IMPORTING, UNCERTAINTY AND THE COSTS OF TRADE

By

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School of Economic Sciences

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To the Faculty of Washington State University:

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IMPORTING, UNCERTAINTY AND THE COSTS OF TRADE

Abstract

by Tim A. Graciano, Ph.D.
Washington State University
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Intermediate goods make up a large share of world trade. However only some producers choose to use imported intermediate inputs while others do not. The decision matters for plant performance. In the data, plants that import are much larger and more productive than non-importers. Yet trade models with heterogeneous firms focus almost exclusively on firms’ export decisions rather than their import decisions. Chapter 1 documents the performance advantage of importers and develops a simple, partial equilibrium model of input choice. Chapter 2 develops an analytically solvable model of a small open economy in which heterogeneous firms make endogenous import decisions. We view decisions about importing as decisions about technology adoption. In the model, firms weigh the benefit of operating a technology that uses imported intermediate goods against the fixed cost of developing trade relationships with foreign input suppliers. Only the most efficient firms choose to import. Trade liberalization leads to a reallocation of resources toward more efficient firms, just as in export-decision models, but also leads to improvements in individual firms’ productive efficiency. Quantitatively, the model captures important aspects of plant-level data — including the large
performance advantage associated with using imported intermediate goods — and generates large increases in trade from small decreases in tariffs. Chapter 3 generalizes the model to a dynamic stochastic general equilibrium setting. Costs of international trade are increasingly modeled as fixed costs paid by individual firms. In a dynamic setting, these fixed costs may take two different forms: costs of starting to trade and costs of continuing to trade. The distinction matters when firms experience idiosyncratic shocks to productivity over time. In the model, there are costs of developing relationships with foreign input suppliers and costs of continuing these relationships. The benefit of using imported inputs lies in a combination of the relative price and the technology embodied in the inputs. The model quantitatively captures many important features of plant-level manufacturing data, including the dynamics of import status, entry and exit rates, the size distribution, and the large performance advantage associated with using imported intermediate inputs.
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To my girls,
Laci and Maria
Availability of firm-level data has introduced economists to a new set of trade facts. Most importantly, trade status matters for firm performance. Firms that trade are much larger and more productive than those that do not. Despite this few firms participate in international trade. To account for these seemingly contradictory facts, trade models often include heterogeneous firms, and fixed costs, where trade decisions are made at the firm level. Yet these models focus almost exclusively on firms’ export decisions rather than their import decisions.

We adopt many of these features, but focus on firms’ import decisions. While import and export decisions may be related in some ways, they involve fundamentally different considerations by firms. In making export decisions, firms must consider the characteristics of foreign markets and the costs involved in entering those markets. In making import decisions, firms must consider how the use of imported intermediate goods will affect their production processes and weigh this against the costs of developing and maintaining business relationships with foreign input suppliers. Central to our approach is the view that the decision to import is really a technology adoption problem.

Firms are heterogeneous along two dimensions. First, they vary in their technical efficiency, and second in their import status. Since firms are heterogeneous in their efficiency the potential benefit they receive from importing is also heterogeneous. The existence of fixed costs ensure that only the most efficient firms import. In export decision models the performance advantage of exporters is due solely to selection. In
contrast, our model has an additional mechanism where adopting a technology that uses imported intermediate goods provides an efficiency gain for the firm. We call this the technology upgrading effect. This effect contributes to the performance advantage of importers and agrees with recent evidence that importers tend to use higher quality inputs. We find that technology upgrading accounts for between 13 and 18 percent of the size difference between importers and non-importers.

Next, we extend this static model to a setting with firms-specific idiosyncratic shocks. In a dynamic setting, fixed costs may take two different forms: costs of starting to trade and costs of continuing to trade. When firms face uncertainty sunk, entry costs cause optimal decisions to become history dependant. Firms want to continuously import, due to costs savings from avoiding repeated entry. Sunk costs cause domestic firms to delay adoption of the importing technology and causes current importers to delay switching back in response to technology shocks. Allowing firms to switch import status in response to shocks highlights the importance of both the intensive and extensive margins of trade.

To match data on trade flows, we need models which generate large increases in trade from small decreases in tariffs. Our model has this potential, as it has both extensive and intensive margins of importing. In models where an extensive margin operates the measured Armington elasticity at the macro level is greater than at the micro level, in this case the within firms.

To test the quantitative implications of our models we calibrate them to data on the Chilean manufacturing sector. The data comes from the annual census of manufacturing plants conducted by Chile’s Instituto Nacional de Estadísticas during the
period 2001 to 2006. The census is detailed: it includes data on plants’ employment, gross output, value added, and expenditures, including expenditures on domestically produced intermediate goods and imported intermediate goods. From 2001 to 2006, the census surveyed a total of 8,014 different manufacturing plants.

The simple structure of the models allow many of the parameters to be calculated directly from the data. The remaining parameters are jointly chosen to match statistics of interest. We are primarily concerned with capturing the following basic facts about plants’ importing behavior.

**Fact 1.** Most plants do not import. Depending on the setting importers are between 13 and 20 percent of our sample.

**Fact 2.** Importers spend more on domestically produced intermediate goods than on imported intermediate goods. In our sample, expenditure on imported intermediate goods is approximately 35 percent of the average importer’s total expenditure on intermediate goods.

**Fact 3.** Importers are much larger than non-importers. In terms of gross output, expenditure on intermediate goods, value added, and employment, importers are, on average, 3.1 to 5.2 times larger than non-importers.
**Fact 4.** Importers are more productive than non-importers. Depending on the setting, importers have between 1.2 to 1.3 times higher value added per worker than non-importers.

The main findings of this research are as follows. Both the static and dynamic models are able to quantitatively capture important aspects of plant-level data, including the large performance advantage associated with using imported intermediate goods. In response to an episode of trade liberalization — either a decrease in the tariff rate or an improvement in the terms of trade — the least efficient firms exit because they can no longer profitably operate at the higher wage, the most efficient non-importers become importers, and technological efficiency increases. The implied performance increase from switching to a technology that uses imported intermediate inputs is between 15 and 21 percent.

The remainder of the dissertation is organized as follows. Chapter 1 documents the performance advantage of importers, and introduces a simple model of technology adoption. Chapter 2 extends the model to a general equilibrium setting. We test the quantitative properties of the model by calibrating it to the data discussed above. Chapter 3 presents a dynamic stochastic general equilibrium model. In addition to matching the set of facts above this model captures the dynamics of import status among plants and provides insights to relative importance of entry and continuation costs.

The modeling framework developed here allows for many interesting extensions. These include the roles of aggregate uncertainty, multiple countries, monopolistic competition, and joint import-export decisions. We leave these extensions for future work. Portions of this dissertation are based on joint work with Mark J. Gibson.
The Decision to Import

If producers that used imported intermediate inputs appeared no different from those that did not, then we might reasonably conclude that the decision to import was of little consequence for the organization of production in an economy. But this is not the case. Importers differ sharply from non-importers.

We document that, in recent surveys of Chilean manufacturing plants, plants that use imported raw materials are much larger and more productive than plants that do not. These findings are robust to a number of different statistical controls. Despite the large performance advantage associated with using imported inputs, only a small fraction of plants do.

Our interpretation of the data is that producers would prefer to use some imported inputs in production — perhaps because they are easier to process, are of higher quality, or offer some other productivity advantage — but that most do not because of barriers to international trade that take the form of fixed costs. We interpret these fixed costs as the costs of developing trade relationships with foreign input suppliers. It is only worthwhile for the largest, most efficient, and most profitable firms to invest in these trade relationships.

