropolis (RWM) Algorithm to generate Markov chains with stationary distributions that correspond to the posterior distributions of interest.
To do so, the algorithm starts by specifying a “candidate” (or jumping) distribution, from which several parameter estimates are drawn and then poses an “acceptance-rejection” rule according to which some of the generated estimates are kept and some are not. To pick the “candidate” distribution, the algorithm builds on the fact that under fairly general regularity conditions, the posterior will be asymptotically normal, and uses the mode and Hessian obtained from the maximization of the posterior kernel to define the mean and variance.

The variance is built as the inverse of the Hessian multiplied by a constant, called the scale factor, which is a crucial parameter, since it determines the acceptance ratio, i.e. the percentage of estimates generated from the jumping distribution that are kept to construct the posterior. The scale factor \( c_0 \) controls the expected distance between the mode and the starting point of the Markov chain. The tuning parameter \( c \) is typically chosen to obtain a rejection rate of about 50%. This allows for an efficient exploration of the posterior distribution at least in the neighborhood of the mode.

1.5.2 Data

We collect nine U.S. time series quarterly data from 1969Q1 to 2015Q1 in which the time series of government data and GDP are taken from the National Income and Product Account, NIPA, while hours worked data are collected from bureau of labor statistics. The government data include government investment in infrastructure, government consumption, capital tax revenues, labor tax revenues, consumption tax revenues, and government transfers. All data collected are nominal. These data are converted into real values by dividing by the GDP deflator. Taking log of all the observables and then use them for estimation inputs.

Hours worked for both sectors are calculated from average weekly hours duration of non-farm business multiplied by the numbers of Civilian employment.

For consumption tax revenues, following Leeper et al. (2009), consumption tax revenues, \( T^c \), include excise taxes and customs duties. The average consumption tax rate is defined as
\[ \tau^c = \frac{T^c}{C - T^c - T^c_s} \]

where \( C \) is personal consumption expenditure on nondurable goods and on services, \( T^c_s \) is state and local sales taxes.

For capital and labor tax rates, following Jones (2002), the average labor income tax rate is calculated as

\[
\tau^l = \frac{\tau^l (W + PRI/2) + CSI}{EC + PRI/2},
\]

\[
\tau^p = \frac{IT}{W + \frac{PRI}{2} + CI},
\]

where \( IT \) is personal current tax revenues, \( W \) is wage and salary accruals, \( PRI \) is proprietors’ income, and \( CI \) is capital income. Capital income is defined as rental income, corporate profits, interest income, and \( \frac{PRI}{2} \).

\( CSI \) is contributions for government social insurance and \( EC \) is compensation of employees. The average capital income tax rate is defined as

\[
\tau^k = \frac{\tau^p CI + CT}{CI + PT},
\]

where \( CT \) is taxes on corporate income and \( PT \) is property taxes.

Capital and Labor Tax Revenues. The capital and labor tax revenues are calculated by multiplying the average tax rate and tax base.

Government consumption expenditure and government gross investment are collected from table 3.9.5.

Transfers are defined as net current transfers, net capital transfers, and subsidies minus the tax residual.

Publicly funded R&D is obtained from National Science Foundation. We include all publicly funded R&D no matter where the research is conducted. The use of a more inclusive data set is important given that R&D undertaken by federal laboratories, colleges and universities, and not-for-profit firms.
1.5.3 Calibration and Prior

Calibration is a strategy for finding numerical values by using economic theory as the basis for restricting a general framework and mapping this framework into measured data. In this model, some parameters are calibrated from the outset, not being include in the estimation process. This can be viewed as imposing a very strict prior consistent with the Bayesian approach in estimation. The important aspects of the parameters that can be a strict prior include those that determine the steady state, having reliable estimates from other sources, and those that are crucial to replicate the main key steady state ratios of the U.S. economy.

The following parameters are calibrated to values as commonly used in the literature. The discount rate, $\beta$, is set at 0.99 to be consistent with an annual steady state interest rate of 4%, and the steady-state depreciation rate of private capital, $\delta = 0.025$. The steady-state depreciation of public capital, $\delta_G$ is set at 0.02. The steady-state fiscal variables are calibrated to sample means of the data set, capital tax rate, $\tau^C = 0.016$, labor income tax rate, $\tau^l = 0.204$, capital income tax rate, $\tau^k = 0.194$, share of public consumption, $\frac{G^C}{Y} = 0.042$, share of public investment in infrastructure, $\frac{G^I}{Y} = 0.009$, share of public investment in R&D, $\frac{G^{RD}}{Y} = 0.011$, and public debt to GDP ratio, $\frac{B}{Y} = 0.361$.

For the parameters that specify the labor shares in output sector and in R&D sector, $\alpha_l$ and $\alpha_k$, we calibrate these values to satisfy the restriction of constant return to scale of the endogenous production function and also of a labor share of 75 percent. That is, we calculate the parameters by solving these two equations,

\[
0.75 = \frac{w_l}{Y} = \alpha_l + \alpha_k \quad \text{and} \quad \alpha_l + 2\alpha_k = 1.
\]

This yields $\alpha_k = 0.25$ and $\alpha_l = 0.5$.

The priors for autoregressive coefficients, $\rho_t$'s, are set to be distributed according to Beta distributions with means of 0.9 and standard deviations of 0.02. The standard deviations of the innovations are assumed to be distributed as Inverse Gamma.
The other remaining parameters shown in Table 1 are important for the behavior of the model and fiscal policy implication. We use a Gamma distribution with parameters that imply prior mean of 2 and a standard deviation of 1. This is based on the Micro-level estimates of labor supply elasticity by Negro and Schorfheide (2010).

The prior for public infrastructure productivity, $\theta$, is obtained from the estimation by Finn (1993). The prior mean is 0.16, and is distributed according to a Beta distribution with a standard deviation of 0.02. The mean of the degrees of nonrivalry with respect to effective capital and labor, $\varphi$ and $\phi$, are both set equal to 0.5, and both are distributed according to a Beta distribution, with the same standard deviations of 0.25.

The priors for the fiscal parameters which response to government debt, $\gamma_i's$, are mostly based on the study by Leeper at al.(2009). These parameters are assumed to be distributed as Gamma distributions with means of 0.4 and standard deviations of 0.2, so that the parameters fall in the range between 0 and 1.25. The prior for the responses of fiscal instruments to output, or output elasticities, $\eta_i's$, are assumed to have Normal distributions. The prior means and standard deviations of government consumption and investment in infrastructure are set at 0.07 and 0.05 respectively while these of government investment in R&D are set at 0.5 and 0.05. The mean of the transfer elasticity is 0.2 and its standard deviation is 0.1. We set the means and standard deviations of tax revenues with respect to output to follow Blanchard and Perotti (2002) with means of 0.5 for both tax rate elasticities and standard deviations of 0.05 for capital tax rate elasticity and 0.25 for labor tax rate elasticity.

1.6 Results

1.6.1 Estimation Results

Table 1 reports the estimation results of 34 model parameters. It contains posterior means and 90% credible intervals, computed from the output of the posterior simulator of the posterior distributions computed with
the MH algorithm. We generate 500,000 draws from the posterior distribution of the parameters of the stochastic growth model. The acceptance rate is 0.549.

Table 1: Prior and posterior distributions of the model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density</td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>Structural parameters</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Tech. growth, $\gamma$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.10</td>
<td>0.005</td>
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<tr>
<td>Labor supply elasticity, $\nu$</td>
<td>Gamma</td>
<td>2.00</td>
<td>1.00</td>
<td>0.118</td>
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<tr>
<td>Productivity of infrastructure, $\theta$</td>
<td>Beta</td>
<td>0.16</td>
<td>0.02</td>
<td>0.163</td>
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<tr>
<td>Degree of nonrivalry wrt. capital, $\psi$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.976</td>
</tr>
<tr>
<td>Degree of nonrivalry wrt. labor, $\phi$</td>
<td>Beta</td>
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<td>0.25</td>
<td>0.371</td>
</tr>
<tr>
<td>Policy parameters</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov. infras. resp.to output, $\eta_{gs}$</td>
<td>Normal</td>
<td>0.07</td>
<td>0.05</td>
<td>0.072</td>
</tr>
<tr>
<td>Gov. consumpt. resp.to output, $\eta_{gs}$</td>
<td>Normal</td>
<td>0.07</td>
<td>0.05</td>
<td>0.055</td>
</tr>
<tr>
<td>Gov. R&amp;D resp.to output, $\eta_{RD}$</td>
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<td>0.50</td>
<td>0.05</td>
<td>0.463</td>
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<tr>
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<td>0.50</td>
<td>0.05</td>
<td>0.479</td>
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<tr>
<td>Labor tax resp.output, $\eta_{cl}$</td>
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<td>0.25</td>
<td>0.582</td>
</tr>
<tr>
<td>Transfer resp.output, $\eta_{tr}$</td>
<td>Normal</td>
<td>0.20</td>
<td>0.10</td>
<td>0.189</td>
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<tr>
<td>Gov. infras. resp to debt, $\gamma_{gs}$</td>
<td>Gamma</td>
<td>0.40</td>
<td>0.20</td>
<td>0.041</td>
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<td>Gov. consumpt. resp to debt, $\gamma_{gs}$</td>
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<td>0.20</td>
<td>0.032</td>
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<td>0.20</td>
<td>0.009</td>
</tr>
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<td>Consumpt. tax resp.debt, $\gamma_{tc}$</td>
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<td>0.20</td>
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</tr>
<tr>
<td>Transfer resp. debt, $\gamma_{tr}$</td>
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<td>0.40</td>
<td>0.20</td>
<td>0.090</td>
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<td>Transfer resp. debt, $\gamma_{tr}$</td>
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<td>0.40</td>
<td>0.20</td>
<td>0.090</td>
</tr>
<tr>
<td>AR(1) coefficients</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leisure preference, $\rho_p$</td>
<td>Beta</td>
<td>0.90</td>
<td>0.02</td>
<td>0.977</td>
</tr>
<tr>
<td>Parameter</td>
<td>Prior distribution</td>
<td>Posterior distribution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------------</td>
<td>--------------------</td>
<td>------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>Mean</td>
<td>Std.</td>
<td>Mean</td>
</tr>
<tr>
<td>Technology, $\rho_A$</td>
<td>Beta</td>
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<td>0.02</td>
<td>0.937</td>
</tr>
<tr>
<td>Gov. infrastructure, $\rho_{gs}$</td>
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<td>0.02</td>
<td>0.945</td>
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<tr>
<td>Gov. consumpt, $\rho_{gc}$</td>
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<td>0.02</td>
<td>0.931</td>
</tr>
<tr>
<td>Gov. R&amp;D, $\rho_{RD}$</td>
<td>Beta</td>
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<td>0.02</td>
<td>0.965</td>
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<tr>
<td>Capital tax, $\rho_{tk}$</td>
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<td>0.927</td>
</tr>
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<tr>
<td>Consumpt tax, $\rho_{tc}$</td>
<td>Beta</td>
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<td>0.02</td>
<td>0.920</td>
</tr>
<tr>
<td>Transfer, $\rho_{\sigma}$</td>
<td>Beta</td>
<td>0.90</td>
<td>0.02</td>
<td>0.922</td>
</tr>
</tbody>
</table>

**Std. of shocks**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leisure preference, $\sigma_I$</td>
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<td>0.10</td>
</tr>
<tr>
<td>Gov. infrastructure, $\sigma_{gs}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Gov. consumpt, $\sigma_{gc}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Gov. R&amp;D, $\sigma_{RD}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Capital tax, $\sigma_{tk}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Labor tax, $\sigma_{tl}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Consumpt tax, $\sigma_{tc}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Transfer, $\sigma_{\sigma}$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
<tr>
<td>Technology, $\sigma_A$</td>
<td>Inv.Gamma</td>
<td>0.10</td>
</tr>
</tbody>
</table>

For the economy growth rate we use a logarithmic transformation of $\gamma$, which can be interpreted as the average quarterly growth rate of the economy and is estimated to be 0.5 percent. The distribution of $\theta$ with the prior mean of 0.16 is not updated in view of data. The estimate is consistent with that of Finn (1993). Other estimated parameters are updated with data. The estimated results are significantly different from the prior distribution. The degree of rivalry with respect to effective capital is estimated to have the posterior mean close to one, indicating a high degree of rivalry, while the degree of rivalry with respect to labor for output sector has a posterior mean of 0.37. This means the aggregate effective capital input causes higher congestion to the public infrastructure.
For policy parameters estimates, all are estimated to be different from zero. The responses of fiscal instruments to the aggregate output are stronger than to the state of government debt. Labor tax rate has a highest procyclical response to the aggregate output, while government consumption and government investment in infrastructure have less responsive, and have lesser responses than government investment in R&D.

The results also show that fiscal instruments stabilize debt even if the estimates here are smaller than the estimates in the literature [Leeper, Plante, and Traum, (2009), Leeper, Walker, and Yang (2010)]. The response of transfers to debt innovations is highest. Other fiscal instruments except labor tax rate also respond to debt but not strongly, indicating that the government can stabilize debt by adjusting the magnitudes of these instruments.

Finally, we consider the persistency and volatility parameters of structural shocks. All the persistent shocks are estimated to lie between 0.92 and 0.98. None of the shocks is excessively persistent in the senses that the prior distribution’s 95 percentile is less than one for any of the shocks, which give an indication on the absence of unit roots in these processes.

1.6.2 Impulse Responses

Figure 1 and 2 plot the impulse responses of labor, consumption, output, investment, government debt, and congestible public infrastructure to a temporary 1 percent increase in government investment in R&D and in infrastructure, respectively. Figure 3 depicts the response to an increase in general government consumption. And figure 4 – 6 depict the responses to an increase in each tax rate.

Consider the first experiment. The increase in government investment in R&D causes more R&D knowledge spillovers for firms. The representative firm can use this additional knowledge together with labor to produce greater variety of capital for output production. The increase in R&D causes the marginal product of both private capital and labor to increase so that profit-maximizing firms demand more private
capital and labor. Thus both labor and private capital increase in the short run, but gradually decrease in the long run so that investment converges to its initial steady state value while labor converges to a level slightly below its original level. Thus, the increase in government spending on R&D does not crowd out private investment; instead it crowds it in. Output follows the same path; output increases in the short run but falls in the long run. However, it eventually converges to a level greater than the initial steady state strongly suggesting there are permanent effects of an increase in R&D spending as capital variety increases.

As output increases, income increases. With higher income, households consume more and aggregate consumption increases to be above the steady state value, even after 10 years. Congestible public infrastructure is also affected by the increase in production inputs. Public infrastructure becomes more rival so that users of infrastructure have less access and face higher costs of using the infrastructure. The congestible infrastructure is below the steady-state value in the short and converges to a point that is also below the initial steady state level; public capital is more congested and less productive in both the short and long runs.

Figure 2 illustrates the impulse responses of labor, consumption, output, investment, government debt, and congestible public infrastructure to a temporary 1 percent increase in government spending on infrastructure. An increase in public investment in infrastructure produces an additional productive public input to private production, which directly leads to greater output. This has a positive wealth effect on private individuals, increasing their lifetime income. Individuals respond by increasing consumption and leisure, i.e., reducing their labor supply. An increase in infrastructure initially crowds out private investment and, therefore, causes private capital to fall in the short run. This process takes about 1 year then private investment starts to increase and stays above the steady-state level for many years. Output increases in both the short run and long run, converging to a point well above the initial steady state. As both capital and labor increase there is greater congestion in the short run and the long run. The stock of government debt first increases in the short run, but then decreases in the long run and converges to a new steady state below
the initial level. So there is greater congestion, but a smaller stock of debt in the long run as the economy responds to the initial increase in infrastructure.

The responses to a temporary 1 percent increase in government consumption are shown in Figure 3. Government spending on consumption has a negative wealth effect on private individuals. Individuals then decrease their consumption and leisure. Therefore, labor supply increases. This also stimulates output as households prefer to work more. But after 1.5 years, output declines due to a decrease in private capital. That is in the long run, an increase in exogenous government spending on consumption crowds out private investment, private capital, and thus causes output to decline. Congested public infrastructure decreases in a short period then goes to steady state level before continues to increase overtime. This is due to the decreases in labor and private capital.

Figure 4 plots the impulse responses to a temporary 1 percent increase in consumption tax rate. The shock lowers consumption. An increase in tax rate on consumption raises unit price of goods as well as total household expenditure. Individuals would consume less and take more leisure to satisfy intratemporal substitutions between consumption and leisure. Therefore, labor supply decreases. Output declines as decreasing in labor supply. All variables tend to go to the steady state levels except output which takes more than ten years to return to its steady state level.

Figure 5 illustrates the effects of a temporary 1 percent increase in labor tax rate. Higher labor tax gives rise to an intratemporal effect which induces its allocation of time towards leisure. This leads households to reduce their labor supply. Consequently, it reduces disposable income and consumption. Reduction in labor supply lowers output. Congested public infrastructure increases as labor supply declines.

Figure 6 reports the impulse responses to a temporary 1 percent increase in capital tax rate. Impacts of a 1 percent increase in capital tax rate leads to that all variables except consumption decrease to be below their steady state levels. Increasing in capital tax rate reduces net return to private capital so that households
decrease their investment. However, consumption tends to rise because households sacrifice their investment for consumption. The consequences of a decrease in labor and private capital reduce congestion.

1.7 Conclusions

This study develops and estimates a DSGE model in which congested public infrastructure and public R&D are the crucial elements of a neoclassical production function in order to investigate the dynamic effects of increasing in government investment in R&D and government investment in infrastructure. Using Bayesian techniques to estimate the model, in overall, we consider the estimation results to be satisfactory. The diagnostic measures seem to indicate that the estimation is robust in the majority of its fields, in particular in what concerns the quality of the numerical posterior kernel maximization and the convergence of the MH algorithm. The data seems to be reasonably informative about most of the parameters. Most of the obtained estimates for the parameters of interest are generally in line with the available literature and, in most cases, seem to make sense from an economic point of view.