To formalize this story, we develop a simple model of an industry in which firms endogenously decide whether or not to use imported inputs. Our model has three main features. First, there is a continuum of heterogeneous firms that take efficiency draws from a probability distribution (along the lines of Hopenhayn (1992)). Second, after obtaining an efficiency draw, each firm has an endogenous choice of two technologies: a
technology that uses only domestic inputs and a technology that uses both domestic and imported inputs. Third, adopting the technology that uses imported inputs requires payment of a fixed cost.

In the model, firms endogenously sort themselves into importers and non-importers. Due to the barrier created by the fixed cost of importing, only the plants with the highest efficiency draws become importers. These are also the largest plants. Quantitatively, we show that for plausible parameters the model can replicate the observed difference in size between importers and non-importers.

The importing behavior of plants has not yet been widely studied, but there is a growing literature. Using Colombian data, Kugler and Verhooven (2009) find evidence that plants import to obtain higher-quality inputs. Using Chilean data, Kasahara and Rodrigue (2008) find that switching from being a non-importer to being an importer can increase a plant’s productivity. Amiti and Konings (2007), examining data from Indonesia, find that lowering tariffs on imported inputs can raise plant productivity through learning, variety, and quality effects. Halpern, Koren, and Szeidl (20059) find, analyzing Hungarian data, that imported inputs increase plant productivity through complementarity and quality channels.

Our findings on the differences between importers and non-importers are similar to findings on the differences between exporters and non-exporters. For instance, Bernard and Jensen (1999), Bernard et al. (2003), and Bernard, Jensen, and Schott (2009) have documented differences between exporters and non-exporters for the United States. These findings have led to a large literature on the decision to export. Roberts and Tybout (1997) made a seminal contribution to the empirical literature on this topic, as did
Melitz (2003) to the theoretical literature. Both papers emphasize fixed costs of exporting.

A similarly large literature has not yet developed with respect to the decision to import. We aim to remedy this and view our work as complementary to that on the decision to export. While exporting and importing decisions may be related, we believe that the underlying considerations by firms are fundamentally different. The decision to export has to do with the demand characteristics of the foreign market, whereas the decision to use imported inputs has to do with the production process at the plant level.

**Data**

We use the annual census of manufacturing plants conducted by Chile’s Instituto Nacional de Estadísticas during the period 2001 to 2006. Previous editions of the survey have been examined by Liu (1993), Levinsohn (1999), and Pavcnik (2002), among others. The data on imported inputs are limited in this survey: we only observe the extent to which plants use imported raw materials. As a result, we cannot analyze the effects of, for instance, imported capital equipment, which may be of greater importance. Nevertheless, the differences between importers and non-importers are striking.

The Chilean census involves a total of 8,014 different manufacturing plants over the period 2001 to 2006. Since our interest is in comparing importers and non-importers, we omit from our analysis the 1,078 plants that changed their import status during the period (see Kasahara and Rodrigue (2008) for an analysis of these types of plants). We categorize the remaining plants as follows. **Importers** are plants that used imported raw materials every year that they appeared in the census during the period; they are 13
percent of our sample. *Non-importers* are plants that did not use imported raw materials in any year that they appeared in the census during the period; they are 87 percent of our sample.

*Comparing importers and non-importers*

We compare plant averages between importers and non-importers over the period 2001 to 2006. All monetary values are in terms of an inflation-adjusted unit of account, the Chilean Unidad de Fomento. Our findings are summarized in Table 1. The table shows that, on average, importers are much larger than non-importers, whether measured by gross output, total materials used, value added, or employment. Importers are also more productive than non-importers. On average, importers have about 30 percent higher value added per worker.

**Table 1. Importer premia**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Ratio of average importer to average non-importer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross output</td>
<td>4.5</td>
</tr>
<tr>
<td>Total materials</td>
<td>4.4</td>
</tr>
<tr>
<td>Employment</td>
<td>4.0</td>
</tr>
<tr>
<td>Assets</td>
<td>7.5</td>
</tr>
<tr>
<td>Assets per worker</td>
<td>2.1</td>
</tr>
<tr>
<td>Value added</td>
<td>5.2</td>
</tr>
<tr>
<td>Value added per worker</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Even within industries that process relatively homogeneous goods, such as agricultural goods, the differences between importers and non-importers are stark. Of the
20 industries at the two-digit ISIC level in our data, the following 5 are likely involved in processing agricultural goods: manufacture of food products and beverages, manufacture of textiles, tanning and dressing of leather, manufacture of wood and wood products, and manufacture of paper and paper products. In table 2, for each of these industries we report the share of plants that use imported raw materials and the extent to which importers differ from non-importers in gross output and value added per worker. The differences in size and productivity between importers and non-importers tend to be more pronounced in these industries than in the manufacturing sector as a whole.

Table 2. Importer premia in industries that process agricultural goods

<table>
<thead>
<tr>
<th>Industry</th>
<th>Ratio of average importer to average non-importer</th>
<th>Importers as a percent of all plants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gross output</td>
<td>Value added per worker</td>
</tr>
<tr>
<td>Manufacture of food products and beverages</td>
<td>10.9</td>
<td>1.5</td>
</tr>
<tr>
<td>Manufacture of textiles</td>
<td>6.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Tanning and dressing of leather</td>
<td>2.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Manufacture of wood and wood products</td>
<td>4.2</td>
<td>2.4</td>
</tr>
<tr>
<td>Manufacture of paper and paper products</td>
<td>15.1</td>
<td>4.6</td>
</tr>
<tr>
<td>All industries</td>
<td>4.5</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Statistically controlling for other plant characteristics

To further test the robustness of our findings on the differences between importers and non-importers, we consider the extent to which these differences may be statistically
accounted for by other plant characteristics. We control for the following: the region in which a plant is located (Chile has 13 regions), the industry to which a plant belongs (the data cover 20 manufacturing industries at the two-digit ISIC level), whether or not a plant exports, the capital intensity of a plant (measured as total assets per worker and sorted into deciles), and whether or not there is any foreign ownership of a plant. Specifically, we perform the following semi-logarithmic regression:

$$\log Y_i = \alpha + \beta \text{Importer}_i + X_i'\delta + \epsilon_i,$$

where $Y_i$ is the dependent variable for plant $i$, Importer$_i$ is a dummy variable for whether or not plant $i$ uses imported raw materials, $X_i$ is a vector of control dummy variables for plant $i$ consisting of the characteristics listed above, and $\epsilon_i$ is an error term. We assume that $\epsilon_i$ satisfies the usual properties. The coefficients to be estimated are $\alpha$, $\beta$, and $\delta$. Table 3 reports the estimated value of the coefficient $\beta$ for the following dependent variables: gross output, total materials, employment, assets, assets per worker, value added, and value added per worker. All differences between importers and non-importers are statistically significant at the one-percent level.
Table 3. Robustness checks on importer premia

<table>
<thead>
<tr>
<th>$Y_i$</th>
<th>$\hat{\beta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross output</td>
<td>1.45***</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
</tr>
<tr>
<td>Total materials</td>
<td>1.55***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
</tr>
<tr>
<td>Employment</td>
<td>0.92***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
</tr>
<tr>
<td>Assets</td>
<td>1.93***</td>
</tr>
<tr>
<td></td>
<td>(0.047)</td>
</tr>
<tr>
<td>Assets per worker</td>
<td>1.05***</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
</tr>
<tr>
<td>Value added</td>
<td>1.37***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
</tr>
<tr>
<td>Value added per worker</td>
<td>0.44***</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses; *** significant at 1%.

Model

We model an industry that consists of measure one of single-plant firms. The industry produces a homogeneous good, the price of which is normalized to one. The industry has access to two different inputs to production: a domestic input and a foreign input. The industry takes the prices of these inputs as given. There are no equilibrium price effects in the model so that the focus is on the decisions of firms.

Technologies

Each firm takes an efficiency draw from a probability distribution $G(\cdot)$. Each firm then has a choice of two technologies: the first technology uses only domestic inputs, while the second technology requires both domestic and foreign inputs. The technologies available to a firm with efficiency draw $a$ take simple functional forms.
The first technology, technology $N$ for *non-importer*, uses only domestic inputs. For a firm with draw $a$, technology $N$ is given by

$$y_N(a) = ad_N(a)\nu,$$  \hspace{1cm} (2)

where $y_N(a)$ is the quantity of output, $d_N(a)$ is the quantity of the domestic input, and $0 < \nu < 1$. The profits of a firm with draw $a$ using technology $N$ are

$$\pi_N(a) = y_N(a) - p_d d_N(a),$$  \hspace{1cm} (3)

where $p_d$ is the price of the domestic input. Profit maximization implies that

$$d_N(a) = \left(\frac{av}{p_d}\right)^{-\frac{1}{\nu}}.$$  \hspace{1cm} (4)

The second technology, technology $I$ for *importer*, uses both domestic and foreign inputs. For a firm with draw $a$, technology $I$ is given by

$$y_I(a) = a\eta \left( \mu d_I(a)^\rho + (1-\mu)f_I(a)^\rho \right)^{\frac{1}{\rho}},$$  \hspace{1cm} (5)

where $y_I(a)$ is the quantity of output, $d_I(a)$ is the quantity of the domestic input, $f_I(a)$ is the quantity of the foreign input, $\eta > 0$, $0 < \mu < 1$, and $\rho < 1$. The firm’s elasticity of substitution between domestic and foreign inputs is $1/(1-\rho)$. The parameter $\eta$ determines the total factor productivity of technology $I$ relative to technology $N$.

Operating technology $I$ requires a fixed cost of $\phi$ units of output, so the profits of a firm with draw $a$ using technology $I$ are

$$\pi_I(a) = y_I(a) - p_d d_I(a) - p_f f_I(a) - \phi,$$  \hspace{1cm} (6)

where $p_f$ is the price of the foreign input. Profit maximization implies that
\[ d_i(a) = (\alpha \eta \nu)^{-\nu} \left( \frac{\mu}{p_d} \right)^{1-\rho} \left( \frac{1}{\mu^{1-\rho}} p_d^{1-\rho} + (1-\mu)^{1-\rho} p_f^{1-\rho} \right)^{v-\rho} \rho^{(1-\nu)} \]  

(7)

\[ f_i(a) = (\alpha \eta \nu)^{-\nu} \left( \frac{1-\mu}{p_f} \right)^{1-\rho} \left( \frac{1}{\mu^{1-\rho}} p_d^{1-\rho} + (1-\mu)^{1-\rho} p_f^{1-\rho} \right)^{v-\rho} \rho^{(1-\nu)} \]  

(8)

**Cutoff for importing**

A firm with efficiency draw \( a \) chooses which technology to use according to the maximum of \( \pi_N(a) \) and \( \pi_I(a) \). Consistent with the data, we focus on the case where parameter values are such that both technologies are used in the industry. In this case, there is an efficiency cutoff for using technology \( I \). A firm with draw \( a \) uses technology \( I \) if \( a \geq a \) and uses technology \( N \) if \( a < a \), where \( a \) solves

\[ \pi_N(a) = \pi_I(a). \]  

(9)

In terms of parameters, the cutoff for importing is

\[ a = \frac{\phi^{1-\nu}}{v'(1-\nu)^{1-\nu} \left( \eta^{1-\nu} \left( \frac{1}{\mu^{1-\rho}} p_d^{1-\rho} + (1-\mu)^{1-\rho} p_f^{1-\rho} \right)^{v(1-\rho)} \rho^{(1-\nu)} - p_d^{1-\nu} \right)} \]  

(10)

This cutoff is increasing in \( \phi \) and \( p_f \) and decreasing in \( \eta \) and \( p_d \). If \( a \) decreases, then firms that were not previously importing switch technologies and start using imported inputs. The model captures, in a simple way, the technology switching that can occur in response to persistent changes in the trade environment.

In contrast to models with a representative firm, our model has both an intensive margin and an extensive margin of importing. The **intensive margin** captures changes in
the use of the foreign input by firms that were previously importing. The *extensive margin* captures changes in the share of firms that import. With regard to exports, Ruhl (2004) and Chaney (2008) find that allowing for an extensive margin is important in accounting for changes following trade liberalization.

**Quantitative analysis of the model**

We calibrate our model to match important facts about the Chilean manufacturing sector. Then we determine the extent to which the model can account for the observed size differences between importers and non-importers.

**Figure 1. Distribution of plants by gross output**

![Figure 1. Distribution of plants by gross output](image)

**Calibration**

We begin by specifying the probability distribution for the efficiency draws. Figure 1 shows the distribution of plants by gross output. The result is suggestive of a
Pareto distribution. Moreover, the Pareto distribution is used in many other studies involving trade and heterogeneous firms (see, for example, Chaney (2008)). Consistent with this, we suppose that each firm’s efficiency is drawn from a Pareto distribution with cumulative distribution function \( G(a) = 1 - a^{-\gamma} \), for \( a \geq 1 \), where the lower bound of one is a normalization and \( \gamma > 2/(1-\nu) \). The restriction on \( \gamma \) ensures that the variance of output in the model is finite.

**Table 4. Calibration**

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Value(s)</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_d, p_f )</td>
<td>1, 1</td>
<td>Normalization of units</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.5</td>
<td>Ruhl (2004)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.85</td>
<td>Atkeson and Kehoe (2005)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.68</td>
<td>Chosen to match the fact that expenditure on imported inputs is 15% of importers’ gross output</td>
</tr>
<tr>
<td>( \phi, \eta, \gamma )</td>
<td>See table 5</td>
<td>Chosen jointly to match 2 facts: (i) importers are 13% of plants and (ii) the coefficient of variation for gross output is 7.3</td>
</tr>
</tbody>
</table>

Table 4 summarizes the calibration. We normalize units so that all of the prices in the model are initially equal to one. This leaves us with six parameters: \( \rho, \nu, \mu, \phi, \eta, \) and \( \gamma \). Following Ruhl (2004), we set \( \rho = 0.5 \) to give an elasticity of substitution of 2. Following Atkeson and Kehoe (2005), we set \( \nu = 0.85 \). We set \( \mu = 0.68 \) so that expenditure on imported inputs is 15 percent of importers’ gross output. We choose the values of \( \phi, \eta, \) and \( \gamma \) jointly to match two facts: (i) importers are 13 percent of plants and (ii) the coefficient of variation for gross output of plants is 7.3. These two facts do
not uniquely pin down the values of $\phi$, $\eta$, and $\gamma$. This is because a higher value of $\phi$ may be offset by a higher value of $\eta$. As a result, we include sensitivity analysis in table 5.