We find that investment in public infrastructure stimulates output, increases labor supply, reduces government debt and congestion in the long run and last long for many years, while in the short run only labor supply and private investment fall for a very short period. An increasing in R&D investment raises all economic activities in the short run and starts to decline in the long run. Consequently, to stimulate the long term economic benefits, with limited public resources, the government should increase investment in infrastructure. But if the economy needs to boot up in the short term, investment in R&D may be more efficient.

Infrastructure congestion explains the diminishing returns to private inputs. Government spending on R&D increases congestion which reduces the long-run growth rate of output. Public R&D causes congestion both from creating varieties of capital i.e. it raises capital stock, and from increasing in labor supply for R&D sector. These factors deplete infrastructure. Whereas, an increase in public spending in infrastructure
reduces congestion in two dimensions. First, it increases availability of infrastructure. Second, public investment crowds out private investment, so private capital declines.

An increase in infrastructure spending, in both short term and long term, significantly boosts economic activity. Infrastructure as roads and other transportation facilities play a crucial role in transport networks connection between producers and consumers to markets. Public infrastructure also increases welfare as consumption increases and labor supply decreases. While public R&D is also an important mean for achieving future growth and maintaining a relevant product in the market. Public R&D generates new ideas that could lead to new or improved products, processes, and services.
1.8 References


1.9 Appendix

Appendix A: Solving the Model

A1. Equilibrium Conditions

(1) Households

A representative household maximizes utility subject to household’s budget constraint and the law of motion for capital.

\[
\text{Max}_{\{c_t, i_t, b_t, k_t\}} \quad U = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \ln c_t - \frac{\left[ l_t / u_t^i \right]^{1+\gamma}}{1+\frac{1}{\nu}} \right\}
\]

s.t.
1. \((1+\tau_t^l)c_t + i_t + b_t = \left(1-\tau_t^l\right)w_t l_t + \left(1-\tau_t^b\right)r^b_t k_{t+1} + R_t^b b_{t+1} + tr_t^c\)
2. \(k_t = (1-\delta)k_{t-1} + i_t\)

First order conditions:

\[
\frac{L_t^C}{\left(u_t^C\right)^{\frac{1+\gamma}{\nu}}} = \frac{(1-\tau_t^l)w_t}{(1+\tau_t^C)c_t}
\]

\[
1 \frac{1}{(1+\tau_t^C)c_t} = \beta E_t \left\{ (1-\delta) + \left(1-\tau_t^{r_{t+1}}\right)\alpha_k \frac{Y_{t+1}}{K_{t+1}} \right\}
\]

(A1)

\[
\frac{1}{(1+\tau_t^C)c_t} \beta E_t \left\{ \frac{R_t^b}{(1+\tau_t^{r_{t+1}})C_{t+1}} \right\}
\]

(A2)

(Following firm’s FOC: \(r_t^k = \frac{\alpha_k}{K_{t+1}}\))

\[
\frac{1}{(1+\tau_t^C)c_t} = \beta E_t \left\{ \frac{R_t^b}{(1+\tau_t^{r_{t+1}})C_{t+1}} \right\}
\]

(A3)

Capital accumulation:

\[
k_t = (1-\delta)k_{t-1} + i_t
\]

(A4)
Household’s budget constraint:

\[(1 + \tau_i^c)C_i + K_i - (1 - \delta)K_{i-1} + B_i = (1 - \tau_i^l)w_iL_i + (1 - \tau_i^k)\alpha_k Y_i + R_i^b B_{i-1} + TR_i\]

(2) Aggregate Production Function

\[Y_i = A_i^{(\alpha_k + \phi \theta)_g + \theta + 1} \left( \eta K_{i-1} G_i^{RD} L_i^{RD} \right)^{\theta_1} \left( L_i \right)^{\theta_1} \left( K_{g,i-1}^g \right)^{\theta} \]

\[K_{g,i-1}^g = \frac{K_{g,i-1}^g}{\eta K_{i-1} G_i^{RD} L_i^{RD}} \left( L_i \right)^{\theta} \]

(3) Firms

\[r_i^k = \alpha_k \frac{Y_i}{K_{i-1}} \]

\[w_i = \alpha_k \frac{Y_i}{L_i^{RD}} \]

\[w_i = \alpha_i \frac{Y_i}{L_i^{RD}} \]

\[L_i^{RD} + L_i^c = L_i \]

(4) Government

Flow of budget:

\[G_i^c + G_i^s + G_i^{RD} + TR_i + R_i^b B_{i-1} = T_i^k + T_i^l + T_i^c + B_i \]

\[T_i^k = \tau_i^k \alpha_k Y_i \]

\[T_i^l = \tau_i^l w_i L_i \]

\[T_i^c = \tau_i^c C_i \]
Public infrastructure’s accumulation:

\[ K_{G,t}^S = (1 - \delta_g) K_{G,t-1}^S + G_t^S \]  
(A16)

Fiscal rule:

\[ G_t^f = \phi^f \left( Y_t \right)^{-\eta^f} \left( B_{t-1} \right)^{-\gamma^f} \mu_t^f \]  
(A17)

\[ G_t^{RD} = \phi^{RD} \left( Y_t \right)^{-\eta^{RD}} \left( B_{t-1} \right)^{-\gamma^{RD}} \mu_t^{RD} \]  
(A18)

\[ G_t^c = \phi^c \left( Y_t \right)^{-\eta^c} \left( B_{t-1} \right)^{-\gamma^c} \mu_t^c \]  
(A19)

\[ \tau_t^k = \phi^k \left( Y_t \right)^{-\eta^k} \left( B_{t-1} \right)^{-\gamma^k} \mu_t^k \]  
(A20)

\[ \tau_t^l = \phi^l \left( Y_t \right)^{-\eta^l} \left( B_{t-1} \right)^{-\gamma^l} \mu_t^l \]  
(A21)

\[ \tau_t^c = \phi^c \left( Y_t \right)^{-\eta^c} \left( B_{t-1} \right)^{-\gamma^c} \mu_t^c \]  
(A22)

\[ TR_t = \phi^{TR} \left( Y_t \right)^{-\eta^{TR}} \left( B_{t-1} \right)^{-\gamma^{TR}} \mu_t^{TR} \]  
(A23)

(5) Aggregate Resource Constraint

\[ Y_t = C_t + I_t + G_t^C + G_t^S + G_t^{RD} \]  
(A24)

A2. Equilibrium Conditions in Terms of Detrended Variables

\[ L_t, L_t^{RD}, I_t^y, R_t^b, \tau_t^k, \tau_t^l, \tau_t^c \]

The variables that are not detrended include

Detrended variables:

\[ C_t = \frac{C_t}{A_t}, \bar{Y}_t = \frac{Y_t}{A_t}, \bar{K}_t = \frac{K_t}{A_t}, \bar{I}_t = \frac{I_t}{A_t}, \bar{B}_t = \frac{B_t}{A_t}, \bar{\omega}_t = \frac{\omega_t}{A_t}, \bar{\bar{K}}_t = \frac{\bar{K}_t}{A_t}, \bar{\bar{K}}_s = \frac{\bar{K}_s}{A_t}, \bar{\bar{G}}_t = \frac{\bar{G}_t}{A_t}, \bar{\bar{G}}_s = \frac{\bar{G}_s}{A_t}, \bar{T}_t^1 = \frac{T_t^1}{A_t}, \bar{T}_t^r = \frac{T_t^r}{A_t}, \bar{TR} = \frac{TR}{A_t} \]
Equilibrium conditions:

Equations (A1)-(A23) are written in terms of the detrended variables

\[
\frac{L_i^{\pi_{t_1}}}{(u_i^{t_1})_t^{\pi_{t_1}}} = \frac{(1 - \tau_i^t)\tilde{w}_i}{(1 + \tau_i^t)\tilde{C}_i} 
\]

\[
\frac{1}{(1 + \tau_i^t)\tilde{C}_i} = \beta E_i \left\{ \frac{(1 - \delta) + (1 - \tau_i^{t+1})\alpha_x \tilde{Y}_i e^{\tau_i^{t+1}}}{(1 + \tau_i^{t+1})\tilde{C}_i e^{\tau_i^{t+1}}} \right\} 
\]

\[
\frac{1}{(1 + \tau_i^t)\tilde{C}_i} = \beta E_i \left\{ \frac{R_i^b}{(1 + \tau_i^{t+1})\tilde{C}_i e^{\tau_i^{t+1}}} \right\} 
\]

\[
\tilde{K}_i = (1 - \delta)\tilde{K}_i e^{-\eta} + \tilde{I}_i 
\]

\[
(1 + \tau_i^t)\tilde{C}_i + \tilde{K}_i - (1 - \delta)\tilde{K}_i e^{-\eta} + \tilde{B}_i = (1 - \tau_i^t)\tilde{w}_i L_i + (1 - \tau_i^t)\alpha_x \tilde{Y}_i + R_i^b\tilde{B}_i e^{-\eta} + \tilde{TR}_i 
\]

\[
\tilde{Y}_i = A_i^{\alpha_x} \left\{ \eta \tilde{K}_i e^{-\eta} \tilde{G}_i^{RD} L_i^{RD} \right\}^{\alpha_x} \left( L_i^{\gamma} \right)^{\alpha_i} \left( \tilde{K}_{g,r}^{RD} e^{-\eta} \right)^{\theta} 
\]

\[
\tilde{K}_{g,r}^{RD} = \frac{A_i^{\alpha_x} \tilde{K}_{g,r}^{RD}}{\left( \eta \tilde{K}_i e^{-\eta} \tilde{G}_i^{RD} L_i^{RD} \right)^{\alpha_x} \left( L_i^{\gamma} \right)^{\alpha_i}} 
\]

\[
\tilde{w}_i = \alpha_x \tilde{Y}_i L_i^{RD} 
\]

\[
\tilde{w}_i = \alpha_x \tilde{Y}_i L_i^{\gamma} 
\]

\[
L_i^{RD} + L_i^{\gamma} = L_i 
\]

\[
\tilde{G}_i^{C} + \tilde{G}_i^{\pi} + \tilde{G}_i^{RD} + \tilde{TR}_i + R_i^b\tilde{B}_i e^{-\eta} = \tilde{T}_i^{t} + \tilde{T}_i^{c} + \tilde{T}_i^{d} + \tilde{B}_i 
\]

\[
\tilde{T}_i^{k} = \tau_i^t \alpha_x \tilde{Y}_i 
\]

\[
\tilde{T}_i^{l} = \tau_i^t \tilde{w}_i L_i 
\]
\[ T_i^e = \tau_i^{*} \tilde{C}_i \] (A14')

\[ \tilde{R}^S_{g.j} = (1 - \delta) \tilde{R}^S_{g.j-1} e^{-\kappa} + \tilde{G}^S_i \] (A15')

\[ \tilde{G}^s_i = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^y} (\tilde{B}_{t-1})^{\kappa_i} u^y_i \] (A16')

\[ \tilde{G}^{RD}_i = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^{RD}} (\tilde{B}_{t-1})^{\kappa_i^{RD}} u^{RD}_i \] (A17')

\[ \tilde{G}^{ge}_i = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^{ge}} (\tilde{B}_{t-1})^{\kappa_i^{ge}} u^{ge}_i \] (A18')

\[ \tau_k^i = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^{y}} (\tilde{B}_{t-1})^{\kappa_i} u^{y}_i \] (A19')

\[ \tilde{c}_i = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^{y}} (\tilde{B}_{t-1})^{\kappa_i} u^{y}_i \] (A20')

\[ \tau_c^i = e^{\varphi_i} (\tilde{B}_{t-1})^{\kappa_i} u^{c}_i \] (A21')

\[ \tilde{T}_i^c = e^{\varphi_i} (\tilde{Y}_i)_{t-1}^{\eta_i^{y}} (\tilde{B}_{t-1})^{\kappa_i} u^{y}_i \] (A22')

\[ \tilde{Y}_i = \tilde{C}_i + \bar{I}_i + \tilde{G}^c_i + \tilde{G}^{RD}_i + \tilde{G}^s_i \] (A23')

**A3. Steady State**

We calculate steady-state values for variables used for estimation. These steady-state values,

\[ \tau^k, \tau^l, \tau^c, \tilde{G}^c, \tilde{G}^s, \tilde{G}^{RD}, \tilde{B}, \tilde{Y}, \tilde{Y}^c, \tilde{Y}^s, \tilde{Y}^{RD} \]

are obtained from the averages of the measured data. The other steady-state values are derived from the model specifications.

\[ \tilde{R}^h = \tilde{Y} \]

\[ \tilde{Y} = \frac{\tilde{Y}}{\beta} - (1 - \delta) \]

\[ \frac{\tilde{Y}}{K} = \frac{\beta}{(1 - \tau^k) \kappa_i} \]
\[
\frac{I}{K} = \gamma - (1 - \delta)
\]

\[
\frac{\bar{W}_L}{K} = (\alpha_i + \alpha_i) \frac{\bar{Y}}{K}
\]

\[
\frac{\bar{T}_k}{K} = \tau^i \alpha_i \left[ \frac{\gamma - (1 - \delta)}{\beta (1 - \tau^i) \gamma_k} \right]
\]

\[
\frac{\bar{T}_k}{K} = \tau^i \alpha_i \frac{\bar{Y}}{K}
\]

\[
\frac{\bar{T}_l}{K} = \tau^i (\alpha_i + \alpha_i) \frac{\bar{Y}}{K}
\]

\[
\frac{\bar{C}}{K} = \frac{\bar{Y}}{K} \left( 1 - \frac{\bar{G}^e}{\bar{Y}} - \frac{\bar{G}^i}{\bar{Y}} - \frac{\bar{G}^{RD}}{\bar{Y}} \right) - \frac{\bar{I}}{K}
\]

\[
\frac{\bar{G}^e}{K^i} = \frac{\gamma - (1 - \delta)}{\gamma}
\]

\[
\frac{\bar{B}}{K} = \left( \frac{\bar{B}}{\bar{Y}} \right) \left( \frac{\bar{Y}}{K} \right)
\]

\[
\frac{\bar{G}^i}{K} = \left( \frac{\bar{G}^i}{\bar{Y}} \right) \left( \frac{\bar{Y}}{K} \right)
\]

\[
\frac{\bar{G}^{RD}}{K} = \left( \frac{\bar{G}^{RD}}{\bar{Y}} \right) \left( \frac{\bar{Y}}{K} \right)
\]
\[ \frac{\bar{G}^{RD}}{K} = \left( \frac{\bar{G}}{\bar{Y}} \right) \left( \frac{\bar{Y}}{K} \right) \]

\[ \frac{\bar{R}}{K} = (1 + \tau^c) \frac{\bar{C}}{K} + \frac{1}{1 + \tau^c} \left( \frac{\bar{B}}{\bar{Y}} \right) \left( \frac{\bar{Y}}{K} \right) \left[ (1 - \tau')(\alpha_k + \alpha_t) + (1 - \tau^t) \alpha_k \right] \left( \frac{\bar{Y}}{K} \right) \]

A4. Log – Linearized System of Equations

\[ \frac{1}{v} \dot{i}_t - \left( \frac{1}{v} + \frac{1}{v} \right) \dot{\hat{u}}_t = \dot{\hat{w}}_t - \frac{\tau'}{1 - \tau'} \dot{\hat{\tau}}_t + \frac{\tau'}{1 + \tau'} \ddot{\hat{c}}_t - \dddot{\hat{c}}_t \quad (A1') \]

\[ \frac{\tau^c}{1 + \tau^c} \dddot{\hat{c}}_t + \dddot{\hat{c}}_t = \frac{(1 - \tau^k) \alpha_k \bar{y}}{K} + \frac{1 - \tau^k - (1 - \tau^k) \alpha_{t} \bar{y}}{K} \left[ E_t \left( \dot{\hat{c}}_{t+1} - \frac{\tau^t}{1 - \tau^t} \dddot{\hat{c}}_{t+1} + \dddot{\hat{c}}_{t+1} \right) - \dddot{\hat{k}}_t \right] = \]

\[ = \frac{\tau^c}{1 + \tau^c} E_t \dddot{\hat{c}}_{t+1} + E_t \dddot{\hat{c}}_{t+1} + E_t \dddot{\hat{c}}_{t+1} - \dddot{\hat{k}}_t \quad (A2') \]

\[ \frac{\tau^c}{1 + \tau^c} \dddot{\hat{c}}_t + \dddot{\hat{c}}_t = \frac{\tau^c}{1 + \tau^c} E_t \dddot{\hat{c}}_{t+1} + E_t \dddot{\hat{c}}_{t+1} + E_t \dddot{\hat{c}}_{t+1} - \dddot{\hat{k}}_t \quad (A3') \]

\[ \dddot{\hat{k}}_t = \left( \frac{1 - \delta}{\gamma} \right) (\dddot{\hat{k}}_{t-1} - \dddot{\hat{a}}_t) + \frac{\bar{I}}{K} \dddot{\hat{c}}_t \quad (A4') \]