**Table 5. Data vs. Model**

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>Relative size of fixed cost for cutoff plant</th>
<th>Ratio of average importer’s gross output to average non-importer’s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As a percent of gross output</td>
<td>As a percent of variable profits</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\eta$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>0.008</td>
<td>1.63</td>
<td>13.41</td>
</tr>
<tr>
<td>0.031</td>
<td>1.66</td>
<td>13.42</td>
</tr>
<tr>
<td>0.064</td>
<td>1.70</td>
<td>13.44</td>
</tr>
</tbody>
</table>

**Results**

To what extent can the model account for the observed size differences between importers and non-importers? As table 5 shows, the model matches the data if we set $\phi = 0.031$, $\eta = 1.66$, and $\gamma = 13.42$. For the cutoff firm, the fixed cost of importing is then equal to 2 percent of gross output and 19 percent of variable profits. We view these magnitudes as plausible. For sensitivity analysis, we also report the results for lower and higher values of $\phi$, $\eta$, and $\gamma$.

**Conclusion**

In contrast to the decision to export, the decision to import has not yet been widely studied. We document the performance advantage associated with using imported
inputs and offer a simple theoretical mechanism that can quantitatively account for this. Our results suggest that the importing behavior of plants is a fruitful area for future research, though more data analysis is needed. The simple model presented here may serve as a point of departure for other researchers. The roles of dynamics and uncertainty are natural extensions of the model. Further applications may include quantitative modeling of terms-of-trade shocks and trade liberalizations. The ultimate goal is to develop a model of trade that can quantitatively account for both the export and import decisions of firms. The challenge is to maintain transparency while doing so.
A large and increasing share of international trade is trade in intermediate goods.\footnote{See, for example, Feenstra (1998) and Hummels, Ishii, and Yi (2001).} Yet almost all models of trade with heterogeneous firms focus on a firm’s decision to export. In these models, a firm decides whether to pay a fixed cost in order to sell its good in a foreign market. This modeling approach began with Melitz (2003) and has been extended in numerous ways.\footnote{Helpman (2006) provides a survey of this literature.} The Melitz model has become one of the workhorse models of international trade, allowing economists to better understand firms’ export decisions and the effects of trade liberalization on an economy’s industrial organization.

We too develop a model with heterogeneous firms and fixed costs of trade, but our focus is on firms’ import decisions. Our model complements Melitz-style models that emphasize firms’ export decisions. While import and export decisions may be related in some ways, they involve fundamentally different considerations by firms. In making export decisions, firms must consider the characteristics of foreign markets and the costs involved in entering those markets. In making import decisions, firms must consider how the use of imported intermediate goods will affect their production processes and weigh this against the costs of developing business relationships with foreign input suppliers.

Our model is motivated by, and consistent with, data on plants’ importing behavior. Using recent plant-level data on Chilean manufacturing firms, we document a
basic set of facts. Importers differ sharply from non-importers. Plants that use imported intermediate goods are much larger and more productive than plants that do not. Despite the apparent performance advantage associated with using imported inputs, only a small fraction of plants do, and, even among importers, imported intermediate goods do not make up a majority of total expenditure on intermediate goods.\footnote{These facts appear to be robust across countries. See, for example, Bernard, Jensen, and Schott (2009) for the United States, Kugler and Verhoogen (2009) for Colombia, Castellani, Serti, and Tomasi (2010) for Italy, and Muûls and Pisu (2009) for Belgium.} We adopt a simple interpretation of these facts: most producers would prefer to use some imported intermediate inputs in production, but there are fixed costs that discourage most from doing so; these fixed costs are the costs of developing business relationships with foreign input suppliers.

Fundamental to our modeling approach is the idea that firms’ import decisions are really decisions about technology adoption. Each firm must decide how using imported intermediate goods will affects its production process and its performance. There is a growing literature providing evidence that using imported intermediate goods enhances firm or plant performance. This includes research by Amiti and Konings (2007), Kasahara and Rodrigue (2008), Halpern, Koren, and Szeidl (2009), and Kugler and Verhoogen (2009). Our aim is to model this phenomenon in a simple way and use the model to better understand, both qualitatively and quantitatively, the effects of trade liberalizations, improvements in the terms of trade, and decreases in trade costs. We keep the model simple enough that there is an exact analytic solution for the equilibrium.

We develop a general equilibrium model of a small open economy with a continuum of single-plant firms. These firms exhibit two forms of heterogeneity: they
differ in their levels of efficiency and in whether or not they use imported intermediate goods. The first form of heterogeneity is the result of random draws, as in Hopenhayn (1992). The second is endogenously decided by each firm. Each firm chooses between two technologies: a technology that uses only domestic inputs and a technology that uses a combination of domestic and imported inputs. The fixed cost of operating the technology that uses imported intermediate inputs is higher, reflecting the additional costs of developing business relationships with foreign input suppliers. The technology that uses imported intermediate goods is superior, in the sense that every firm would choose it if there were no additional fixed cost of doing so. The total benefit of using this technology is increasing in a firm’s scale of operation, while the operating cost is fixed.

In the model, as in the data, importers are very different from non-importers. Because of the fixed cost of importing, only the firms with the highest efficiency draws choose to import. This is the usual selection effect emphasized by Melitz (2003) in the context of exporting. In addition, our model has what we refer to as a technology upgrading effect: a firm that opts for the technology using some imported inputs over the technology using only domestic inputs increases its output, employment, expenditure on intermediate inputs, and variable profits. The technology upgrading effect is analogous to “learning by importing,” where the very act of importing leads to improved firm performance. Kasahara and Rodrigue (2008), among others, provide evidence of this effect.

To obtain the quantitative implications of the theory, we calibrate the model using the Chilean manufacturing data described earlier. The model does a good job of replicating the basic facts that we document, including the large performance advantage
associated with importing. Moreover, the calibrated model allows us to determine the relative contributions of the selection and technology upgrading effects in accounting for this performance advantage.

We use the model to qualitatively and quantitatively analyze the effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction. These sorts of changes lead to a process of reallocation across firms: the least efficient firms exit, the most efficient non-importers become importers, and aggregate technological efficiency increases. In many ways this process of reallocation resembles what Melitz (2003) finds in his export-decision model. Here the effects of the reallocation following trade liberalization are augmented by the increased superiority of the technology that uses imported inputs. This feature of the model agrees well with the evidence of, for example, Amiti and Konings (2007) and Halpern, Koren, and Szeidl (2009).

Comparing standard trade models with the data, Yi (2003) stresses the need to develop models that can generate large increases in trade in response to small decreases in tariffs. Our model does this. Ruhl (2004) and Chaney (2008) stress the importance of the extensive margin in Melitz-style export-decision models. We stress the importance of the extensive margin in our import-decision model. In response to trade liberalization, many non-importers switch technologies to become importers. With this extensive margin, we can generate the sort of large aggregate Armington elasticity (the elasticity of substitution between imported and domestic goods) found in the data without assuming unusually large elasticities at the level of an individual importer.

There are few other general equilibrium models that incorporate the importing decisions of firms. Ramanarayanan (2007) builds a dynamic model in which entering
firms make irreversible decisions about their import status in the presence of aggregate and idiosyncratic uncertainty. He uses the model to contrast the effects of business-cycle shocks and trade liberalizations on the Armington elasticity. Kasahara and Lapham (2008) consider both the decision to export and the decision to import. They develop a dynamic model in which firms face stochastic fixed costs of importing in addition to a fixed cost of exporting. By contrast, we isolate the decision to import and develop a simple, static, non-stochastic, competitive model that has an exact analytic solution. This allows for a high degree of transparency in our analysis and provides a modeling framework that can be readily extended and applied.

The paper is organized as follows. In the next section we discuss our data and some basic facts on importing behavior. In the third section, we develop the model. In the fourth and fifth sections, we qualitatively and quantitatively analyze the model. The sixth section concludes.

Data

Here we document a basic set of facts that we would like our model to be able to quantitatively replicate. These facts concern the extent to which producers use imported intermediate goods and how producers that use imported intermediate goods differ from those that do not. We take these facts from the annual census of manufacturing plants conducted by Chile’s Instituto Nacional de Estadisticas during the period 2001 to 2006.4 The census is detailed: it includes data on plants’ employment, gross output, value added,

4 A previous version of this census was examined by Liu (1993), Levinsohn (1999), Pavcnik (2002), and Kasahara and Rodrigue (2008), among others.
and expenditures, including expenditures on domestically produced intermediate goods and imported intermediate goods. All monetary values in the census are expressed in an inflation-adjusted unit of account, the Chilean Unidad de Fomento.

From 2001 to 2006, the census surveyed a total of 8,014 different manufacturing plants. To be consistent with our model, which is static, we omit from our sample the plants that changed their import status over this period.\footnote{These plants are 13 percent of the original sample. For an empirical analysis of plants that switch import status, see Kasahara and Rodrigue (2008). Chapter 3 develops a quantitative dynamic general equilibrium model with switchers.} We categorize the remaining 6,936 plants as follows. \textit{Importers} are plants that purchased imported raw materials every year that they participated during the period; they are 13 percent of our sample. \textit{Non-importers} are plants that did not purchase imported raw materials in any year that they participated during the period; they are 87 percent of our sample. When we calculate averages, we average over every relevant plant-year observation. We document four basic facts about plants’ importing behavior.

**Fact 1.** Most plants do not import. Importers are only 13 percent of our sample.

**Fact 2.** Importers spend more on domestically produced intermediate goods than on imported intermediate goods. In our sample, expenditure on imported intermediate goods is 39 percent of the average importer’s total expenditure on intermediate goods.
Fact 3. Importers are much larger than non-importers. In terms of gross output, expenditure on intermediate goods, value added, and employment, importers are, on average, 4.0 to 5.2 times larger than non-importers.

Fact 4. Importers are more productive than non-importers. Importers have 1.3 times higher value added per worker than non-importers.

These facts may seem contradictory. There appears to be a large performance advantage associated with importing, yet most plants do not import and those that do typically spend more on domestically produced intermediate goods than on imported intermediate goods. We next develop a model that has the potential to account for these facts.

Model

There is a small open economy that competitively produces a single good. This good, which serves as the numéraire, may used in four different ways: for consumption, for export, for payment of fixed costs, and as an intermediate good. The good is produced by a continuum of heterogeneous firms. The economy imports a single intermediate good, which it cannot produce, from the rest of the world. Because the economy is small relative to the rest of the world, it takes the relative price of this good as given. The government may impose an ad valorem tariff on the imported good.
Consumer

There is a representative consumer in the economy who is endowed with \( L \) units of labor. The consumer supplies labor inelastically and spends all income on consumption. The consumer’s budget constraint is

\[
C = wL + T ,
\]

where \( C \) is consumption, \( w \) is the wage, and \( T \) is the lump-sum rebate of tariff revenue.

Firms

There is a continuum of single-plant firms in the economy. These firms exhibit two forms of heterogeneity: they differ in their levels of efficiency and in whether or not they use imported intermediate goods. The first form of heterogeneity is the result of random draws, while the second is endogenously decided by each firm.

A firm’s actions are as follows. After paying the fixed cost of entry, the firm takes an efficiency draw from a probability distribution. The firm then has three options: not to operate, to operate using a technology that does not require imported inputs, or to operate using a technology that requires imported inputs. All firms face decreasing returns to scale, so firms with different efficiency levels can coexist, with each firm operating at its optimal scale. Each firm does, however, face a fixed cost of operating, so the firms with the worst draws may choose not to operate at all.

Fundamental to our model is the characterization of firms’ technologies. Let technology \( N \) be the technology of a non-importer and let technology \( I \) be the technology of an importer. The technologies are similar, but differ along important dimensions. Technology \( N \) uses only labor and the domestically produced intermediate
good as inputs, while technology \( I \) uses labor, the domestically produced intermediate good, and the imported intermediate good as inputs. For each technology, the extent of diminishing returns is determined by the parameter \( \nu \), where \( 0 < \nu < 1 \). The total factor productivity with which a firm operates technology \( N \) is given by its efficiency draw, \( a \), while the total factor productivity with which a firm operates technology \( I \) is given by \( a\eta \), where \( \eta > 0 \). Operating either technology requires payment of a fixed cost. We assume that the fixed cost of operating technology \( I \) is greater than the fixed cost of operating technology \( N \). This assumption captures the idea that there are additional costs involved in developing business relationships with foreign input suppliers relative to domestic input suppliers. Next we specify the two technologies.

First consider a firm with efficiency \( a \) operating technology \( N \). The firm’s output is given by

\[
y_N(a) = a^{1-\nu} \psi_N(\ell_N(a), d_N(a))^\nu, \tag{12}
\]

where \( \psi_N(\cdot, \cdot) \) is a standard production function with constant returns to scale, \( \ell_N(a) \) is the input of labor, and \( d_N(a) \) is the input of the domestically produced intermediate good. The firm’s profits are

\[
\pi_N(a) = y_N(a) - w\ell_N(a) - d_N(a) - \phi_N, \tag{13}
\]

where \( w \) is the wage and \( \phi_N \) is the fixed cost of operating. To maximize profits, the firm chooses \( \ell_N(a) \) and \( d_N(a) \) to solve

\[
\nu a^{1-\nu} \psi_N(\ell_N(a), d_N(a))^{\nu-1} \psi_{N\ell}(\ell_N(a), d_N(a)) - w = 0 \tag{14}
\]

\[
\nu a^{1-\nu} \psi_N(\ell_N(a), d_N(a))^{\nu-1} \psi_{Nd}(\ell_N(a), d_N(a)) - 1 = 0, \tag{15}
\]
where $\psi_{Nk} = \partial \psi_N / \partial k$, $k = \ell, d$.

Now consider a firm with efficiency $a$ operating technology $I$. The firm’s output is given by

$$y_I(a) = (a\eta)^{1-v} \psi_I \left( \ell_I(a), d_I(a), f_I(a) \right)^{\nu-1},$$  

(16)

where $\psi_I(\cdot, \cdot, \cdot)$ is a standard production function with constant returns to scale, $\ell_I(a)$ is the input of labor, $d_I(a)$ is the input of the domestically produced intermediate good, and $f_I(a)$ is the input of the imported intermediate good. The firm’s profits are

$$\pi_I(a) = y_I(a) - w\ell_I(a) - d_I(a) - (1 + \tau)pf_I(a) - \phi_I,$$

(17)

where $p$ is the (exogenous) relative price of the imported good, $\tau$ is the country’s ad valorem tariff on imports, and $\phi_I$ is the fixed cost of operating. To maximize profits, the firm chooses $\ell_I(a)$, $d_I(a)$, and $f_I(a)$ to solve

$$v(\eta)^{1-v} \psi_I \left( \ell_I(a), d_I(a), f_I(a) \right)^{\nu-1} \psi_{\ell_I} \left( \ell_I(a), d_I(a), f_I(a) \right) - w = 0$$  

(18)

$$v(\eta)^{1-v} \psi_I \left( \ell_I(a), d_I(a), f_I(a) \right)^{\nu-1} \psi_{d_I} \left( \ell_I(a), d_I(a), f_I(a) \right) - 1 = 0$$  

(19)

$$v(\eta)^{1-v} \psi_I \left( \ell_I(a), d_I(a), f_I(a) \right)^{\nu-1} \psi_{f_I} \left( \ell_I(a), d_I(a), f_I(a) \right) - (1 + \tau)p = 0,$$

(20)

where $\psi_{rk} = \partial \psi_I / \partial k$, $k = \ell, d, f$.