(We don’t use log-linearized household’s budget constraint for estimation, thus there is no(A5’’)

presented)

\[ \dot{\hat{y}}_t = \alpha_k \left( \dot{\hat{k}}_{t-1} - \dot{\hat{a}}_t + \dot{\hat{g}}^{RD}_{t-1} + \dot{\hat{l}}^{RD}_{t-1} \right) + \alpha_t \dot{\hat{y}}_t + \theta \left( \dot{\hat{k}}^{cong}_{g,1} - \dot{\hat{a}}_t \right) + 2 \phi \theta \dot{\hat{A}}_t \quad (A6’’)
\]

\[ \dot{\hat{g}}^{cong}_{g,1} = \dot{\hat{g}}^{cong}_{g,1} - \phi \left( \dot{\hat{k}}_{t-1} - \dot{\hat{a}}_t + \dot{\hat{g}}^{RD}_{t-1} + \dot{\hat{l}}^{RD}_{t-1} \right) - \phi \dot{\hat{y}}_t - 2 \phi \dot{\hat{A}}_t \quad (A7’’)
\]

\[ \dot{\hat{w}}_t = \dot{\hat{y}}_t - \dot{\hat{l}}^{RD}_t \quad (A8’’)
\]

\[ \dot{\hat{w}}_t = \dot{\hat{y}}_t - \dot{\hat{l}}^{RD}_t \quad (A9’’)
\]
Fiscal financing rules:

\[
\left( \frac{\alpha_k}{\alpha_k + \alpha_i} \right) \dot{i}^{RD} + \left( \frac{\alpha_j}{\alpha_k + \alpha_j} \right) \dot{i}^y = \hat{i},
\]

(A10"")

\[
\frac{\ddot{G}^c}{K} \hat{g}^c + \frac{\ddot{G}^s}{K} \hat{g}^s + \frac{\ddot{G}^{RD}}{K} \hat{g}^{RD} + \frac{\ddot{R}}{K} \hat{i} + \frac{1}{\beta K} (\hat{R}_{t-1}^c + \hat{b}_{t-1} - \hat{a}_t) = \frac{\ddot{\tau}^k}{K} \hat{t}^k + \frac{\ddot{\tau}^l}{K} \hat{t}^l + \frac{\ddot{\tau}^c}{K} \hat{t}^c + \frac{\ddot{B}}{K} \hat{b}_t
\]

(A11"")

\[
\hat{i}^k = \hat{\dot{i}}^k + \hat{\dot{y}}_t
\]

(A12"")

\[
\hat{i}^l = \hat{\dot{i}}^l + \hat{\dot{w}}_t + \hat{i}_t
\]

(A13"")

\[
\hat{i}^c = \hat{\dot{i}}^c + \hat{\dot{c}}_t
\]

(A14"")

\[
\hat{k}^g = \left( \frac{1 - \delta_{\dot{a}}}{\gamma} \right) (\hat{k}^g_{t-1} - \hat{a}_t) + \frac{\ddot{G}^g}{K} \hat{g}^g_t
\]

(A15"")

\[
\hat{g}^s = -\eta_{gs} \hat{y}_t - \gamma_{gs} \hat{b}_{t-1} + u^g_t
\]

(A16"")

\[
\hat{g}^{RD} = -\eta_{RD} \hat{y}_t - \gamma_{RD} \hat{b}_{t-1} + u^{RD}_t
\]

(A17"")

\[
\hat{g}^c = -\eta_{gc} \hat{y}_t - \gamma_{gc} \hat{b}_{t-1} + u^c_t
\]

(A18"")

\[
\hat{t}^k = \eta_{tk} \hat{y}_t + \gamma_{tk} \hat{b}_{t-1} + u^{tk}_t
\]

(A19"")

\[
\hat{t}^l = \eta_{tl} \hat{y}_t + \gamma_{tl} \hat{b}_{t-1} + u^{tl}_t
\]

(A20"")

\[
\hat{t}^c = \gamma_{tc} \hat{b}_{t-1} + u^{tc}_t
\]

(A21"")

\[
\hat{r} = -\eta_{r} \hat{y}_t - \gamma_{r} \hat{b}_{t-1} + u^r_t
\]

(A22"")
Resource constraint:

\[
\frac{\ddot{Y}}{\dot{K}} = \frac{\ddot{C}}{K} \hat{\delta}_t + \frac{\ddot{I}}{K} \hat{\gamma}_t + \frac{\ddot{G}^c}{K} \hat{\mu}_t + \frac{\ddot{G}^s}{K} \hat{\nu}_t + \frac{\ddot{G}^{RD}}{K} \hat{\rho}_t
\]

(A23″)

Exogenous shocks:

Technology shock:  \( \hat{A}_t = \rho_A \hat{A}_{t-1} + \sigma_A \varepsilon_t \)  \hspace{1cm} (A24)

Labor supply shock:  \( u^l_t = \rho_{u^l} u^l_{t-1} + \sigma_{u^l} \varepsilon_t \)  \hspace{1cm} (A25)

Government consumption shock:  \( u^{gc}_t = \rho_{u^{gc}} u^{gc}_{t-1} + \sigma_{u^{gc}} \varepsilon_t \)  \hspace{1cm} (A26)

Government investment in infrastructure shock:  \( u^{gs}_t = \rho_{u^{gs}} u^{gs}_{t-1} + \sigma_{u^{gs}} \varepsilon_t \)  \hspace{1cm} (A27)

Government investment in R&D shock:  \( u^{RD}_t = \rho_{u^{RD}} u^{RD}_{t-1} + \sigma_{u^{RD}} \varepsilon_t \)  \hspace{1cm} (A28)

Capital income tax shock:  \( u^{rk}_t = \rho_{u^{rk}} u^{rk}_{t-1} + \sigma_{u^{rk}} \varepsilon_t \)  \hspace{1cm} (A29)

Labor income tax shock:  \( u^{ri}_t = \rho_{u^{ri}} u^{ri}_{t-1} + \sigma_{u^{ri}} \varepsilon_t \)  \hspace{1cm} (A30)

Consumption tax shock:  \( u^{rc}_t = \rho_{u^{rc}} u^{rc}_{t-1} + \sigma_{u^{rc}} \varepsilon_t \)  \hspace{1cm} (A31)

Transfer shock:  \( u^{tr}_t = \rho_{u^{tr}} u^{tr}_{t-1} + \sigma_{u^{tr}} \varepsilon_t \)  \hspace{1cm} (A32)

The process \( a_t \):  \( \hat{a}_t = \hat{A}_t - \hat{A}_{t-1} \)  \hspace{1cm} (A33)

The model contains 32 unknowns and 33 equations consisting of endogenous and exogenous variables:

Endogenous variables (22):  \( L, L^{RD}, L^c, Y, K, I, B, R^k, w, K^{comb}, K^c, G^s, G^{RD}, G^c, T^k, T^l, T^c, \tau, \tau^l, \tau^c, TR \)

Exogenous variables: shocks (9+1):  \( A, u^l, u^{gs}, u^{RD}, u^{gc}, u^{rk}, u^{ri}, u^{rc}, u^{tr}, \) and the process \( a_t \).
Model parameters (41): \[ \theta = \left\{ \gamma, \beta, \nu, \delta, \alpha_k, \theta, \phi, \phi, \delta, \eta_{g_1}, \eta_{g_2}, \eta_{r_1}, \eta_{r_2}, \eta_{g_3}, \eta_{g_4}, \eta_{r_3}, \eta_{r_4}, \theta, \beta, \nu, \alpha_k, \gamma, \beta, \nu, \delta, \alpha_k, \theta, \phi, \phi, \delta, \eta_{g_1}, \eta_{g_2}, \eta_{r_1}, \eta_{r_2}, \eta_{g_3}, \eta_{g_4}, \eta_{r_3}, \eta_{r_4} \right\} \]

Appendix B: Estimation Results

Table B1: Policy and transition functions

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>C</th>
<th>Y</th>
<th>I</th>
<th>B</th>
<th>Congested KG</th>
</tr>
</thead>
<tbody>
<tr>
<td>K(-1)</td>
<td>-0.0582</td>
<td>0.6029</td>
<td>0.0577</td>
<td>-2.7646</td>
<td>-0.0770</td>
<td>-0.8716</td>
</tr>
<tr>
<td>B(-1)</td>
<td>-0.0009</td>
<td>0.0037</td>
<td>-0.0018</td>
<td>-0.0188</td>
<td>0.9597</td>
<td>0.0159</td>
</tr>
<tr>
<td>RB(-1)</td>
<td>-0.0005</td>
<td>0.0049</td>
<td>-0.0003</td>
<td>-0.0272</td>
<td>1.0102</td>
<td>0.0006</td>
</tr>
<tr>
<td>KG(-1)</td>
<td>0.0065</td>
<td>0.0740</td>
<td>0.1596</td>
<td>0.6823</td>
<td>-0.1576</td>
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</tr>
<tr>
<td>UL(-1)</td>
<td>0.9977</td>
<td>0.2388</td>
<td>0.5079</td>
<td>2.1530</td>
<td>-0.5014</td>
<td>-1.1145</td>
</tr>
<tr>
<td>UGS(-1)</td>
<td>-0.0003</td>
<td>0.0025</td>
<td>-0.0001</td>
<td>-0.0724</td>
<td>0.0243</td>
<td>0.0003</td>
</tr>
<tr>
<td>UGRD(-1)</td>
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<td>0.0285</td>
<td>0.0867</td>
<td>0.3590</td>
<td>-0.0549</td>
<td>-0.9094</td>
</tr>
<tr>
<td>UGC(-1)</td>
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<td>0.0015</td>
<td>-0.1129</td>
<td>0.1078</td>
<td>-0.0032</td>
</tr>
<tr>
<td>UTAUK(-1)</td>
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<td>0.0881</td>
<td>-0.0049</td>
<td>-0.4936</td>
<td>-0.1229</td>
<td>0.0108</td>
</tr>
<tr>
<td>UTAUL(-1)</td>
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<td>-0.0208</td>
<td>-0.0127</td>
<td>0.0237</td>
<td>-0.3953</td>
<td>0.0278</td>
</tr>
<tr>
<td>UTAUC(-1)</td>
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<td>-0.0073</td>
<td>-0.0004</td>
<td>0.0353</td>
<td>-0.0314</td>
<td>0.0009</td>
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<tr>
<td>UTR(-1)</td>
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<td>-0.0614</td>
<td>0.3841</td>
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<tr>
<td>A(-1)</td>
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<td>0.0431</td>
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<td>0.0384</td>
<td>-0.1154</td>
</tr>
</tbody>
</table>
Figure 1. Estimated impulse responses to a 1 percent increase in government investment in R&D

Figure 2. Estimated impulse responses to a 1 percent increase in government investment infrastructure
Figure 3. Estimated impulse responses to a 1 percent increase in government consumption

Figure 4. Estimated impulse responses to a 1 percent increase in consumption tax rate
Figure 5. Estimated impulse responses to a 1 percent increase in labor tax rate

Figure 6. Estimated impulse responses to a 1 percent increase in capital tax rate
Figure 7. Univariate MH convergence diagnosis
Figure 8. Prior (gray line) and posterior (black line) distributions.
Figure 9. Mode check plots
CHAPTER TWO

DYNAMIC EFFECTS OF TRANSPORT INFRASTRUCTURE INVESTMENT, DENSITY, AND CONGESTION IN THAILAND

2.1 Introduction

This study examines and estimates the dynamic effects of increasing public transport infrastructure, shocks to population density, and the effects of other fiscal policy shocks on several key economic variables including aggregate output, aggregate consumption, aggregate investment, aggregate labor supply, congested transport infrastructure, and total population density. The analysis focuses on transport infrastructure investment and congestion effects due to the use of aggregate production inputs and total population density in Thailand. We employ a dynamic stochastic general equilibrium (DSGE) model for a small economy and use the Bayesian techniques to estimate the dynamic effects. The fiscal policies include investment in infrastructure, government consumption, taxes on income and consumption, transfers to consumers, and government debt, and are chosen to follow some simple policy rules designed to smooth out the business cycle, guarantee solvency for the government’s budget.

We make three contributions to the literature. First, Thai data is used to estimate a small scale DSGE model and the model is used to study the dynamic impact of shocks to population density, infrastructure, and fiscal policy. Second, we include congestion in the model to net out the congestion costs in estimating the contribution of infrastructure to GDP. Clearly, the more the transport facilities are congested, the lower their contribution to the productivity of the private sector. Third, we include population density itself in the model as a cause of congestion; the greater the density of population, the more difficult it is for transportation infrastructure to maintain it’s productivity. We also include another source of congestion, namely, use of private capital and labor in production as a cause of congestion. As it turns out, one of our more interesting results is that population density has a greater impact on congestion than use of private
inputs in production. This may have different policy implications than if production inputs were the only cause of congestion.

Transportation is a key ingredient to sustainable economic growth [Esfahani and Ramirez (2003), Harmatuck, (1996), Holleyman, (1996), Nadiri, and Mamuneas, (1996)]. Transportation infrastructure expands the productive capacity of a nation, both by increasing the mobilization of available resources, and by enhancing the productivity of those resources. It can enter in the production process as direct input and as an unpaid factor of production.

However, public infrastructure is most likely not a pure public good; most such facilities are congested and congestion causes rivalry; the more congested a facility is, the lower its productivity. As shown in several studies [Barro and Sala-i-Martin (1995), Glomm and Ravikumar (1997), Fisher and Turnovsky (1998) for example], public services are characterized by some degree of congestion. These studies found that public infrastructure is congested by aggregate private capital stocks and aggregate labor input used in production, which reduces efficiency. We argue that for the Thai economy, the population density due to the economic activities of the people in the nation also causes congestion. Thailand is a small economy but its density is high especially for the area with intensive public infrastructure. Not only do the labor and capital inputs cause congestion, but also the frequency of infrastructure usage also affects the productivity of public transport infrastructure.

Thailand is a small-scale economy that has been facing transport infrastructure congestion. The transportation system including highways, roads, trains, and sky trains, is all used intensively. For example, in the Bangkok city covering 606 square miles, with 14 million people in 2017 (World Population Review) is a densely populated area with about 23,102 people per square mile. Transportation infrastructure in this area is congested due to 1) economic activities of production inputs and 2) total density which is defined as the economic activities of all people in the economy. The total population density causes infrastructure congestion and economic growth may induce a greater volume of economic activities, which causes density
to increase and in turn causes more congestion. For example, as incomes rise and prosperity improves, people may use the streets more by going out to dinner more often, making shopping trips, visiting friends, ordering goods that have to be shipped and delivered using the infrastructure, and some may move to the urban area to enjoy such amenities and look for work.

Thailand’s transportation system has suffered from insufficient investment in recent years. Currently, Thailand is implementing a megaproject of public infrastructure investment to modernize key infrastructure of the country including airports, trains, motorways, and ports. It is expected that this will stimulate economic growth in GDP to 3 percent in 2018. From the Ministry of Transport of Thailand, the Thai government has allocated about $50 billion for this plan.

We estimate a small scale DSGE model for the Thai economy. The endogenous growth model is developed to incorporate congestion when estimating the contribution of public infrastructure to GDP. We also include the population density to incorporate a separate congestion effect. We estimate the impulse responses to density shocks, shocks due to improved infrastructure, and fiscal policy shocks. The estimated elasticity of transport infrastructure to output is 0.260. The estimated degrees of congestion with respect to total density, capital, and labor are 0.324, 0.310, and 0.289 respectively. These results suggest that all factors have congested public infrastructure, and that density may be the dominant influence.

In terms of the impulse responses to a shock to density, or public infrastructure, a shock to population density causes all key variables of the economy, e.g., consumption, income, investment, labor supply and public infrastructure, to decline and stay below the steady state levels. The infrastructure shock initially crowds out private investment in the short run, and therefore, causes private capital to decline in the short run. But then it reverses and stays above the steady-state level for several years. The other variables also increase in both the short run and consumption and income remain above their initial steady state values even after ten years. Therefore, a shock to public infrastructure stimulates long-term economic growth and increases labor. Density increases in the short run but returns to its initial steady state.
The remainder of the study is organized as follows: in section 2 we review the related literature; in section 3 we explain the theoretical framework of a DSGE model, congestion, total density and fiscal policy; in section 4 we present the data construction; in section 5 we present calibration and prior; in section 6, we estimate and report the estimation results; and finally, in section 7, we wrap up our main conclusions.

2.2 Literature Review

This study employs Bayesian methods to estimates a DSGE model for a small and developing country emphasized on public transport infrastructure, congestion, and density. The related literature follows.

2.2.1 Population Density and Public Infrastructure Congestion

Factors affecting the density of population can be explained following growth theory which relates transportation infrastructure investment to economic growth and population change. Boarnet, (1997), and Eberts, (1990), apply neoclassical growth theory to consider transportation infrastructure as an input into the production process, which enhances the productivity of other inputs such as labor, or acts as a household amenity to attract workers. Also, Chi, Voss and Deller (2006) argue that transportation is an indirect cause of population change. Highways are one of many factors affecting economic growth and population change. At the county and municipal levels, highways influence population change indirectly through economic growth or decline, employment change, altered sociodemographic structure, and environmental change. The covariant between highways and population change varies in different construction stages as well as across rural, suburban, and urban areas.

Beyzatlar and Kuştepeli (2011) examine the relationships between railway infrastructure and economic growth, and between railway infrastructure and population density in Turkey. The results from cointegration and causality tests imply that there is a positive long run relationship between railway length and population density and between railway length and real GDP per capita. Railway length causes real GDP per capita to increase only in the long run but it causes population density to increase both in the long
and the short run. These results confirm the theoretical framework that improvements in transportation infrastructure lead to higher income and higher population.

Carlino and Mills (1987) and McHugh and Wilkinson (1988) investigate the factors affecting US county population and employment growth during the 1970s. Total employment, manufacturing employment and population density are positively affected by the presence of limited-access highways. They find that transportation can influence population change by several paths: economic growth or decline, employment and sociodemographic structure.