Given its efficiency draw, each firm decides whether to operate and, if so, which technology to use. Firms’ operating decisions are as follows. For a firm with efficiency draw $a$, the decision rule for operating technology $N$ is given by the indicator function

$$I_N(a) = \begin{cases} 1 & \text{if } \pi_N(a) \geq 0 \text{ and } \pi_N(a) > \pi_I(a) \\ 0 & \text{otherwise} \end{cases}$$  

(21)

and the decision rule for operating technology $I$ is given by the indicator function
\[ t_i(a) = \begin{cases} 
1 & \text{if } \pi_i(a) \geq \max \left[ \pi_N(a), 0 \right] \\
0 & \text{otherwise} 
\end{cases} \]  \quad (22)

The cost of firm entry is \( \phi_M \) units of output. This entitles the firm to an efficiency draw from probability distribution \( G(\cdot) \). The expected value of entry must equal the cost of entry, so

\[ \int t_N(a)\pi_N(a)dG(a) + \int t_i(a)\pi_i(a)dG(a) = \phi_M \]  \quad (23)

(Though (23) is typically referred to as a free-entry condition, it actually pins down the wage here. The expected value of entry is not decreasing in the measure of entrants, as it would be in a model with monopolistic competition.) This condition ensures that there are no aggregate profits in the economy. Let \( M \) denote the measure of entrants.

**Market clearing**

Define aggregate use of the domestically produced intermediate good as

\[ D = M \left( \int t_N(a)d_N(a)dG(a) + \int t_i(a)d_i(a)dG(a) \right) \]  \quad (24)

Define aggregate use of the foreign intermediate good as

\[ F = M \int t_i(a)f_i(a)dG(a) \]  \quad (25)

Define aggregate output as

\[ Y = M \left( \int t_N(a)y_N(a)dG(a) + \int t_i(a)y_i(a)dG(a) \right) \]  \quad (26)

International balance of payments requires that

\[ X = pF \]  \quad (27)
where \( X \) is aggregate exports. Tariff revenue is rebated to the consumer as a lump sum, so
\[
T = \tau pF. \tag{28}
\]
Clearing in the labor market requires that
\[
M \left( \int t_N(a)\ell_N(a)dG(a) + \int t_i(a)\ell_i(a)dG(a) \right) = L. \tag{29}
\]
Finally, clearing in the goods market requires that
\[
C + D + X + M \left( \phi_N + \phi_i \int t_N(a)dG(a) + \phi_i \int t_i(a)dG(a) \right) = Y. \tag{30}
\]

**Equilibrium**

Here we define an equilibrium and specify an algorithm for calculating it.

**Definition.** A *competitive small open economy equilibrium* is a list of aggregate measures \( \hat{C}, \hat{D}, \hat{X}, \hat{F}, \hat{Y}, \) and \( \hat{M} \); a wage \( \hat{w} \); a transfer \( \hat{T} \); and firm decision rules \( \hat{\ell}_N(a), \hat{\ell}_i(a), \hat{\pi}_N(a), \hat{\pi}_i(a), \hat{\nu}_N(a), \hat{\nu}_i(a), \hat{\ell}_N(a), \hat{\ell}_i(a), \hat{d}_N(a), \hat{d}_i(a), \hat{f}_N(a), \hat{f}_i(a), \hat{i}_N(a), \) and \( \hat{i}_i(a) \) such that (11)-(30) hold.

The equilibrium is straightforward to calculate using the following algorithm. Taking \( w \) as given, solve for \( \hat{\ell}_N(a) \) and \( \hat{\nu}_N(a) \) using (14) and (15), solve for \( \hat{\ell}_i(a) \), \( \hat{d}_i(a) \), and \( \hat{f}_i(a) \) using (18)-(20); calculate \( \hat{\ell}_N(a) \) and \( \hat{\nu}_N(a) \) using (12) and (16); calculate \( \hat{\pi}_N(a) \) and \( \hat{\pi}_i(a) \) using (13) and (17); and calculate \( \hat{i}_N(a) \) and \( \hat{i}_i(a) \) using (21) and (22). Solve for \( \hat{w} \) using (23). Solve for \( \hat{M} \) using (29). Calculate \( \hat{D}, \hat{F}, \) and \( \hat{Y} \)
using (24)-(26). Calculate \( \hat{X} \) and \( \hat{T} \) using (27) and (28). Finally, calculate \( \hat{C} \) using (11). By Walras’s Law, (30) holds.

**Qualitative analysis**

Here we make assumptions regarding functional forms and parameter values so that we can obtain an exact analytic solution for the equilibrium. We consider various qualitative properties of the model and then analyze the effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction.

**Further assumptions**

First, we choose functional forms for the constant-returns-to-scale components of the production technologies. We let

\[
\psi_N(\ell, d) = \ell^\alpha d^{1-\alpha}
\]

(31)

\[
\psi_f(\ell, d, f) = \ell^\alpha \left( \mu d^\rho + (1 - \mu) f^\rho \right)^{\frac{1-\alpha}{\rho}}.
\]

(32)

With these Cobb-Douglas functional forms, the elasticity of substitution between labor and intermediate goods is one for each firm. This is consistent with our data, in the sense that expenditure shares for labor and intermediate goods by Chilean manufacturing plants over the period 2001 to 2006 were roughly constant. We can think of importers as using a composite intermediate good, the quantity of which is given by

\[
z_f(a) = \left( \mu d_f(a)^\rho + (1 - \mu) f_f(a)^\rho \right)^{\frac{1}{\rho}}.
\]

(33)

Here the elasticity of substitution between domestic and imported intermediate goods is \( 1/(1 - \rho) \). The price of a unit of the composite intermediate good is then
\[ P = \left( \mu^{1+\rho} + (1-\mu)^{1+\rho} \left( (1+\tau)\rho \right)^{\frac{1-\rho}{\rho}} \right)^{\frac{1-\rho}{\rho}}. \]  

(34)

Second, we choose a functional form for the distribution of efficiency draws. We follow Chaney (2008) in letting the distribution be Pareto:

\[ G(a) = 1 - (\theta / a)^\gamma, \]

(35)

\( a \geq \theta \), where \( \theta > 0 \) and \( \gamma > 1 \). The size distribution of firms in the model is proportional to the distribution of efficiency draws. As figure 2 shows, the size distribution of plants in the data is consistent with a Pareto distribution.

**Figure 2. Plant size distribution (Static)**

Third, we restrict our attention to the case where not all entering firms choose to operate and not all operating firms choose to import. That is, we assume that parameter values are such that \( \theta < a_s < a_f \), where \( a_s \) satisfies
\[ \pi_N(a_N) = 0, \tag{36} \]

and \( a_j \) satisfies

\[ \pi_I(a_j) = \pi_N(a_j). \tag{37} \]

We refer to \( a_N \) as the cutoff for operating and to \( a_j \) as the cutoff for importing. Under these assumptions, there is an exact analytic solution for the equilibrium, which we provide in Appendix A. In Appendix B, we provide the solution under the assumption that the economy is in autarky.

**Costs and benefits of importing**

Each operating firm weighs the cost of importing against the benefit. In our model, the net cost of importing is fixed at \( \phi_I - \phi_N \) units of output for each firm. (Our assumption that \( a_N < a_J \) guarantees that \( \phi_I - \phi_N > 0 \).) In contrast, the total benefit of importing is increasing in the firm’s scale of operation. A firm that switches from technology \( N \) to technology \( I \) increases its output, employment, expenditure on intermediate goods, and variable profits (profits gross of fixed costs) by a factor of \( B \), where

\[ B = \eta P \frac{(1-a)^\nu}{1+\nu}. \tag{38} \]

(Our assumption that \( a_N < a_J \) guarantees that \( B > 1 \).) We refer to \( B \) as the benefit of importing. An implication of (38) is that, in the context of this model, it does not matter whether the benefit of importing comes from a lower price or an efficiency advantage; only the combination of these factors matters. Imported intermediates may be more
expensive than domestic intermediates \((P > 1)\), but there will be a benefit from using them if \(\eta\) is sufficiently high. Alternatively, imported intermediates may have an undesirable effect on firm efficiency \((\eta < 1)\), but there will be a benefit from using them if \(P\) is sufficiently low. The findings of Kugler and Verhoogen (2009) suggest that the first case is more consistent with the data than the second.

Sources of differences between importers and non-importers

In the data, importers are very different from non-importers. In the model, there are two causes of this: a selection effect and a technology upgrading effect. Because of the fixed cost, only the firms with the highest efficiency draws choose to become importers. This is the selection effect. If a firm chooses to use technology \(I\), then its output, employment, expenditure on intermediate goods, and variable profits are larger by a factor of \(B\) than if the firm had chosen to use technology \(N\). This is the technology upgrading effect.

We can measure the contribution of each effect to the relative size of importers (the measure of size can be gross output, employment, total expenditure on intermediate goods, or variable profits). In the absence of any selection effect, technology upgrading would result in importers being \(B\) times larger than non-importers. The selection effect determines the extent to which the size ratio is greater than \(B\). Let \(S\) be size of the average importer relative to the size of the average non-importer. Using a logarithmic decomposition to account for this ratio, the share due to the technology upgrading effect is given by \(\log B / \log S\); the remaining share is due to the selection effect. Later, our calibration procedure will pin down the magnitudes.
Aggregate technological efficiency

Along with the benefit of importing, an important statistic in the model — it shows up in the calculation of every equilibrium object — is

\[
A = \frac{1}{\gamma \phi_M} \left( \int_{\omega} a \, dG(a) + B \int_{\omega} \infty \, a \, dG(a) \right).
\] (39)

We refer to \( A \) as an aggregate technological efficiency index because the expression in parentheses is a weighted average of firms’ efficiency draws. The relative weight on importers, \( B \), accounts for the benefit of using technology \( I \). Implicitly, the weight on firms that choose not to operate is zero. Since the cutoffs \( a_N \) and \( a_I \) are equilibrium objects, we can simplify (39) to express aggregate technological efficiency entirely in terms of parameters:

\[
A = \theta \left( \phi_N^{1-\gamma} + (B-1)^\gamma (\phi_I - \phi_N)^{1-\gamma} \right)^{1/\gamma}.
\] (40)

Aggregate technological efficiency affects many important aspects of the economy. For example, aggregate output can be expressed as

\[
Y = \frac{A^\alpha}{\alpha} KL^\alpha\gamma,
\] (41)

where \( K \) is given in Appendix A. The wage and the measure of entrants are also proportional to \( A^{(1-\gamma)/\alpha} \), as is social welfare if \( \tau = 0 \). In addition, the cutoffs for operating and importing are proportional to \( A \):

\[
a_N = A \phi_N
\] (42)

\[
a_I = \frac{A(\phi_I - \phi_N)}{B-1}.
\] (43)
Notice that the cutoff for importing is increasing in the cost of importing and decreasing in the benefit of importing.

*Effects of tariff reduction, terms-of-trade improvement, and trade-cost reduction*

In our experiments, we consider the effects of trade liberalization, as given by a decrease in $\tau$; an exogenous improvement in the terms of trade, as given by a decrease in $p$; and a reduction in trade costs, as given by a decrease in $\phi$. Since our model is static, we view it as capturing the long-term effects of permanent changes. These three types of changes have many qualitative effects in common.

**Proposition.** Tariff reduction, an improvement in the terms of trade, or a decrease in the cost of importing has the following effects: (i) the cutoff for operating increases, (ii) the cutoff for importing decreases, (iii) the (real) wage increases, (iv) output increases, (v) firm entry increases, and (vi) social welfare increases.

Proving the proposition just involves finding the signs of various partial derivatives using Appendix A, so we omit it here. A decrease in either $\tau$ or $p$ decreases the price of the composite intermediate input, $P$, which increases the benefit of importing, $B$, which leads to an increase in aggregate technological efficiency, $A$ (see (34), (38), and (40)). A decrease in $\phi$ does not change $P$ or $B$ but, rather, directly increases $A$. 

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As the proposition indicates, all the changes result in a reallocation of resources across firms. The least efficient firms exit because they can no longer profitably operate at the higher wage, the most efficient non-importers become importers, and technological efficiency increases. With a decrease in \( \tau \) or \( p \), the effects of reallocation are augmented because, for a given efficiency draw, there is an increase in the optimal scale at which technology \( I \) is operated and a decrease in the optimal scale at which technology \( N \) is operated. By contrast, with a decrease in \( \phi \), both technologies are operated at smaller scales due to the increase in the wage.

Yi (2003) stresses the need to develop trade models that can generate large increases in trade in response to small decreases in tariffs. Our model has this potential, as it has both extensive and intensive margins of importing. The change in total imports resulting from a change in \( \tau \) or \( p \) can be decomposed into changes on three margins: (i) the measure of entrants, (ii) the cutoff for importing, and (iii) use of the imported good by existing operators of technology \( I \). Specifically,

\[
d \log F = d \log M + (1 - \gamma) d \log a_j + d \log Q,
\]

where

\[
Q = f_i(a) / a.
\]

(As Appendix A shows, \( f_i(a) \) is proportional to \( a \), so \( Q \) does not depend on \( a \).) The percentage change in existing importers’ use of the imported intermediate good is equal to the percentage change in \( Q \). Thus the first two margins are extensive, while the third is intensive. Importantly, with the presence of extensive margins, the Armington elasticity — the elasticity of substitution between domestic and imported goods — is
greater at the macro level than at the micro level. As a result, the model has the potential to generate a large Armington elasticity at the macro level without assuming an unusually large elasticity at the micro level.

**Quantitative analysis**

Here we calibrate the model using the data on the Chilean manufacturing sector discussed earlier. Then we use the calibrated model to perform a number of counterfactual numerical experiments.

**Calibration**

Our strategy for calibrating the model is as follows. First, we normalize certain parameters that do not affect the quantitative findings in which we are interested. Then we take some parameter values from the literature. Finally, we select the remaining parameter values to match important statistics on plants’ importing behavior.

As normalizations, we set the labor endowment, $L$; the lower bound on the Pareto distribution, $\theta$; and the cost of entry, $\phi_M$, to one. As (38) indicates, it does not matter whether the benefit of importing comes from a lower price or an efficiency advantage. Consequently, we normalize the price of the composite intermediate good, $P$, to one and initially allow the size of the benefit of importing to be determined by the parameter $\eta$. To obtain $P = 1$, we set the tariff rate, $\tau$, to be consistent with the data and then choose the relative price of the imported intermediate good, $p$. The World Bank’s World
dataBank reports that Chile’s average tariff rate in 2001 was 8 percent, so we set \( \tau = 0.08 \). Then we set \( p = 0.11 \) to obtain \( P = 1 \).