2.2.2 Fiscal Policy in Developing Countries

Perotti (2004), Riascos and Végh (2003), and Talvi and Végh (2005) argue that there is a fundamental difference between how fiscal policy is conducted in developing countries compared to industrial countries. While fiscal policy in developed countries is countercyclical, fiscal policy in developing countries is tentatively procyclical. There are two main factors that might explain this. First, developing countries may face imperfect international credit markets that prevent them from borrowing in bad times. This limits the government’s ability to stimulate GDP. Second, it may be difficult to implement countercyclical policy due to political economy considerations that encourage wasteful spending. However, Jha et all (2014) assess whether countercyclical fiscal policy can support future growth in developing Asia where many countries implemented fiscal stimulus measures to support domestic demand during the global crisis of 2008. The examination is based on identifying shocks by restricting the contemporaneous relation between fiscal and non-fiscal variables. They find that in developing Asia, tax cuts have a greater countercyclical impact on output than government spending. This implies there is some scope for countercyclical tax adjustments so long as fiscal sustainability is not compromised.
Budina and Tuladhar (2010) study the post-crisis of 2008 fiscal priorities in the ASEAN-5 economies—Indonesia, Malaysia, Philippines, Singapore and Thailand. The paper highlights the need for fiscal policies to address infrastructure gaps, stimulate private consumption and expand social safety nets. Creating fiscal space or fiscal surplus to address these challenges will require raising revenues and reorienting public spending rather than increasing borrowing. Supporting structural reforms and aiming to stimulate private infrastructure investment, could help address long-term growth challenges, while easing the burden on fiscal policy.

Moreno - Dodson (2012) analyzes the importance of fiscal policies and the help of automatic stabilizers in Thailand when the 2008 global crisis hit. Thailand had a fiscal surplus and a relatively low public debt–to–GDP ratio. The availability of fiscal space allowed the government to introduce a large and timely stimulus package. The other advantage of Thailand was that the value of its automatic spending stabilizers was larger than in other countries in the region. The first stimulus package basically included measures with a direct impact on low-income groups, whereas the second package mainly targeted infrastructure spending. With additional spending on infrastructure, the share of public investment increased to 8 percent of GDP from 7 percent starting in 2009. As a result of higher expenditure and lower taxes, the fiscal deficit jumped from 0.1 percent of GDP in 2008 to 3.2 percent of GDP in 2009, higher than its historical trend. Thanks to automatic spending stabilizers, the deficit is expected to decline gradually to 1.4 percent by 2014. In 2011, the government had already begun a gradual exit from fiscal policy support.

2.3 A Dynamic Stochastic General Equilibrium (DSGE) Model

We apply a dynamic stochastic general equilibrium model with a version of the basic neoclassical model that permits a variety of fiscal interventions to study the economic consequences of changes in fiscal policy focusing on transport infrastructure congested by production inputs and total density. The model's structural elements consist of households’ preference, technology that incorporates public transport infrastructure and congestion, and resource constraints for both private and public agents, together with rules governing public
finance.

2.3.1 Households

Identical households receive the income from supplying labor $l_t$, renting capital $k_{t-1}$, the return on government bond holding $R_{t-1}^b b_{t-1}$, and a transfer payment $tr_t$ from the government. They use this income for consumption $c_t$, investment $i_t$, holding bond for next period $b_t$, and tax payments. The households pay income tax and consumption tax in each period.

A representative household maximizes the discounted expected life-time utility from choosing the levels of consumption, hours worked, investment, and bond holding, subject to the household’s budget constraint (1) and the law of motion for capital (2).

$$\max_{\{c_t, l_t, i_t, k_t, b_t\}_{t=0}^\infty} U = E_T \sum_{t=0}^\infty \beta^t \left\{ \ln c_t - \frac{l_t / l_t^1}{1 + \frac{1}{\nu}} \right\}$$

Subject to

$$\left(1 + \tau^c_t\right) c_t + i_t + b_t = (1 - \tau^{inc}_t)(w_t l_t + r^k_t k_{t-1}) + R_{t-1}^b b_{t-1} + tr_t, \quad (1)$$

$$k_t = (1 - \delta) k_{t-1} + i_t. \quad (2)$$

The labor supply shock, $u_t^l = \rho_t u_{t-1}^l + \sigma_t \varepsilon_t^l$, follows an AR(1) process, $\rho_t \in [0,1]$, $\sigma_t$ is the standard deviation, $\varepsilon_t^l \sim iid N(0,1)$. Future consumption and labor are discounted using the subjective discount factor, $\beta \in (0, 1)$. $\nu$ is the Frisch aggregate labor supply elasticity, $r^k_t$ is the rate of return on capital, $w_t$ is the wage rate, $\delta$ is the depreciation rate for capital, $\tau^{inc}$ and $\tau^c$ are the income tax and consumption tax rates respectively.
The first order conditions follow.

\[
\frac{L_t^{1/v}}{(u_t)^{1+1/v}} = \frac{(1-t_c^{in})w_t}{(1+t_c^{in})c_t},
\]

(3)

\[
\frac{1}{(1+t_c^{in})c_t} = \beta E_t \left\{ \frac{(1-\delta)+(1-t_c^{in})r_{t+1}^k}{(1+t_c^{in})c_{t+1}} \right\},
\]

(4)

\[
\frac{1}{(1+t_c^{in})c_t} = \beta E_t \left\{ \frac{r_t^p}{(1+t_c^{in})c_{t+1}} \right\}.
\]

(5)

The initial conditions on capital and government bonds are

\[K_0 = K_0,\]

(6)

\[B_0 = 0.\]

(7)

The transversality condition is given by the following two equations,

\[
\lim_{t \to \infty} E_t [\beta^t \tilde{u}(c_t)k_t]=0,
\]

(8)

\[
\lim_{t \to \infty} E_t [\beta^t \tilde{u}(c_t)b_t]=0.
\]

(9)

2.3.2 Technology

2.3.2.1 Public Transport Infrastructure, Density, and Congestion in Thailand

The economy consists of identical firms and identical households. A representative household allocates its time for work and leisure to maximizes their life time utilities. Population density is the number of people per unit of area. Total density is defined as the total economic activities of all people in the economy.

In period \(t\), all households can access the public infrastructure \(K_{g,t-1}\), and it is congested by private capital input \(K_{t-1}\), labor input \(L_t\), and total density \(D_t\). The congested transport infrastructure is defined as
\[ K_{g,t-1}^{\text{cong}} = \frac{K_{g,t-1}}{(d_t)^\psi (k_{t-1})^\varphi (l_t)^\phi}. \]  

(10)

The parameters \( \psi, \varphi, \phi \geq 0 \) are the degrees of rivalry with respect to total density, private capital and labor respectively.

The total density increases with the economic growth as we described above. Consequently, we define the total density to be endogenously determined by the ratio of current GDP and one period lagged GDP. The \( U_t^d \) is an exogenous shock in population density which is assumed to follow stationary AR(1) processes,

\[ D_t = \alpha_d \left( \frac{y_t}{y_{t-1}} \right) U_t^d, \]  

(11)

where \( \alpha_d \) is the marginal effect of the GDP ratio, and \( U_t^d = \rho_d U_{t-1}^d + \sigma_d \varepsilon_t^d \).

The law of motion for public infrastructure is

\[ K_{g,t} = (1 - \delta_g)K_{g,t-1} + G_t^I, \]  

(12)

where \( \delta_g \) is the transportation infrastructure depreciation rate, \( G_t^I \) is the government investment in transport infrastructure.

2.3.2.2 Firms

A representative firm chooses the levels of capital \( (k) \) and labor \( (l) \) to produce a homogenous output to maximize its profit given congested public transport infrastructure defined in (10) and (11). The specification of the production function follows,

\[ y_t = (A_t)^{1-\alpha-a_g} (k_{t-1})^\alpha (l_t)^{1-\alpha} (K_{g,t-1}^{\text{cong}})^{a_g}, \]  

(13)

where \( K_{g,t-1}^{\text{cong}} \) is the congested public transport infrastructure (10), \( \alpha, 1-\alpha, \) and \( \alpha_g \) are the capital share, labor share, and public infrastructure productivity respectively. This production function exhibits constant
return to scale over private inputs and exhibit increasing return to scale over both private and public inputs since $\alpha_g$ is greater than zero.

The first order condition for the firm are,

$$r_t^k = \frac{\alpha Y_t}{K_{t-1}}$$

$$w_t = (1 - \alpha) \frac{Y_t}{L_t}.$$  

Let $K = Nk, L = Nl, Y = Ny$ be aggregate capital, labor, and output. Combining the equations of the technology,

$$Y_t = (A_t)^{1-\alpha-\phi a_g} (K_{t-1})^{\phi a_g} (L_t)^{1-\alpha-\phi a_g} (D_t)^{-\psi a_g}.$$  

This production function exhibits long-run growth if $\alpha - \phi \alpha_g + \alpha_g = \alpha + (1 - \phi)\alpha_g = 1$. We also need to impose assumptions of positive and diminishing marginal products on the aggregate production function to ensure the existence of interior equilibria that are globally asymptotically stable. The marginal product of capital is positive and diminishing if

$$MP_K = (\alpha - \phi \alpha_g) \frac{Y_t}{K_{t-1}} > 0,$$

$$\frac{\partial MP_K}{\partial K} = (\alpha - \phi \alpha_g) (\alpha - \phi \alpha_g - 1) \frac{Y_t}{(K_{t-1})^2} < 0.$$  

Equation (17) implies $\alpha - \phi \alpha_g > 0$ and (18) implies $\alpha - \phi \alpha_g - 1 < 0 \Rightarrow \phi - \phi \alpha_g < 1$. Therefore, $0 < \alpha - \phi \alpha_g < 1$.

Similarly, marginal products of labor are positive and diminishing if

$$MP_L = (1 - \alpha - \phi a_g) \frac{Y_t}{L_t} > 0,$$
\[
\frac{\partial MPL}{\partial L} = (1 - \alpha - \phi \alpha_g)(1 - \alpha - \phi \alpha_g - 1) \frac{Y_t}{(\alpha_t)^2} < 0. \tag{20}
\]

Equation (19) implies \(1 - \alpha - \phi \alpha_g > 0\) and (20) implies \(1 - \alpha - \phi \alpha_g - 1 < 0 \Rightarrow \alpha + \phi \alpha_g > 0\).

2.3.3 Fiscal Policy

2.3.3.1 Fiscal Policy in Thailand

Following the data from Fiscal Policy Office, Ministry of Finance, during the period between 2007 and 2016, there were discretionary policies applied in response to the economic activity. Figure 1 shows the size of the government (measured by public spending) between 2007 and 2016 averaged 18.3 percent of GDP, whereas the average public revenue was 26.9 percent of GDP. The spike in public spending was attributed to spending on economic services. In 2012, the sharp increase was due to spending on general public services, and a defense spending hike raised public spending to 20.2 percent of GDP.

Figure 1. Government expenditure and revenue ratios (% of GDP)

Source: Ministry of Finance, Thailand

Thailand’s fiscal policy has been countercyclical both on the revenue and expenditure sides. Public revenue and expenditure are countercyclical because of automatic tax stabilizers, and government spending
stabilizers respectively. With growth acceleration, budget deficits decline and the fiscal surplus is positive during these periods due to successful fiscal consolidation to reduce budget deficits and public debt by growth-enhancing policies and cuts in public wages. The government spending on infrastructure, as percentage of GDP was quite low comparative to the revenues. The long-term trend is persistent with the expenditure ratio of between 2 and 5 percent during this period. However, the government is implementing greater fiscal stimulus, which mainly includes transport-related infrastructure projects. This should not only boost sentiment but also ease business conditions in the medium-term. This might cause the fiscal deficit to deteriorate in 2017, before improving in 2018.

2.3.3.2 The Government Budget Constraint

Government spending can be classified into three categories: 1) consumption, 2) investment in infrastructure, and 3) transfers. The government budget constraint takes the following standard form in discrete time:

\[ G_t^c + G_t^i + TR_t + R_{t-1}^B B_{t-1} = T_{t}^{inc} + T_{t}^c + B_t, \]  
\[ T_{t}^{inc} = \tau_t^{inc} Y_t, \]  
\[ T_{t}^c = \tau_t^c C_t, \]

where \( G_t^c, G_t^i, TR_t, R_{t-1}^B B_{t-1}, T_{t}^{inc}, T_{t}^c, B_t \) are the aggregate levels of government consumption, investment in infrastructure, transfer payments, debt payments, income tax revenues, consumption tax revenues, and debt, respectively.

2.3.3.3 The Fiscal Rules

The fiscal rules in Thailand are countercyclical to smooth out the business cycle, uncertain shocks, generate solvency for the fiscal sector, and guarantee that the intertemporal budget constraint of the government is satisfied [see Jansen (2004); Jansen and Khannabha (2009); Levine et al (2009); Sangubhan and
The fiscal rules in this study are set to follow the studies by Talvi and Végh (2005), Leeper et al. (2010), and also adjusted to the fiscal policy management in Thailand by Jittungsakulwith (2014). Each fiscal instrument responds to changes in its own lagged value. The fiscal revenue instruments also respond to changes in government debt, while fiscal expenditure instruments respond only to changes in the GDP. The cyclical policy may raise the tax rates when the level of debt increases and may reduce government spending when the level of GDP increases. In terms of deviations from steady state levels, the rules are,

\[ g_t^i = \omega_{gi} g_{t-1}^i - \eta_{gi} y_t + u_t^{gi}, \]

\[ g_t^c = \omega_{gc} g_{t-1}^c - \eta_{gc} y_t + u_t^{gc}, \]

\[ t_{\text{inc}} = \omega_{\text{inc}} t_{\text{inc},t-1} + \gamma_{\text{inc}} b_{t-1} + u_{t}^{\text{inc}}, \]

\[ t_t^c = \omega_{tc} t_{t-1}^c + \gamma_{tc} b_{t-1} + u_{t}^{tc}, \]

\[ t_{tr} = \omega_{tr} t_{tr,t-1} - \eta_{tr} y_t + u_{t}^{tr}. \]

The government spending instruments are allowed to adjust to the cyclical position of the economy \((\eta_{gi}, \eta_{gc}, \eta_{tr} > 0)\), while the tax instruments are allowed to adjust to the cyclical position of the government debt \((\gamma_{\text{inc}}, \gamma_{tc} > 0)\). To keep persistency in fiscal policy, we set the coefficients of the lagged values to be less than unity: \(\omega_{gi}, \omega_{gc}, \omega_{\text{inc}}, \omega_{tc}, \omega_{tr} < 1\).

AR(1) components are denoted as \(u_t^{gc}, u_t^{gi}, u_t^{\text{inc}}, u_t^{tc}, u_t^{tr}\) representing the exogenous shocks to government instruments: government spending on consumption, infrastructure investment, income tax rate, consumption tax rate, and transfers, respectively.
2.4 The Data

We collected Thailand time series quarterly data from 2007Q1 to 2015Q1 including hours worked, population in Bangkok city, GDP, government investment in transport infrastructure, government investment in R&D, government consumption, income tax revenues, consumption tax revenues, and government transfers. The data of consumption and investment were collected from the Office of the National Economic and Social Development Board used for calculating the steady state values as calibrated parameters.

Hours worked were obtained from the survey of employed persons with 15 years and over taken from the National Statistical Office, Ministry of Information and Communication Technology. Population density were calculated from total population in Bangkok city divided by the area of 1,568.7 square kilometers, collected from the National Statistical Office, Ministry of Information and Communication Technology.

The GDP and government data were taken from GFMIS, Ministry of Finance of Thailand. Consumption and income tax rates were calculated as follows.

Consumption tax revenues, $T^c$, include excise taxes and customs duties. The average consumption tax rate is defined as

$$\tau^c = \frac{T^c}{C - T^c},$$

where $C$ is personal consumption expenditure on nondurable goods and on services.

Income tax includes personal income tax (PIT) and corporate income tax (CIT). All earned income from capital gains is taxed the same as regular income. The average income tax rate is calculated as:

$$\tau^{inc} = \frac{CIT + PIT}{W + CI},$$

where $W$ is wage and salary accruals, $CI$ is capital income.
We collected the data in nominal terms, and they are divided by the GDP deflator to be real values.

2.5 Calibration and Prior

2.5.1 Calibration

We calibrate the values of some parameters that can be strict priors including those determining the steady-state obtained from the model specification, those having reliable estimates from other sources and commonly used in the literature, and those that are crucial to replicate the main steady state ratios of the Thai economy.

Table 1 presents the parameters that are calibrated to Thailand’s economic conditions. We choose the prior means for the discount rate and depreciation rates to be consistent with a capital share of 0.4, an investment-to-output ratio of about 29%, and an annual interest rate of 3%. These choices yield values of $\beta = 0.99$ and $\delta = 0.02$. 

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Depreciation rate of private capital, $\delta$</td>
<td>0.02</td>
</tr>
<tr>
<td>Depreciation rate of public capital, $\delta_g$</td>
<td>0.02</td>
</tr>
<tr>
<td>Capital share, $\alpha$</td>
<td>0.30</td>
</tr>
<tr>
<td>Labor share, $1-\alpha$</td>
<td>0.70</td>
</tr>
<tr>
<td>Consumption tax rate, $\tau^c$</td>
<td>0.07</td>
</tr>
<tr>
<td>Income tax rate, $\tau^{inc}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Share of public consumption, $G^c / Y$</td>
<td>0.115</td>
</tr>
<tr>
<td>Share of public infrastructure investment, $G^I / Y$</td>
<td>0.025</td>
</tr>
<tr>
<td>Public debt to GDP ratio, $B / Y$</td>
<td>0.272</td>
</tr>
</tbody>
</table>
\( \delta = 0.02 \) in quarterly terms. We assume that the depreciation rate of public infrastructure is equal to that of private capital, \( \delta = 0.02 \). We set \( \alpha \) equal to 0.3, which implies a labor share of 70 percent.

The steady-state fiscal variables are calibrated to sample means of the dataset, the consumption tax rate, \( \tau^c = 0.07 \), the income tax rate, the \( \tau^{inc} = 0.15 \), the share of public consumption, \( G^c/Y = 0.115 \), the share of public investment in infrastructure, \( G^i/Y = 0.025 \), and the public debt to GDP ratio, \( B/Y = 0.272 \).

### 2.5.2 Priors

The prior distributions for the remaining parameters are shown in the first three columns of Table 2. These priors are similar to those commonly used in the literature [see Christiano et al. (2011), Sangaré (2016), Montolio, D. and Solé-Ollé, A. (2009), Adolfson et al. (2007), and An and Schorfheide (2007) for example]. The Frisch elasticity of labor supply, \( \nu \), is evaluated at 2 (Sangaré, 2016). Following Pholphirul (2005), the share of capital in the domestic production \( \alpha \) is set to 0.3.

The prior means of the degrees of congestion with respect to population density, capital, and labor inputs were set following the study by Montolio and Solé-Ollé (2009). They estimated the productivity of public transport infrastructure and the congestion parameter for public transportation considering the total usage of roads as a factor congesting infrastructure, and obtained the estimates of 0.27 and 0.25 respectively. Following this study, we set the same prior means of degrees of congestion with respect to total density, capital, and labor inputs of 0.25, and the prior mean of 0.27 for capital productivity.

The prior distributions for policy parameters were chosen to be fairly diffuse and cover a reasonably large range of parameter values, as well as consistent with the studies by Jansen (2004), Jansen and Khannabha (2009), Jha et al. (2014) and Sangaré (2016). The fiscal responses to lagged instruments are assumed to have a Beta distribution with the mean of 0.8 and standard deviation of 0.2. The fiscal instruments’ elasticities with respect to output are assumed to have Normal distributions. The means of the government
spending elasticity is 0.15 for government investment in infrastructure, transfer, and government consumption. The fiscal responses to government debt are assumed to have a Gamma distribution with a mean of 0.20 and standard deviation of 0.1.

2.6 Results

2.6.1 Estimation Results

Table 2 reports the estimation results of interested parameters for the Thai economy. It contains posterior means and 90% credible intervals\(^5\), computed from the output of the posterior simulator of the posterior distributions computed with the MH algorithm\(^6\). We generate 500,000 draws from the posterior distribution of the parameters of the stochastic growth model. The acceptance rate is 53.7 percent which gives good estimation results (Following Del Negro and Schorfheide (2005,2010), and An, S., and Schorfheide, F. (2007), an acceptance rate should be around 50%).

\(^5\) The Bayesian technique calculates the probability of different values of the parameter given the data. This new probability distribution is called the "a posteriori probability". Bayesian approaches can summarize their uncertainty by giving a range of values on the posterior probability distribution that includes 90% of the probability. This is called a "90% credibility interval."

\(^6\) The Metropolis–Hastings algorithm is a Markov chain Monte Carlo (MCMC) method for obtaining a sequence of random samples from a probability distribution by specifying a “candidate” (or jumping) distribution, from which several parameter estimates are drawn and then poses an “acceptance-rejection” rule.
Table 2: Prior and posterior distributions of the model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Density</td>
<td>Mean</td>
</tr>
<tr>
<td><strong>Structural parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology growth, $\gamma$</td>
<td>Normal</td>
<td>0.01</td>
</tr>
<tr>
<td>Labor supply elasticity, $v$</td>
<td>Gamma</td>
<td>2.00</td>
</tr>
<tr>
<td>Productivity in public infrastructure, $\alpha_g$</td>
<td>Beta</td>
<td>0.27</td>
</tr>
<tr>
<td>Degree of nonrivalry wrt. density, $\psi$</td>
<td>Beta</td>
<td>0.25</td>
</tr>
<tr>
<td>Degree of nonrivalry wrt. capital, $\varphi$</td>
<td>Beta</td>
<td>0.25</td>
</tr>
<tr>
<td>Degree of nonrivalry wrt. labor, $\phi$</td>
<td>Beta</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Policy parameters</strong></td>
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<td></td>
</tr>
<tr>
<td>Gov infras. resp. lagged, $\omega_{gi}$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>Gov conspt. resp. lagged, $\omega_{gc}$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>Transfer resp. lagged, $\omega_{tr}$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>Income tax resp. lagged, $\omega_{\tau inc}$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>Conspt tax resp. lagged, $\omega_{\tau tc}$</td>
<td>Beta</td>
<td>0.80</td>
</tr>
<tr>
<td>Gov. infras. resp. to output, $\eta_{gi}$</td>
<td>Normal</td>
<td>0.15</td>
</tr>
<tr>
<td>Gov. conspt. Resp.to output, $\eta_{gc}$</td>
<td>Normal</td>
<td>0.15</td>
</tr>
<tr>
<td>Transfer resp. to output, $\eta_{tr}$</td>
<td>Normal</td>
<td>0.15</td>
</tr>
<tr>
<td>Income tax resp. to debt, $\gamma_{\tau inc}$</td>
<td>Gamma</td>
<td>0.20</td>
</tr>
<tr>
<td>Conspt. tax resp. to debt, $\gamma_{\tau tc}$</td>
<td>Gamma</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>AR(1) coefficients</strong></td>
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<td></td>
</tr>
<tr>
<td>Density, $\rho_d$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Leisure preference, $\rho_l$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Technology, $\rho_A$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Gov. infrastructure, $\rho_{gi}$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Gov. conspt., $\rho_{gc}$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Income tax rate, $\rho_{\tau inc}$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Conspt. tax rate, $\rho_{\tau tc}$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
<tr>
<td>Transfer, $\rho_{tr}$</td>
<td>Beta</td>
<td>0.85</td>
</tr>
</tbody>
</table>
For the economy growth rate, we use a logarithmic transformation of $\gamma$, which can be interpreted as the average quarterly growth rate of the economy and is estimated to be 1.01 percent. The distribution of productivity of transport infrastructure $\alpha_p$ with the prior mean of 0.27 is little updated in view of data with the estimated result of 0.266. Other estimated parameters are updated with data. The estimated results are significantly different from the prior distributions.

Transport infrastructure has contributed to output with the elasticity of 26 percent. As Thailand is a small country and the economy is developing, transport infrastructure has been used mainly in the production process such as delivering production inputs and outputs to trade markets. Also, there is almost no other private transportation efficiently used for production. If accounting for congestion, the estimates show that the degrees of congestion with respect to total density, capital, and labor are 0.324, 0.310, and 0.289 respectively. We put the same prior distributions for these three factors since there is no previous estimates of the degrees of congestion for each factor separately, only the estimated parameter of overall usage of 0.25 is available.

The results estimated for the Thai economy show that all factors have congested public infrastructure. The total density causes the highest congestion, while the labor input is estimated to have lowest impact on congestion. This is of some importance for public policy purposes and one of the most important implications of our research. Private capital is also estimated to increase rivalry of using the infrastructure. The estimates imply that transport infrastructure has been used intensively by production inputs and by total economic activities, as measured by population density, causing rivalry and depletion. This reduces the productivity of the infrastructure over time.

The policy parameters are updated with data and all are estimated to be different from zero. First, the estimates of the fiscal instruments except for the income tax responding to their lagged are quite low between 0.121-0.270 much different from the prior means of 0.80 equally. These results imply that the Thai government has not set the fiscal policy by changing the consumption tax rate and spending to follow their
lagged values. However, the income tax policy seems the most persistent among other fiscal instruments with the estimate of 27 percent. Second, the counter-cyclical responses of expenditure instruments to the aggregate output are close to each other. Given the same prior means of 0.15, the estimated posterior means of responses of investment in infrastructure, consumption, and transfer payment to output are 0.156, 0.110, and 0.136 respectively. Government consumption expenditure has the lowest procyclical response to aggregate output. Third, the estimates of procyclical responses of income and consumption taxes to the state of government debt are 0.064 and 0.124 which are quite low compared to their prior means of 0.2. The tax instruments are less responsive to debt. The government have stabilized debt by adjusting the magnitudes of both income and consumption tax but with the small sizes.

Finally, we consider the persistency and volatility parameters of structural shocks. All the persistent shocks are updated well with data with the estimates of between 0.516 and 0.685, except that of the total density which give high persistency of 0.853.

### 2.6.2 Impulse Responses

In this section, we demonstrate the impulse responses of population density, congested public transport infrastructure, labor, consumption, output, and private investment to a 1 percent increase in 5 main exogenous shocks as follows.
Figure 2. Estimated impulse responses to a 1 percent increase in population density.

Figure 2 illustrates the impulse responses of density, congestible public infrastructure, labor, consumption, output, and investment to an exogenous, temporary 1 percent increase in total density. All of the key variables decrease in response and stay below their steady state levels for quite some time. The decrease is largest for output and consumption, and smaller for labor supply, investment, and public infrastructure. An exogenous increase in density reduces output immediately by about 0.8 percent which is not trivial, especially given the source of the shock, population density. The endogenous response of population density after the initial shock is to decrease gradually over time. Eventually, output, consumption, labor supply, and investment increase. However, consumption is still below its initial steady state value even after 10 years. With the estimated degree of congestion with respect to the density of 0.324 in Table 2, the congested transport infrastructure reduces productivity significantly when density increases. Wages and
labor supply fall, which is consistent with the marginal product of public capital falling and a fall in the marginal product of labor.

**Figure 3. Estimated impulse responses to a 1 percent increase in public infrastructure investment.**

Figure 3 illustrates the economy’s response to a temporary 1 percent increase in government spending on infrastructure. Given the population density, the infrastructure becomes less congested. With the positive estimated value of productivity of public infrastructure, output, consumption, and labor supply all increase. The endogenous response of public infrastructure after the initial shock is positive in the short run, but falls in the long run to its steady state value; an increase in public infrastructure raises the productivity of labor and thus labor supply increases in the short run but eventually falls in the long run due to the improvement in productivity of labor and the associated wealth effect. These responses cause an increase in household income. Individuals respond to this increase in their wealth by increasing consumption and eventually leisure.
An increase in infrastructure in Thailand also crowds out private investment, and therefore causes private capital to decline in the short run. This process takes a year, then private investment starts to increase and stay above the steady-state level for several years as well as that of aggregate output. Therefore, an increase in public infrastructure stimulates long-term economic growth and increases labor supply. As labor and capital decrease in the long run to their steady state values, there is less congestion due to production. This increases access to public infrastructure. As a result, public infrastructure is less rival and more productive at the margin.

Figure 4. Estimated impulse responses to a 1 percent increase in income tax.

Figure 4 illustrates the impulse responses to a temporary 1 percent increase in the income tax rate. An increase in this tax rate reduces domestic economic activity which is consistent with conventional theory. The density declines as capital, labor, and output decline, but then increases as these variables increase. Since density and other production inputs initially decline, public transportation infrastructure becomes less
rival. This increases output productivity causing output and the inputs to increase after the initial shock. Eventually, all of the variables converge to their initial steady state values except for consumption which remains high due to increased transfers. Public infrastructure increases in the short run, but eventually falls in the long run.

**Figure 5. Estimated impulse responses to a 1 percent increase in consumption tax.**

Figure 5 plots the impulse responses to a temporary 1 percent increase in the consumption tax rate. Increasing the tax rate on consumption raises the unit price of goods as well as total household expenditure. Individuals consume less and take more leisure to satisfy infratemporal substitution between consumption and leisure. Therefore, labor supply decreases. The initial impact on density is relatively small due to a small change in labor supply, but the increase in investment in private capital. The response of density stays above its steady state value for a short period and then declines to the steady state level in the long run,
actually becoming negative after about 12 quarters but still converges to its steady state value. The public transport infrastructure becomes more congested due to the initial increase in capital and density. Investment initially increases because households sacrifice consumption for investment. However, eventually, investment converges to a point below its steady state value after ten years.

2.6.3 Sensitivity Analysis

We set the values of the degrees of nonrivalry with respect to density $\psi$, capital $\varphi$, and labor $\phi$ to be the same at 0.25. We do the sensitivity analysis setting the values of the degree of non-rivalry with respect to density to vary from 0.25 by a 20% change, and set two values at 0.20 and 0.25 which are the same for both the degrees of non-rivalry with respect to capital and labor.

We compare the impulse responses of density, congested public infrastructure, labor, consumption, output, and investment to a temporary 1 percent increase in density, and in government spending on infrastructure, given the different values of prior means of sensitive parameters. The results are presented in Figure 6 and 7.
Figure 6. Impulse responses to a 1 percent increase in density

\((\psi, \varphi, \phi) = (0.20, 0.20, 0.20)\)

\((\psi, \varphi, \phi) = (0.20, 0.20, 0.25)\)
$$\psi, \varphi, \phi = (0.20, 0.25, 0.20)$$

$$\psi, \varphi, \phi = (0.20, 0.25, 0.25)$$
\[(\psi, \varphi, \phi) = (0.25, 0.20, 0.20)\]

\[(\psi, \varphi, \phi) = (0.25, 0.20, 0.25)\]
\[(\psi, \varphi, \phi) = (0.25, 0.25, 0.20)\]

\[(\psi, \varphi, \phi) = (0.25, 0.25, 0.25)\]
$$(\psi, \varphi, \phi) = (0.30, 0.20, 0.20)$$

$$(\psi, \varphi, \phi) = (0.30, 0.20, 0.25)$$
\[(\psi, \varphi, \phi) = (0.30, 0.25, 0.20)\]

\[(\psi, \varphi, \phi) = (0.30, 0.25, 0.25)\]
Figure 7. Impulse responses to a 1 percent increase in infrastructure investment.

\[(\psi, \varphi, \phi) = (0.20, 0.20, 0.20)\]

\[(\psi, \varphi, \phi) = (0.20, 0.20, 0.25)\]
\((\psi, \varphi, \phi) = (0.20, 0.25, 0.20)\)

\((\psi, \varphi, \phi) = (0.20, 0.25, 0.25)\)
\((\psi, \varphi, \phi) = (0.25, 0.20, 0.20)\)

\((\psi, \varphi, \phi) = (0.25, 0.20, 0.25)\)
\((\psi, \varphi, \phi) = (0.25, 0.25, 0.20)\)

\[(\psi, \varphi, \phi) = (0.25, 0.25, 0.25)\]
\((\psi, \phi, \theta) = (0.30, 0.20, 0.20)\)

\((\psi, \phi, \theta) = (0.30, 0.20, 0.25)\)
$((\psi, \varphi, \phi) = (0.30, 0.25, 0.20))$

$((\psi, \varphi, \phi) = (0.30, 0.25, 0.25))$
For each shock, there are 12 cases sensitive to the degrees of congestion. The results show that there are only small differences in magnitude for the responses to both shocks in density and in infrastructure investment. However, the degrees of sensitivity of responses to density and to infrastructure investment are quite different. The impulse responses to infrastructure investment are more sensitive than the impulse responses to density.

2.7 Conclusion

This study applies a DSGE model to data from Thailand and uses Bayesian techniques to estimate the dynamic effects of congestion of public transport infrastructure from two sources, production activities and total population density for the Thai economy. Public transport infrastructure is congested more by population density than production activities. This indicates that public investments in transport infrastructure do not increase in a simple linear manner, but in a much more complicated manner.

The model includes optimizing consumers and firms, market clearing conditions in the factor markets and output market, policy rules for the government's choice of an income tax rate, a consumption tax rate, a transfer to consumers, and an investment in public infrastructure, as well as a government budget constraint that allows for government debt to balance the budget. The policy rules balance the budget in a long run sense, to maintain debt at a reasonable level, and to also be countercyclical.

We estimate the model using Thai data and study the impact of a number of policy experiments on the Thai economy including a temporary increase in population density, a temporary increase in public infrastructure, and temporary shocks to the income tax rate and the consumption tax rate. In general, both density and the effective stock of public infrastructure both respond in a significant manner. For example, an exogenous temporary increase in population density causes a significant decrease in output, consumption, labor supply, private investment, and effective public infrastructure. Our model predicts that it can be a number of years before the Thai economy recovers. This is an entirely new result to our
knowledge and has serious policy implications. We also show that a temporary increase in public infrastructure has the opposite impact on output, consumption, and labor supply; output, consumption, and labor supply all increase in response to a temporary increase in public infrastructure. This strongly implies that one sensible policy response is to increase public infrastructure spending when there is an increase in population density.
2.8 References


2.9 Appendix

Appendix A: Solving the model

A1. Equilibrium conditions

(1) Households

A representative household maximizes utility subject to household’s budget constraint and the law of motion for capital.

\[
\max_{\{c_t, l_t, b_t, k_t\}} u = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{\left[ \frac{l_t}{u_t} \right]^{1+\frac{1}{\psi}}}{1 + \frac{1}{\psi}} \right\}
\]

Subject to

\[
(1 + \tau^c_t)c_t + i_t + b_t = (1 - \tau^inc_t)(w_t l_t + \tau^k_t k_{t-1}) + R^b_{t-1} b_{t-1} + \tau r_t,
\]

\[
k_t = (1 - \delta)k_{t-1} + i_t.
\]

The first order conditions follow.

\[
\frac{L^i_t}{(u_t)^{1+\frac{1}{\psi}}} = \frac{(1 - \tau^inc_t)w_t}{(1 + \tau^c_t)c_t}, \quad (A_1)
\]

\[
\frac{1}{(1 + \tau^c_t)c_t} = \beta E_t \left\{ \frac{(1 - \delta) + (1 - \tau^{inc}_{t+1}) d_t^{K^{c+1}}}{(1 + \tau^{inc}_{t+1})c_{t+1}} \right\}, \quad (A_2)
\]

\[
\frac{1}{(1 + \tau^c_t)c_t} = \beta E_t \left\{ \frac{r^b_t}{(1 + \tau^{inc}_{t+1})c_{t+1}} \right\}. \quad (A_3)
\]

The capital accumulation follows.

\[
K_t = (1 - \delta)K_{t-1} + l_t. \quad (A_4)
\]
The household's budget constraint is
\[(1 + \tau_t)C_t + I_t + B_t = (1 - \tau_t^{inc})(w_tL_t + r_t^kK_{t-1}) + R_t^bB_{t-1} + TR_t.\] 
\(\text{(A5)}\)

(2) Aggregate production function

The aggregate production function is
\[y_t = (A_t)^{1-\alpha\theta}(k_{t-1})^\alpha (l_t)^{1-\alpha} (K_{g,t-1}^{cong})^{\alpha\theta},\] 
\(\text{(A6)}\)

where
\[K_{g,t-1}^{cong} = \frac{K_{g,t-1}}{(\psi(k_{t-1})\phi(l_t))\phi},\] 
\(\text{(A7)}\)

\[D = \alpha_d \left(\frac{y_t}{y_{t-1}}\right) U_t^d.\] 
\(\text{(A8)}\)

(3) Firms

The equilibrium conditions for a rental rate of capital and wage are
\[r_t^k = \alpha \frac{y_t}{k_{t-1}},\] 
and
\[w_t = (1 - \alpha) \frac{y_t}{l_t}.\] 
\(\text{(A9)}\)

(4) Government

The flow of government budget follows.
\[G_t^c + G_t^i + TR_t + R_t^bB_{t-1} = T_t^{inc} + T_t^c + B_t,\] 
\(\text{(A10)}\)

where
\[T_t^{inc} = \tau_t^{inc}Y_t,\] 
\(\text{(A11)}\)

\[T_t^c = \tau_t^cC_t,\] 
\(\text{(A12)}\)
Public infrastructure's accumulation follow.

\[ K_{g,t} = (1 - \delta)K_{g,t-1} + G_t^l \]  \hspace{1cm} (A13)

(5) Aggregate Resource constraint

\[ Y_t = C_t + I_t + G_t^c + G_t^l \]  \hspace{1cm} (A14)

A2. Equilibrium Conditions in Terms of Detrended Variables

The variables that are not detrended include \( L_t, R_t^b, \tau_t^{inc}, \tau_t^c \).

The variables that are detrended include \( C_t, Y_t, K_t, I_t, B_t, w_t, K_{g,t}, K_{g,t}^{cong}, G_t^c, G_t^c, \tau_t^c, T_t^c, TR_t \).

Let \( \bar{X}_t \) be a detrended variable. Thus, \( \bar{X}_t = \frac{x_t}{\lambda_t} \).

Equilibrium conditions

The equilibrium conditions (equations \( A_1 \) \( \rightarrow \) \( A_{14} \)) are written in terms of the detrended variables as the following.

\[ \frac{L_t^{1/v}}{(u_t^{1/v}) - Y_t} = \frac{(1 - \tau_t^{inc})g_t}{(1 + \tau_t^c)\lambda_t} \]  \hspace{1cm} (A1')

\[ \frac{1}{(1 + \tau_t^c)\lambda_t} = \beta E_t \left\{ \frac{(1 - \delta + (1 - \tau_t^{inc})\delta + \tau_{t+1}^c \lambda_{t+1}^a \lambda_{t+1}}{(1 + \tau_{t+1}^c)\lambda_{t+1}^a \lambda_{t+1}^a} \right\} \]  \hspace{1cm} (A2')

\[ \frac{1}{(1 + \tau_t^c)\lambda_t} = \beta E_t \left\{ \frac{R_t^b}{(1 + \tau_{t+1}^c)\lambda_{t+1}^a \lambda_{t+1}^a} \right\} \]  \hspace{1cm} (A3')

\[ \bar{R}_t = (1 - \delta)\bar{R}_{t-1} e^{-\alpha_t} + I_t. \]  \hspace{1cm} (A4')

\[ (1 + \tau^c)\lambda_t + \bar{R}_t - (1 - \delta)\bar{R}_{t-1} e^{-\alpha_t} + \bar{B}_t = (1 - \tau_t^{inc})(\bar{Y}_t) + R_{t-1}^b \bar{B}_{t-1} e^{-\alpha_t} + T\bar{R}_t \]  \hspace{1cm} (A5')

\[ \bar{Y}_t = (\bar{R}_{t-1} e^{-\alpha_t})^a (L_t)^{1-a} (\bar{R}_{g,t-1} e^{-\alpha_t})^{ag} \]  \hspace{1cm} (A6')
\[ R_{g,t-1}^{cong} = \frac{A_t^{-\sigma} K_{g,t-1}}{(d_t)^{\psi}(K_{t-1}e^{-\alpha_t})^{\psi}(u_t)^{\phi}} \]  
(A7')

\[ D_t = \alpha_d \left( \frac{\gamma e^{\alpha t}}{\gamma_t} \right) U_t^d \]  
(A8')

\[ \bar{w}_t = (1 - \alpha) \frac{\gamma_t}{l_t} \]  
(A9')

\[ \bar{c}_t + \bar{c}_i + \bar{p}R_t + R_{g,t-1}^b B_{t-1} e^{-\alpha_t} = \bar{t}_t^{inc} + \bar{c}_t + B_t, \]  
(A10')

\[ \bar{t}_t^{inc} = \tau_t^{inc} \frac{\gamma_t}{\gamma} \]  
(A11')

\[ \bar{c} = \tau_c \bar{c}_t \]  
(A12')

\[ \bar{R}_{g,t} = (1 - \delta_g) \bar{R}_{g,t-1} e^{-\alpha_t} + \bar{g}_t \]  
(A13')

\[ \bar{y}_t = \bar{c}_t + \bar{I}_t + \bar{c}_t^c + \bar{c}_t^i \]  
(A14')

**A.3: Steady State**

We calculate the steady-state values for variables used for estimation. These steady-state values including \( \tau^{inc}, \tau_c, \bar{c}^c, \bar{c}_t^i, \bar{p}, \bar{\gamma}, \) are obtained from the averages of the measured data. The other steady-state values are derived from the model specifications. There are

\[ R^b = \frac{\gamma}{\beta} \]

\[ \bar{y} = \frac{\gamma}{\beta} - (1 - \delta) \]

\[ \frac{\bar{y}}{\bar{R}} = \frac{(1 - \tau^{inc}) \gamma \alpha'}{(1 - \gamma)} \]

\[ \frac{\bar{f}}{\bar{R}} = \frac{\gamma(1-\delta)}{\gamma} \]

\[ \frac{\tau^{inc}}{\bar{R}} = \tau^{inc} \frac{\gamma}{\bar{R}} \]

\[ \frac{\bar{c}}{\bar{R}} = \frac{\gamma}{\bar{R}} \left( 1 - \frac{\bar{c}_t^c}{\bar{\gamma}} - \frac{\bar{c}_t^i}{\bar{\gamma}} \right) + \frac{\bar{f}}{\bar{R}} \]
\[
\frac{\tau^c}{K} = \tau^C \frac{c}{\bar{K}} ,
\]
\[
\frac{\tilde{G}^i}{K_g} = \frac{\gamma - (1 - \delta_g)}{\gamma},
\]
\[
\frac{\beta}{K} = \left( \frac{\beta}{\bar{V}} \right) \left( \frac{\bar{V}}{\bar{K}} \right),
\]
\[
\frac{\tilde{G}^c}{K} = \left( \frac{\tilde{G}^c}{\bar{V}} \right) \left( \frac{\bar{V}}{\bar{K}} \right),
\]
\[
\frac{\tilde{c}^i}{K} = \left( \frac{\tilde{c}^i}{\bar{V}} \right) \left( \frac{\bar{V}}{\bar{K}} \right),
\]
\[
\frac{\tau_R}{K} = (1 + \tau^C) \frac{c}{K} + \frac{f}{K} + \left( \frac{\beta - 1}{\beta} \right) \left( \frac{\beta}{\bar{V}} \right) \left( \frac{\bar{V}}{\bar{K}} \right) - (1 - \tau^{inc}) \left( \frac{\bar{V}}{\bar{K}} \right).
\]

**A.4: Log–Linearized System of Equations**

Equations (A1‘) – (A14‘) are log-linearized as the following.

\[
\frac{1}{v} \dot{\lambda}_t - \left( 1 + \frac{1}{v} \right) \lambda^l_t = \tilde{\omega}_t - \frac{\tau^{inc}}{1 - \tau^{inc}} \tilde{x}^{inc}_t - \frac{\tau^c}{1 + \tau^c} \tilde{x}^c_t - \lambda_t \tag{A1''}
\]

\[
\frac{\tau^c}{1 + \tau^c} \tilde{x}^c_t + \lambda_t + \left[ \frac{(1 - \tau^{inc})\alpha \gamma \frac{\bar{V}}{\bar{K}}}{(1 - \delta) + (1 - \tau^{inc})\alpha \gamma} \right] \left[ E_t \left( \hat{\lambda}_{t+1} - \frac{\tau^{inc}}{1 - \tau^{inc}} \tilde{x}^{inc}_{t+1} + \tilde{\alpha}_{t+1} \right) - \tilde{K}_t \right] = \frac{\tau^c}{1 + \tau^c} E_t \tilde{x}^c_{t+1} + E_t \lambda_{t+1} + E_t \tilde{\alpha}_{t+1} \tag{A2''}
\]

\[
\frac{\tau^c}{1 + \tau^c} \tilde{x}^c_t + \lambda_t = \frac{\tau^c}{1 + \tau^c} E_t \tilde{x}^c_{t+1} + E_t \lambda_{t+1} + E_t \tilde{\alpha}_{t+1} - \tilde{R}_t \tag{A3''}
\]

\[
\tilde{K}_t = \left( \frac{1 - \delta}{\gamma} \right) \left( \tilde{K}_{t-1} - \tilde{\alpha}_t \right) + \frac{f}{\bar{K}} \tilde{\eta}_t \tag{A4''}
\]

\[
\dot{\gamma}_t = \alpha \left( \tilde{K}_{t-1} - \tilde{\alpha}_t \right) + (1 - \alpha) \dot{\lambda}_t + \alpha_g \left( \tilde{K}^{cong}_{g,t-1} - \tilde{\alpha}_t \right) \tag{A5'}
\]

\[
\tilde{K}^{cong}_{g,t-1} = \tilde{K}_{g,t-1} - \psi \dot{\alpha}_t - \varphi \left( \tilde{K}_{t-1} - \tilde{\alpha}_t \right) - \phi \dot{\lambda}_t - \varphi \dot{\alpha}_t \tag{A6''}
\]
\( \hat{a}_t = y_t - y_{t-1} + u_{t}^{d} \)  

(A7"")

\( \omega_t = y_t - \ell_t \)  

(A8"")

\( \frac{c^c}{R} g_t^c + \frac{c^i}{R} g_t^i + \frac{r}{R} \ell_t + \frac{1}{\beta R} (\hat{K}_{t-1} + \hat{b}_{t-1} - \hat{a}_t) = \frac{\tau^{inc}}{R} t_t^{inc} + \frac{\tau^{c}}{R} c_t^c + \frac{\beta}{R} b_t \)  

(A9"")

\( \hat{t}_t^{inc} = \hat{t}_t^{inc} + \gamma_t \)  

(A10"")

\( \hat{t}_t^{c} = \hat{t}_t^{c} + \beta_t^{c} \)  

(A11"")

\( \hat{k}_{g,t} = \left( \frac{1-\delta_g}{\gamma} \right) (\hat{k}_{g,t-1} - \hat{a}_t) + \frac{g^i}{R_g} g_t^i \)  

(A12"")

\( \frac{y}{R} y_t = \frac{c}{R} c_t + \frac{1}{R} \ell_t + \frac{c^c}{R} g_t^c + \frac{c^i}{R} g_t^i \)  

(A13"")

Fiscal Rules

\( g_t^i = \omega g_i g_{t-1}^i - \eta_{gi} y_t + u_t^{gi} \)  

(A14"")

\( g_t^c = \omega g_c g_{t-1}^c - \eta_{gc} y_t + u_t^{gc} \)  

(A15"")

\( \hat{t}_t^{inc} = \omega_{tinc} t_{t-1}^{inc} + \gamma_{tinc} \hat{b}_{t-1} + u_t^{inc} \)  

(A16"")

\( \hat{t}_t^{c} = \omega_{tinc} c_{t-1}^{inc} + \gamma_{tinc} \hat{b}_{t-1} + u_t^{inc} \)  

(A17"")

\( \hat{r}_t = \omega_{tr} \hat{r}_{t-1} - \eta_{tr} y_t + u_t^{tr} \)  

(A18"")

Technology shock: \( \hat{A}_t = \rho_A \hat{A}_{t-1} + \sigma_A e^{\hat{A}}_t \)  

(A19"")

Labor supply shock: \( u_t^l = \rho_l u_{t-1}^l + \sigma_l e^{\hat{A}}_t \)  

(A20"")

Government consumption shock: \( u_t^{gc} = \rho_{gc} u_{t-1}^{gc} + \sigma_{gc} e^{\hat{A}}_t \)  

(A21"")

Government investment in transport infrastructure shock: \( u_t^{gi} = \rho_{gi} u_{t-1}^{gi} + \sigma_{gi} e^{\hat{A}}_t \)  

(A22"")

Income tax shock: \( u_t^{inc} = \rho_{inc} u_{t-1}^{inc} + \sigma_{inc} e^{\hat{A}}_t \)  

(A23"
Consumption tax shock: \( u^c_t = \rho_{tc} u^c_{t-1} + \sigma_{tc} \epsilon^c_t \)  

(A24'')

Transfer shock: \( u^{tr}_t = \rho_{tr} u^{tr}_{t-1} + \sigma_{tr} \epsilon^{tr}_t \)  

(A25'')

Density shock: \( u^d_t = \rho_d u^d_{t-1} + \sigma_d \epsilon^d_t \)  

(A26'')

The process \( a_t: \hat{a}_t = \hat{A}_t - \hat{A}_{t-1} \)  

(A27'')

The model contains 27 unknowns and 27 equations consisting of endogenous and exogenous variables as the following.

Endogenous variables (18) are \( L, C, Y, K, I, B, D, K^c_{g cong}, K_g, G^i, G^c, T^{inc}, T^c, T^{inc}, \tau^c, TR, w, R^b \).

Exogenous variables: shocks (8+1) are \( A, u^i, u^{gc}, u^{r inc}, u^{r}, u^{tr}, u^d \), and the process \( a_t \).

The model parameters (36) are

\[
\Theta = \{ \gamma, \beta, \delta, \alpha_i, \alpha_g, \psi, \phi, \delta_i, \eta_{gi}, \eta_{gc}, \eta_{tr}, \gamma_{rinc}, \gamma_{tc}, \omega_{gi}, \omega_{gc}, \omega_{r}, \omega_{rinc}, \omega_{tc} \} \\
\rho, \rho_A, \rho_{gc}, \rho_{gi}, \rho_{rinc}, \rho_{tc}, \rho_{tr}, \rho_d, \sigma_{A}, \sigma_{gc}, \sigma_{gi}, \sigma_{rinc}, \sigma_{tc}, \sigma_{tr}, \sigma_d \}
\]
Appendix B. Estimation Results

Figure 8. Mode Check Plots
Figure 9. Prior and Posterior
Orthogonalized Shocks

Figure 10. Estimated impulse responses to a 1 percent increase in labor supply

Figure 11. Estimated impulse responses to a 1 percent increase in transfer payment
CHAPTER THREE
SUBSTITUTION ELASTICITIES BETWEEN PUBLIC R&D AND HETEROGENOUS LABOR

3.1 Introduction

This study is aimed at analyzing the effects of public R&D on differentiated labor for the US economy compared to the Thailand economy. We study four kinds of labor: male skilled labor, female skilled labor, male unskilled labor, and female unskilled labor working for final goods sector and for intermediate goods sector including R&D and capital varieties productions. We focus on R&D production and gender labor. Two sectors of production are analyzed in which the final goods firms produce homogenous final output to the economy, and intermediate goods firms invest in R&D knowledge and utilize public R&D to produce blueprints and then produce variety of capital.

As evidence of the ratios of female to male labor participation rate of the United States and The Thailand economies during the 1990s and 2010s, the participation rate in the United States has increased from 75% to 82%, while that of the Thailand has declined from 87% to 80%. The wages of low-skilled workers have fallen relative to those of high-skilled workers in both economies. The most widely cited explanation for the rise in wage inequality is skill-biased technical change. Research and development enhances new innovation and this technology affects the labor market in terms of the participation rate ratio of male and female workers, and wage inequality.

The model of this study consists of two levels of production. Intermediate goods production uses public R&D, private investment in machines, and skilled labor inputs to produce varieties of capital. Final goods production demands these capital varieties to produce homogenous final outputs by using unskilled workers. This model allows for three channels of influence of public R&D on employment: 1) direct complementary or substitution effects in the R&D sector, 2) indirect effects caused by the demand in capital varieties for final goods production, and 3) indirect effects on wages due to changes in productivity.
We examine the elasticities of substitution between technology derived from increasing public R&D spending and demands for heterogeneous labor inputs to answer our research questions. Is public R&D a complement or substitute for skilled male and female labor? Does the impact of public R&D affect the inputs in the same way in a developing country versus a developed country?

The contributions to the literature of this paper should be emphasized in three dimensions. First, we extend the model of two sectors production by incorporating gender and skilled labor inputs with public R&D and estimate the elasticities of substitution between such inputs. Second, the results provide policy recommendations in reducing wage inequality and improvement in labor market. Third, we compare the effects of public R&D on skilled and unskilled labor inputs between developed and developing countries to analyze the matter of skilled bias technology.

We find that for the R&D sector, the elasticities of substitution between the two inputs of the two nested levels for the Thai economy and for the United States are the same in terms of complementarity and substitution. The estimates for public R&D and skilled male and female inputs are negative meaning that they are complements. Skilled labor increases with public R&D spending. Both male and female labor are complementarity to public R&D. For the final goods sector, all the inputs are substitutes for the Thai economy but the for the United States, unskilled female and variety capital are complements. Final goods production uses capital varieties derived from production of public R&D sector, and male and female unskilled workers. Unskilled male and female workers can be substitutes as the process does not require specialty or skills that are related to gender in Thailand.

The remainder of the study is organized as follows: in section 2 we review the related literature; in section 3 we explain the theoretical framework including two sectors of production; in section 4 we present the data construction; in section 5 we present the elasticity of substitution for a CES production function with three and four inputs; in section 6, we estimate and report the estimation results; and finally, in section 7, we wrap up our main conclusions.
3.2 Literature Review

Romer (1990) examines theoretically how technological change that arises from investment decisions made by profit-maximizing agents affects growth. The economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector. A research sector uses human capital and the existing stock of knowledge to produce new knowledge. An intermediate-goods sector uses the designs from the research sector together with forgone output to produce the large number of producer durables. A final-goods sector uses labor, human capital, and the set of producer durables that are available to produce final output. The study shows that an economy with a larger total stock of human capital will experience faster growth.

Acemoglu (2009) employs the lab-equipment model: the endogenous growth model with expanding varieties of inputs and an R&D technology that only uses output for creating new inputs. Only investment in equipment is required for research, rather than the employment of skilled or unskilled workers or scientists. The final good firm uses labor and varieties of machines produced by the intermediate good firms. While each monopolist as an intermediate good firm invents new machine variety via spending on that new machine and on R&D to satisfy the demand for that machine derived from the final good firm’s profit maximizing.

Freire-Serén (2000) analyzes the impacts of aggregate R&D expenditure on long-run economic growth. The government plays a role in subsidizing the physical capital accumulation by reducing the production cost of the intermediate goods. This study develops R&D based model of economic growth, where the core economy consists of differentiated capital stock expanded by R&D expenditure, a private input the consumers choose. The R&D technology assumes (1) the productivity of R&D expenditure positively depends on the level of ideas, and (2) the aggregate level of the R&D expenditure drives the marginal productivity of the R&D expenditure at the firm level. This study uses Cobb-Douglas production function consisting of human capital, labor force, and variety of capital. Intermediate goods sector purchases R&D
technology and assumes the capital good is equal to consumers’ savings. This paper uses pooled cross-section data for a sample of 21 OECD countries with five observations for each country, which correspond to the time period 1965-1990 at five-year intervals to estimate the parameters. The results show that the estimated coefficients of human capital, physical capital and R&D-expenditure are positive and significant. Concerning R&D-activity, the estimation shows a strong positive correlation between the growth of total R&D-expenditure and the growth of the GDP.

Voyvoda and Yeldan (2015) study the productivity growth of the Canadian economy using a dynamic general equilibrium model driven by analytics of endogenous growth and investigate the viable policy options and assess the interactions between knowledge driven growth, acquisition of human capital, and the role of strategic public policy. They analyze alternative public policies aimed at fostering the development of human capital (investment in education) and those at enhancing investments in innovation (capital varieties). Based on calibration to the Canadian economy, the results show that Canadian economy is falling short of its potential in (business) technological innovation and the most welfare enhancing policy is to have a complementary mix of education and R&D subsidization designed to avoid the trade-offs that emerge in the short run.

Zachariadis (2003) studies the impact of R&D on economic growth by applying Schumpeterian endogenous growth framework without scale effects. This paper estimates a system of three equations implied by a model of R&D-induced growth in steady state. These equations relate R&D intensity to patenting, patenting to technological progress, and technological progress to economic growth. By using a panel of 10 U.S. manufacturing industries during the period 1963–88, and instrumenting the contemporaneous explanatory variables using their lagged values and applying three-stage least squares, the results show that R&D intensity has a positive impact on the rate of patenting. The rate of patenting is then shown to drive technological progress which in turn drives the growth rate of output per worker. Moreover, the evidence points to technological spillovers from aggregate research intensity to industry-level innovation success.
Kristkova (2013) analyses the Private R&D Effects in a CGE Model with Capital Varieties on economic growth by applying a Computable General Equilibrium (CGE) model following Romer’s theory of endogenous growth. He models R&D activity as the form of two specific R&D production sectors including private and public R&D. The private R&D sector is assumed to represents the research efforts of private businesses to produce new designs. The companies involved in private R&D operate in an environment of monopolistic competition i.e. each R&D firm produces a different design and therefore a different variety of capital. The results show that the dynamics of GDP growth are positively related to the production of capital varieties and the elasticity of substitution between homogenous and variety capital.

Archibald and Pereira (2003) analyze the effects of public and private research and development (R&D) on private-sector output, employment, and investment using a multivariate time-series approach. The analysis is based on the impulse response function results associated with a five-variable vector auto-regressive (VAR) system. They find that in the long term, public R&D does not affect employment but crowds in private investment, whereas private R&D is a substitute for both.

Gómez and Sequeira (2014) develop an endogenous growth model with R&D, physical capital, and human capital with several externalities. They assume three production sectors in the economy: a competitive final-good sector, a monopolistic intermediate-goods sector, and a competitive R&D sector. The government sector is also included in which income, consumption, and firm’s profits are taxed. The revenue raised is used to subsidize intermediate-goods production costs, R&D costs and education costs, as well as to provide lump-sum transfers to agents representing welfare programmes. The model is calibrated to the US economy and used to quantitatively evaluate the effect on growth and welfare of implementing different budget-neutral policies. They find that subsidies to research are the most welfare-increasing amongst the budget-neutral policies.
3.3 Theoretical Framework

We apply the expanding verity model of growth framework (Acemoglu, 2009) to incorporate heterogenous labor inputs in order to examine whether skilled male and female labor are complements or substitutes for R&D, and whether unskilled male and unskilled female labor are complements or substitutes for capital. The economy consists of two sectors of production: R&D sector and final goods production. There are labor inputs differentiated by skill and gender, which include skilled male labor \( H^m_1 \), skilled female labor \( H^f_1 \), unskilled male labor \( L^m_1 \), and unskilled female labor \( L^f_1 \). We assume without loss of generality that households are identical in that each household supplies four types of labor inputs. Therefore, the economy admits a representative household. Population is constant, and labor is supplied elastically.

The theoretical model of study includes a final goods sector and a R&D sector that produces innovation leading to capital improvement that increases the varieties of capital as intermediate goods for a final goods production. The innovation leads to an improvement to capital implying an increase in productivity of capital for output sector. We use this procedure to illustrate how this productivity is derived and how it enters the production function as new efficient capital. This term is defined so that it accounts for the varieties of capital in which we expect to increase productivity explicitly by the number of varieties.

3.3.1 Final Goods Sector

In this section, we show how capital variety is derived from final goods production, and show that the economy obtains the same optimal level of each variety given some assumptions. There is a number of machine varieties or capital varieties \( k_t(i), i = 1, ..., N_t \) used in the production of a homogenous final good. Final goods firms are identical. A representative final goods firm employs unskilled male labor \( l^m_t \), unskilled female labor \( l^f_t \), and varieties of capital \( k_t(i) \) to produce a homogenous final good.

The study applies two-level nested Constant Elasticity of Substitution (CES) function proposed by Sato (1967) and Henningsen (2011). This production function increased in popularity in the field of macro-
econometrics, where input factors needed further differentiation. We use the CES function as a generalization of the Cobb-Douglas function to allow for any constant elasticity of substitution. Since all final goods firms have the same technology to produce a homogenous good, the aggregate production function takes the form

\[
Y_t = \left\{ \alpha_t \left[ \alpha_{tm} (L_t^m)^\mu + (1 - \alpha_{lm})(L_t^f)^\mu \right]^\frac{\mu}{\rho} + (1 - \alpha_t) \sum_{i=1}^{N_t} (k_t(i))^{\rho} \right\}^{\frac{1}{\rho}},
\]

(1)

where \( \alpha_t, \alpha_{tm} \in [0,1] \) determine the optimal distribution of the inputs, \( N_t \) is the index of varieties of capital input available to this economy at time \( t \), and \( \rho, \mu \in [-1,0) \cup (0, \infty) \) are the parameters that determine the elasticity of substitution. Equation (1) has constant returns to scale over all inputs.

A representative final goods firm maximizes its profit by choosing the levels of \( k_t(i), L_t^m, \) and \( L_t^f \).

\[ \max_{k_t(i), L_t^m, L_t^f} \left\{ \alpha_t \left[ \alpha_{tm} (L_t^m)^\mu + (1 - \alpha_{lm})(L_t^f)^\mu \right]^\frac{\mu}{\rho} + (1 - \alpha_t) \sum_{i=1}^{N_t} (k_t(i))^{\rho} \right\}^{\frac{1}{\rho}} - \sum_{i=1}^{N_t} p_t^k(k_t(i)) - w_t^m L_t^m - w_t^f L_t^f \]

(2)

where \( p_t^k(i) \) refers to the price of machine of variety \( i \), and \( w_t^m, w_t^f \) are the wages for unskilled male and female labor, and the price of the final good is normalized to be one.

The first order conditions follow.

\[ (Y_t)^{1-\rho} \cdot \alpha_t \left[ \alpha_{tm} (L_t^m)^\mu + (1 - \alpha_{lm})(L_t^f)^\mu \right]^\frac{\rho-\mu}{\mu} \alpha_{tm} (L_t^m)^{\mu-1} = w_t^m \]

(3)

\[ (Y_t)^{1-\rho} \cdot \alpha_t \left[ \alpha_{tm} (L_t^m)^\mu + (1 - \alpha_{lm})(L_t^f)^\mu \right]^\frac{\rho-\mu}{\mu} (1 - \alpha_{tm})(L_t^f)^{\mu-1} = w_t^f \]

(4)
\[(Y_t)^{1-p} \cdot (1 - \alpha_t)k_t(i)^{p-1} = p_t^k(i)\]  \hspace{1cm} (5)

The first order conditions above give the ratio for labor demand,

\[
\frac{L^f_t}{L^m_t} = \left( \frac{\alpha_{lm} w^f_t}{(1-\alpha_{lm}) w^m_t} \right)^{1/\mu}. \hspace{1cm} (6)
\]

Thus, the ratio of labor demanded also depends on the substitution parameter between two types of labor, $\mu$. The higher the value of $\mu$, the greater the percentage change in the labor ratio.

The first order condition for $k_t(i)$ is

\[
k_t(i) = \left[ \frac{\gamma p (1-\alpha_t)}{k(i)} \right]^{1/1-p} Y_t. \hspace{1cm} (7)
\]

Equation (7) shows that the demand for variety is differentiated by its price in equilibrium. We do not explicitly solve $k_t$ to be a function of only prices of inputs and parameters, but we will have the aggregate output, $Y_t$, as an index in that the optimal capital is demanded as a function of the aggregate output and the price of capital variety, $p_t^k(i)$.

### 3.3.2 R&D Sector

The engine of productivity here is process innovation that leads to capital varieties. We follow the theoretical concepts of the R&D input, and the technological progress that arises from increasing in varieties of capital. First, Aghion and Howitt (1998, 418) suggest that R&D intensity, rather than the R&D stock or the number of scientists and engineers, is the proper measure for the R&D input in the innovation function within the context of the endogenous growth model without scale effects we consider here. R&D is considered to be an input in the process innovation and that bring about gains in productivity.
Capital is modeled here as a heterogeneous input which accumulates the varieties, $k_{t}(i)$. The intermediate firm in R&D sector invests in variety and obtains ‘blueprints'\(^7\). And according to the instructions therein, produces a new capital variety. The number of new capital varieties produced at period $t$ is equal to the number of new blueprints produced in the same period. Following Acemoglu (2009), we assume that $k$ depreciate fully after use; thus, they can be interpreted as generic inputs, intermediate goods, machines, or even capital. This assumption simplifies the exposition considerably. The number of the capital varieties in the economy at time $t$ is equal to the number of blueprints available in the economy.

Each intermediate goods firm is a monopolist who uses public R&D expenditure ($G_{t}^{rd}$), spending on machines ($i_{t}(i)$), scientists and other skilled workers which are defined as skilled male labor ($h_{t}^{m}(i)$), and skilled female labor ($h_{t}^{f}(i)$) to produce capital variety $i$. Thus, we start working with individual firm’s equilibrium conditions. Then we aggregate to use aggregate data for estimation.

Each monopolist holds the patent for machine variety $i$. The production structure on the capital varieties follows a nested CES function, where different production factors are combined to create varities of capital. The output flow of capital variety $i$ in period $t$ is given by

\[^7\] We can also define separated sectors of one for intermediate goods, and one for R&D. The R&D sector sell blueprints to intermediate firms. The R&D sector is competitive. Researchers produce blueprints. Blueprints are protected by perpetual patents. Innovators auction their blueprints to a large number of potential buyers, thus absorbing all the profits of the intermediate good sector. But there is free entry in the R&D sector, which drive net profits in that sector to zero.
\[ k_t(i) = \left\{ \alpha_v \left[ \alpha_{rd} (G_t^{rd})^\theta + (1 - \alpha_{rd}) (i_t(i))^{\phi} \right]_{\phi} + (1 - \alpha_v) \left[ \alpha_{hm} (h_t^m (i))^{\pi} + (1 - \alpha_{hm}) (h_t^f (i))^{\pi} \right]_{\pi} \right\}^{\frac{1}{\rho}}, \]  

where \( \alpha_v, \alpha_{rd}, \alpha_{hm} \in [0,1] \) determine the optimal distribution of the inputs, and \( \theta, \varphi, \pi \in [-1,0) \cup (0, \infty) \) are the parameters that determine the elasticities of substitution between inputs.

The capital is obtained as a valued added capital due to varieties created in the two-level nested function. The lower level of the production function includes two bundles: public R&D together with private investment are bundled with the parameter determining elasticity of substitution of \( \theta \), and skilled male and female labor are bundled with the parameter determining elasticity of substitution of \( \pi \). The upper level is the composite bundle of the two bundles of the lower level.

We assume that all firms are symmetric and they all set the same price to sell their respective capital varieties. That is we set \( p_t^k (i) = p_t^k \) and \( k_t (i) = k_t \) \( \forall i, t \). Under these conditions the final good production function (eq.(1)) becomes

\[ Y_t = \left\{ \alpha_t \left[ \alpha_{lm} (L_t^m)^\mu + (1 - \alpha_{lm}) (L_t^f)^\mu \right]_{\mu}^{\frac{1}{\rho}} + (1 - \alpha_t) N_t (K_t)^{\rho} \right\}^{\frac{1}{\rho}}. \]  

In period \( t \), there is the total amount of varities \( K_t \), and the patents granted in period \( t \) is \( N_t \). With the symmetry of intermediate firms assumed above, we then rewrite

\[ k_t = \frac{K_t}{N_t}, \]

and plug this into equation (8), the final goods production function in aggregate term is

\[ Y_t = \left\{ \alpha_t \left[ \alpha_{lm} (L_t^m)^\mu + (1 - \alpha_{lm}) (L_t^f)^\mu \right]_{\mu}^{\frac{1}{\rho}} + (1 - \alpha_t) (N_t)^{1-\rho} (K_t)^{\rho} \right\}^{\frac{1}{\rho}}. \]
This production function implies that the number of variety of capital affects output. We derive \( N_t \) in order to adjust the final goods production function to also account for the productivity of variety of capital. Thus, we adjust equation (10) to allow for the increasing returns to scale of the entire economy. By defining a proxy for variety capital variable, the estimation equation is

\[
Y_t = \left\{ \alpha_t \left[ \alpha \left( L_t^m \right)^\mu + (1 - \alpha \left( L_t^f \right)^\mu \right]^{\frac{\rho}{\mu}} + (1 - \alpha_t) (R_t)^{\frac{\omega}{\rho}} \right\}^{\frac{1}{\omega}},
\]

where \( R_t \) is the total variety of capital, and \( \omega \) is the return to scale parameter.

As we assume that each firm charges the same price, the demand for each firm is also the same. Consequently, investment and labor employed by each firm are also the same. Hence, if the total R&D is \( G_{t}^d \), total investment is \( I_t \), total skilled male and female labor inputs are \( H^m_t \) and \( H^f_t \), we have

\[
k_t = \frac{K_t}{N_t} = \left\{ \alpha_y \left[ \alpha_{rd} \left( \frac{G_{t}^d}{N_t} \right)^{\theta} + (1 - \alpha_{rd}) \left( \frac{I_t}{N_t} \right)^{\theta} \right]^{\frac{\phi}{\theta}} + (1 - \alpha_y) \left[ \alpha_{hm} \left( \frac{H^m_t}{N_t} \right)^{\pi} + (1 - \alpha_{hm}) \left( \frac{H^f_t}{N_t} \right)^{\pi} \right]^{\frac{1}{\phi}} \right\}^{\frac{1}{\phi}}.
\]

### 3.4 Data Construction

Only new (flow) variety capital is productive in that even if it still exists in the actual data it is no more productive after one period use (fully depreciated). Thus we use the flow data of variety capital.

We define the following variables for estimation and to be matched to the actual data.

\( \bar{K}_t \) is the sum of equipment and intellectual property products from Table 1.1. Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods for the U.S. data. This is a stock of variety. Thus we obtain the flow of variety from

\[
K_t = K_t^{stock} - (1 - \delta_k) K_{t-1}^{stock},
\]

and we use the depreciation rate \( \delta_k \) of 0.02.
$N_t$ is the number of variety capital, which is approximated by the number of patents granted in period $t$.

$I_t$ is the total investment in variety capital, which is the total investment in equipment and intellectual property products (Table 5.3.5. Private Fixed Investment by Type, from Bureau of Economic Analysis).

We use the data in aggregate terms for all variables including GDP, government investment in R&D, patents, capital stock, investment, skilled male workers, skilled female workers, unskilled male workers, unskilled female workers.

### 3.4.1 The U.S. Data

The US time series annual data from 1992 to 2015 were collected and constructed as described above.

Skilled and unskilled labor

Skilled and unskilled labor is disaggregated by the level of education. Skilled labor is the employment level of workers who are 25 years and over with a bachelor degree and over. Unskilled labor is derived from employment level of workers with some college or associate degree, high school graduates, no college, and less than a high school diploma with 25 years & over. The skilled workers are employed for immediate goods production while unskilled workers work for final goods production. These data were collected from the Bureau of Labor Statistics (BLS).

Hours worked for male (female) labor:

$$L = \frac{\text{Hours} \times \text{Emp}}{100}$$

Hours = non farm business, all male (female) persons, average weekly hours duration

Index: 1922=100, seasonally adjusted.
Emp = civilian employment: male (female) 25 years & over, measured in thousands, seasonally adjusted.

The government R&D was collected from the Bureau of Economic Analysis.

The available patents data consist of patents granted allocated in the year in which the application was filed with the U.S. Patent and Trademark Office.

3.4.2 The Thai data

We collected Thailand time series annual data from 2001 to 2015 including GDP, government investment in R&D, patents, capital stock, investment, skilled male workers, skilled female workers, unskilled male workers, unskilled female workers. The GDP data was taken from GFMIS, Ministry of Finance of Thailand.

Skilled and unskilled labor is disaggregated by the level of education. Skilled labor is the employment level of workers who are 15 years and over with bachelor degree and over. Unskilled labor is derived from employment level of workers with the degree lower than bachelor. The employment levels were obtained from the survey of employed persons by the National Statistical Office, Ministry of Information and Communication Technology.

Government investment in R&D and the number of patents were collected the Office of the National Research Council of Thailand. Capital stock and investment were collected from the Office of the National Economic and Social Development Board.

3.5 Estimation

We use a nonlinear least square estimator to estimate the parameters of the model for the U.S. economy and the Thai economy. In order to examine substitution or complementarity between inputs, we calculate the substitution elasticities which can be recovered from the estimated coefficients. We use the formula for
calculating the Hicks-Mcfadden elasticity of substitution for three-inputs and four-inputs nested CES functions (Sato (1967) and Henningsen (2011)).

(1) Three-inputs nested CES functions,

\[ Y_t = \gamma \left\{ \delta [\delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1}]^{\frac{\rho}{\rho_1}} + (1 - \delta) (X_3)^{-\rho} \right\}^{\frac{1}{\rho}} \]

Hicks-Mcfadden elasticity of substitution is given by

\[ \sigma_{c,k} = \left\{ \frac{1}{(1 - \rho_1) \left( \frac{1}{\theta_c} - \frac{1}{\theta} \right)} + (1 - \rho_2) \left( \frac{1}{\theta_k} - \frac{1}{\theta} \right) + (1 - \rho) \left( \frac{1}{\theta^*} - \frac{1}{\theta} \right) \right\}^{-1}, \quad \text{for } c = 1, 2; k = 3 \]

\[ (1 - \rho_1)^{-1}, \quad \text{for } c = 1; k = 2 \]

with

\[ \theta^* = \delta [\delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1}]^{\frac{\rho}{\rho_1}} (Y_t)^{\rho} \]

\[ \theta = (1 - \delta) (X_3)^{-\rho} (Y_t)^{\rho} \]

\[ \theta_1 = \delta \delta_1 (X_1)^{-\rho_1} [\delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1}]^{\frac{\rho_1 - \rho}{\rho_1}} (Y_t)^{\rho} \]

\[ \theta_2 = \delta (1 - \delta_1) (X_2)^{-\rho_1} [\delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1}]^{\frac{\rho_1 - \rho}{\rho_1}} (Y_t)^{\rho} \]

\[ \theta_3 = \theta \]

(2) Four-inputs nested CES functions

First, let rewrite the two-level CES functions in terms of a function of four inputs \( X_1, X_2, X_3, X_4 \),

\[ Y_t = \gamma \left\{ \delta [\delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1}]^{\frac{\rho}{\rho_1}} + (1 - \delta) [\delta_2 (X_3)^{-\rho_2} (1 - \delta_2) (X_4)^{-\rho_2}]^{\frac{\rho}{\rho_2}} \right\}^{\frac{1}{\rho}} \]

The elasticity of substitution is given by
\[ \sigma_{c,k} = \begin{cases} 
\frac{1}{\theta_c} + \frac{1}{\theta_k} & (1 - \rho_1) \left( \frac{1}{\theta_c} - \frac{1}{\theta_k} \right) + (1 - \rho_2) \left( \frac{1}{\theta_k} - \frac{1}{\theta_k} \right) + (1 - \rho) \left( \frac{1}{\theta_k} - \frac{1}{\theta_k} \right), 
\text{for } c = 1,2; k = 3,4, \\
(1 - \rho_1)^{-1}, & \text{for } c = 1; k = 2 \\
(1 - \rho_2)^{-1}, & \text{for } c = 3; k = 4 
\end{cases} \]

with

\[ \theta^* = \delta \left[ \delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1} \right] \frac{\rho}{\rho_1} (Y_t)^\rho \]

\[ \theta = (1 - \delta) \left[ \delta_2 (X_3)^{-\rho_2} (1 - \delta_2) (X_4)^{-\rho_2} \right] \frac{\rho}{\rho_2} (Y_t)^\rho \]

\[ \theta_1 = \delta \delta_1 (X_1)^{-\rho_1} \left[ \delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1} \right] \frac{\rho_1 - \rho}{\rho_1} (Y_t)^\rho \]

\[ \theta_2 = \delta (1 - \delta_1) (X_2)^{-\rho_1} \left[ \delta_1 (X_1)^{-\rho_1} + (1 - \delta_1) (X_2)^{-\rho_1} \right] \frac{\rho_1 - \rho}{\rho_1} (Y_t)^\rho \]

\[ \theta_3 = (1 - \delta) \delta_2 (X_3)^{-\rho_2} \left[ \delta_2 (X_3)^{-\rho_2} + (1 - \delta_2) (X_4)^{-\rho_2} \right] \frac{\rho_2 - \rho}{\rho_2} (Y_t)^\rho \]

\[ \theta_4 = (1 - \delta)(1 - \delta_2) (X_4)^{-\rho_2} \left[ \delta_2 (X_3)^{-\rho_2} + (1 - \delta_2) (X_4)^{-\rho_2} \right] \frac{\rho_2 - \rho}{\rho_2} (Y_t)^\rho \]

Then we determine the substitutes or complementarity of each pair of inputs for two sectors of production following these criteria.

if \( \sigma_{c,k} > 0 \) \( \Rightarrow \) \( X_c \) and \( X_k \) are substitutes

if \( \sigma_{c,k} < 0 \) \( \Rightarrow \) \( X_c \) and \( X_k \) are complements

The parameters that are not estimated are contained in the following Table.
Table 1. Parameter calibration for R&D sector.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public R&amp;D/Private investment Coeff.</td>
<td>$\alpha_v$</td>
<td>0.60 0.60</td>
</tr>
<tr>
<td>Public R&amp;D Coeff.</td>
<td>$\alpha_{rd}$</td>
<td>0.50 0.50</td>
</tr>
<tr>
<td>Skilled male labor Coeff.</td>
<td>$\alpha_{hm}$</td>
<td>0.60 0.55</td>
</tr>
</tbody>
</table>

We assume that public R&D and private investment in capital variety give the same contribution to valued added capital variety. Thus $\alpha_{rd} = 0.5$. For the distribution parameters $\alpha_v$, and $\alpha_{hm}$, the calibration is derived from these ratios,

$$\frac{\alpha_v}{1-\alpha_v} = \text{capital income/skilled labor income},$$

and

$$\frac{\alpha_{hm}}{1-\alpha_{hm}} = \text{skilled male labor income/skilled female labor income}.$$

3.6 Results

We estimate the elasticities of substitution between production inputs for two sectors of production. The final goods production and the intermediate goods production give the different elasticities of substitution and there are large differences between two economies for estimated elasticities of substitution and the parameters of the model.

3.6.1 Final Goods Sector

The estimates of parameters and elasticities of substitution for the final goods production function (eq.11) for both the U.S. and the Thai economy are shown in the table 2 and 3 respectively. All parameters estimated...
except for the determinant for elasticity of substitution between unskilled labor and variety of capital ($\rho$) are very close to each other and the estimates are about unity in the two countries.

The estimate also shows the increasing return to scale for both economies with the estimates of 1.3675 and 1.3048 in the US and Thailand respectively. This are consistent with the endogenous growth model with capital variety produces an increasing return to scale for an aggregate economy derived from increasing in the number of verities. Thus, public R&D contributes to the economy in two directions. First, it increases marginal products skilled male and skilled female labor. This means the wage income also increases. Second, As the variety capital enters into the final goods production function, even if the final goods firms do not observe these varieties, the variety increases productivity and thus increases marginal product of unskilled labor as well.
Table 2: Estimates of parameters for final goods production

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>U.S.</td>
</tr>
<tr>
<td>Unskilled male labor/Unskilled female labor Coeff.</td>
<td>$\alpha_l$</td>
<td>0.7994</td>
</tr>
<tr>
<td>Variety Capital Coeff.</td>
<td>$1 - \alpha_l$</td>
<td>0.2006</td>
</tr>
<tr>
<td>Unskilled male labor Coeff</td>
<td>$\alpha_{lm}$</td>
<td>0.6862</td>
</tr>
<tr>
<td>Unskilled female labor Coeff</td>
<td>$1 - \alpha_{lm}$</td>
<td>0.3138</td>
</tr>
<tr>
<td>Determinant for elasticity of substitution, Unskilled male labor/Unskilled female labor</td>
<td>$\mu$</td>
<td>28.4131</td>
</tr>
<tr>
<td>Determinant for elasticity of substitution, Unskilled labor/Variety capital</td>
<td>$\rho$</td>
<td>2.4342</td>
</tr>
<tr>
<td>Return to scale</td>
<td>$\omega$</td>
<td>1.2504</td>
</tr>
</tbody>
</table>

The estimates of the elasticities of substitution for the final goods production are presented in Table 3. Final goods production uses capital varieties derived from production of public R&D sector, and male and female unskilled workers to produce a homogenous final output. Unskilled labor performs as bundling intermediate goods. Unskilled male labor and unskilled female labor are substitutes, as well as unskilled male labor and variety capital are also substitutes for both economies.
The estimated elasticities of substitution between unskilled female and variety capital are opposite for both countries. The unskilled female and variety capital inputs are complements in the US but substitutes in Thailand. These inputs in Thailand are all substitutes which do not depend on the first or the second nested levels of production meaning that the final output production process can adjust using labor or capital for bundling intermediate goods if the relative input prices change. Also, unskilled male and female workers can be substitutes as the process does not require specialty or skills that are related to gender.

However, the final goods production process is estimated to be different between the two economies in that in the US, final goods firms tend to use unskilled female with variety capital. This process is the same as in the Leontief production function in which all inputs are used at a constant ratio to produce one unit of output. We can think of this estimation to be consistent with the reality that, the final goods firm may need female unskilled labor to work with the machines for example, they need persons for observing, arranging, scheduling and setting up the machines while they are working. This requires more female unskilled labor than unskilled male labor because these kinds of task need female worker specific performance and also the wage costs are lower for women. Thus the final goods firm will hire more female workers than male workers.

In Thailand, machines and unskilled labor can be substitutes since they all have similar performances and the gender wage gap is small. The unskilled labor has the same capacity in working for final bundling intermediate goods. The final goods firms have flexibility to use any of the three inputs, or use all three inputs to bundle a homogeneous final output.

Both developed and developing countries have estimated elasticities of substitution smaller than unitary. This indicates that there is relatively small possibility of substituting labor and capital in both countries. The lowest substitution possibility is found for unskilled male and female in Thailand and highest substitution possibility is found for unskilled female and capital in the same economy.
Table 3. Estimated elasticities of substitution

<table>
<thead>
<tr>
<th>Description</th>
<th>Elasticity of substitution</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unskilled male labor/Unskilled female labor</td>
<td>$\sigma_{1,2}$</td>
<td>0.0340</td>
</tr>
<tr>
<td>Unskilled male labor/Variety Capital</td>
<td>$\sigma_{1,3}$</td>
<td>0.0346</td>
</tr>
<tr>
<td>Unskilled female labor/ Variety Capital</td>
<td>$\sigma_{2,3}$</td>
<td>-0.2979</td>
</tr>
</tbody>
</table>

3.6.2 R&D sector

Tables 4 and 5 present the estimates of parameters and elasticities of substitution of the capital variety production function (eq 12). This production uses four inputs: public R&D, private investment in capital, skilled male and female labor.

The parameters estimated in Table 4 are positive and negative and all are greater than -1. The estimated determinant for elasticity of substitution between two bundles of inputs ($\varphi$), between public R&D-investment bundle and skilled labor bundle, is positive for the US economy, but is negative for the Thai economy. The other two estimated parameters ($\theta, \pi$) also have the opposite sign for the two economies. This shows that the two economies have different complementarity and substitution between each input bundles.
Table 4: Estimates of parameters for capital variety production

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Estimate</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Determinant for elasticity of substitution, ( \theta )</td>
<td></td>
<td></td>
<td>0.2126</td>
<td>-0.0819</td>
</tr>
<tr>
<td>Public R&amp;D/Private investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determinant for elasticity of substitution, ( \varphi )</td>
<td></td>
<td></td>
<td>6.0330</td>
<td>-0.1429</td>
</tr>
<tr>
<td>R&amp;D/Skilled labor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determinant for elasticity of substitution, ( \pi )</td>
<td></td>
<td></td>
<td>-0.1582</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

Table 5 reports the estimated elasticities of substitution for the four inputs in R&D sector with the comparison between elasticities of substitution for the United States and for the Thai economies. The estimates give different values of substitution elasticity but the same in terms of the relation among inputs. The estimates show that public R&D and skilled male labor, and public R&D and skilled female labor are complements, with negative values of elasticity of substitution. That is public R&D spending as subsidies on R&D projects to private firms in inventing new innovations or varieties of capital or machines as well as R&D induced by public incentive schemes requires skilled labor inputs (or scientists) to incorporate into production. On the supply side, because most technologies once invented, can be used by many firms and workers at low marginal cost, this induces the labor market to have more skilled workers. The market for skill-complementary technologies is then larger. The producers of capital varieties therefore able to obtain higher profits, and more effort will be devoted to the invention of skill-complementary technologies.
Following our estimates, this happens in both developed and developing countries like the United States and Thailand. This is consistent with the study by Griliches (1969) showing that skill" or "schooling" is more complementary with capital than unskilled or un-schooled labor. Increasing public R&D leads to innovation technology and that needs skilled workers either male or female. The investment in capital from private sector is also complementary to skilled labor. The other two pairs of inputs consisting of skilled male and female labor, and public R&D and investment in capital from private sector are substitutes. The estimated absolute values of the substitution elasticity show more elastic for a skilled male and female labor substitution than that of public R&D and private investment.

However, the values of substitution elasticities between public R&D and private investment are different in both economies. It is more elastic in Thailand. This means for any one percentage change in the ratio of R&D and investment cost, intermediated firms will increase a comparative lower cost input to obtain the same output. The estimate of the elasticity of substitution between skilled male and skilled female labor in Thailand is close to the estimate for the US economy. The skilled workers are substitutes in both countries and the degrees of substitution are around unity which is consistent with the one in Cobb Douglas production for the aggregate economy.
Table 5. Estimates of elasticities of substitution for capital variety production

<table>
<thead>
<tr>
<th>Description</th>
<th>Elasticity of substitution</th>
<th>Estimate U.S.</th>
<th>Estimate Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public R&amp;D/Private investment</td>
<td>( \sigma_{1,2} )</td>
<td>0.825</td>
<td>1.0892</td>
</tr>
<tr>
<td>Public R&amp;D/Skilled male labor</td>
<td>( \sigma_{1,3} )</td>
<td>-0.0022</td>
<td>-0.0018</td>
</tr>
<tr>
<td>Public R&amp;D/Skilled female labor</td>
<td>( \sigma_{1,4} )</td>
<td>-0.0034</td>
<td>-0.2640</td>
</tr>
<tr>
<td>Private investment/ Skilled male labor</td>
<td>( \sigma_{2,3} )</td>
<td>-0.0020</td>
<td>-0.3814</td>
</tr>
<tr>
<td>Private investment/ Skilled female labor</td>
<td>( \sigma_{2,4} )</td>
<td>-0.0031</td>
<td>-0.9176</td>
</tr>
<tr>
<td>Skilled male labor/Skilled female labor</td>
<td>( \sigma_{3,4} )</td>
<td>1.1880</td>
<td>1.1000</td>
</tr>
</tbody>
</table>

3.7 Conclusion

This study applies a two-sector model of production in which intermediate goods production uses public R&D, private investment in machine, and skilled labor inputs to produce varieties of capital. Final goods firms use capital varieties and unskilled male and female labor. We differentiate the labor inputs into four types in order to investigate the substitution elasticity mainly between public R&D and differentiated labor inputs and compare them between developed and developing economy: the United States vs. Thailand. Interestingly, the skilled labor inputs are complement to public R&D spending in both economies. In addition, unskilled female and variety capital are complements in the United States but substitutes in Thailand.

The estimates also suggest that increasing public R&D raises productivities and also increases return to capital and labor from the increasing return to scale for the entire economy. Firms distribute these implicit
returns to inputs in terms of wages and capital prices. Due to these results, for the U.S., if the government allocates more subsidies to the private R&D sector, the economy can be stimulated, increasing in capital varieties which contribute to output. Also, this increases labor supply as the complementarity inputs of labor and R&D. Also, for Thailand, a small and developing economy, support in public R&D can be particularly beneficial, since it stimulates the variety capital and then enhances the economic growth.
3.8 References