We take the values of \( \rho \) and \( \nu \) from the literature. There is debate over the elasticity of substitution between domestic and imported goods, the Armington elasticity. Ruhl (2004) tries to resolve this debate and argues in favor of an elasticity of two at the micro level (measurements at the macro level differ when an extensive margin is involved). Following this, we set \( \rho = 0.5 \) so that an importer’s elasticity of substitution between domestic and foreign inputs is two. The parameter \( \nu \) determines the degree of decreasing returns at the firm level. Calibrating a competitive model with heterogeneous firms operating decreasing-returns-to-scale technologies, Atkeson and Kehoe (2005) find that \( \nu = 0.85 \) is consistent with data on U.S. manufacturing plants; we adopt this value here.

The values of \( \alpha \) and \( \mu \) are selected to match expenditure shares in the data. In the data, expenditure on labor as a share of expenditure on both labor and intermediate goods is 0.34, so we set \( \alpha = 0.34 \). Among plants that import, expenditure on imported intermediate goods as a share of total expenditure on intermediate goods is 0.39; we set \( \mu = 0.78 \) to match this.

The remaining four parameters are the fixed cost of operating technology \( N \), \( \phi_N \); the fixed cost of operating technology \( I \), \( \phi_I \); the TFP of technology \( I \) relative to technology \( N \), \( \eta \); and the shape parameter of the Pareto distribution, \( \gamma \). We jointly select the values of these four parameters so that the following four statistics hold in the model: (i) 13 percent of operating plants use imported intermediate goods, (ii) the
average gross output of importers relative to non-importers is 4.5, (iii) the coefficient of variation for gross output is 6.0, and (iv) 90 percent of entrants choose to operate.\textsuperscript{6} The resulting parameter values are $\phi_N = 1.05$, $\phi_I = 1.66$, $\eta = 1.21$, and $\gamma = 2.02$. To place these numbers in context, consider the following. The fixed cost of operating technology $N$ is 10 percent of the average non-importer’s gross output and 68 percent of its variable profits. The fixed cost of operating technology $I$ is 4 percent of the average importer’s gross output and 24 percent of its variable profits. Since $B = 1.21$, switching from technology $N$ to technology $I$ increases a firm’s gross output, employment, expenditure on intermediate goods, and variable profits by 21 percent. Our value for the shape parameter of the Pareto distribution is consistent with the findings of Del Gatto, Mion, and Ottaviano (2007). Table 6 summarizes the calibration.

\textsuperscript{6} The last statistic is not based on data (we do not observe plants that do not operate), but the quantitative results in which we are interested are not sensitive to the particular percentage chosen. We simply need the cutoff for operating to be binding.
Table 6. Summary of the calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\phi_M$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$p$</td>
<td>0.11</td>
<td>Chosen so that $P = 1$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
<td>Ruhl (2004)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.85</td>
<td>Atkeson and Kehoe (2005)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.08</td>
<td>World Bank’s World dataBank</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.34</td>
<td>Expenditure on labor as a share of total expenditure on labor and intermediate goods</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.78</td>
<td>Importers’ expenditure on imported intermediate goods as a share of total expenditure on intermediate goods: 0.39</td>
</tr>
<tr>
<td>$\phi_N, \phi_I, \eta, \gamma$</td>
<td>1.05, 1.66, 1.21, 2.02</td>
<td>Jointly chosen to match 4 statistics: (i) importers are 13 percent of operating plants, (ii) gross output of average importer relative to gross output of average non-importer is 4.5, (iii) coefficient of variation for gross output is 6.0, and (iv) 90 percent of entrants choose to operate</td>
</tr>
</tbody>
</table>

The calibrated model allows us to account for the average importer being 4.5 times larger, in terms of gross output, than the average non-importer. Using the decomposition discussed in the previous section, with $B = 1.21$ the share due to the selection effect is 87.4 percent and the share due to the technology upgrading effect is 12.6 percent.

The calibration guarantees that the model satisfies Facts 1 to 3. We did not use Fact 4 in the calibration, but the calibrated model is not far off. In the data, importers are 1.3 times more productive than non-importers, as measured by value added per worker.
In the calibrated model, importers are still substantially more productive than non-importers, by a factor of 1.2. It is worth pointing out that the ratio would be one if we did not take into account fixed costs of operation (as noted by Atkeson and Kehoe (2005)).

But our interpretation of the fixed costs is that they are expensed costs of setting up business relationships with input suppliers, so they must be subtracted from gross output to obtain value added. Though $\phi_1 > \phi_N$, expenditure on fixed costs relative to average output is smaller for importers than non-importers. Consequently, measured value added per worker is higher among importers than among non-importers. Table 7 provides a comparison between the facts in the data and in the model.

**Table 7. Data vs. Model (Static)**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importers as a share of operating plants (%)</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Average importer’s expenditure on imported intermediate goods as a share of total expenditure on intermediate goods (%)</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>Gross output of average importer relative to average non-importer</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Value added per worker of importers relative to non-importers</td>
<td>1.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

**Experiments**

We use the calibrated model to perform four numerical experiments: (i) elimination of the *ad valorem* tariff, (ii) an exogenous improvement in the terms of trade, (iii) a reduction in fixed costs, and (iv) going from autarky to free trade.

We set up the first two experiments to be of the same magnitude. We consider (i) the elimination of the 8 percent tariff in the benchmark calibration and (ii) an equivalent
reduction in the terms of trade (a 7.4 percent decrease in $p$). Table 8 presents the percentage changes in a number of statistics of interest. These two experiments have almost identical quantitative results. The only differences are with respect to the changes in exports, consumption, and tariff revenue. The main difference between the two experiments is that the tariff reduction results in the loss of all tariff revenue, while the terms-of-trade improvement increases tariff revenue. Our index of social welfare is the consumption level. Because of the changes in tariff revenue, the welfare gain from tariff elimination is much smaller than the welfare gain from the terms-of-trade improvement. The terms-of-trade improvement allows the economy to import just as much as after the tariff elimination, but without exporting as much output.

Both experiments result in a large increase in trade: the quantity of imports increases by 98.8 percent. Using the decomposition given by (44), we find that 4.7 percent of the increase is due to the change in the measure of entrants, 69.7 percent is due to the change in the cutoff for importing, and 25.6 percent is due to the change in existing importers’ use of the imported intermediate good. Thus the extensive margins account for the majority of the increase. In the benchmark calibration, of the firms that enter, 10 percent choose not to operate, 78 percent choose to operate technology $N$, and 12 percent choose to operate technology $I$. Following the change, entry increases by 3.3 percent and, of the firms that enter, 21 percent choose not to operate, 49 percent choose to operate technology $N$, and 30 percent choose to operate technology $I$ (see Table 9).

When there are extensive margins, the Armington elasticity at the macro level is greater than the Armington elasticity at the micro level. Here the Armington elasticity at the level of an individual firm is 2, while the measured aggregate Armington elasticity is
10.5. This large aggregate Armington elasticity is consistent with the empirical findings of researchers who estimate it using data from trade liberalizations. As Ruhl (2004) notes, these researchers typically find Armington elasticities ranging from 4 to 15. This experiment makes clear that the extensive margins play an important role in generating large increases in trade from small decreases in tariffs.

**Table 8. Three experiments**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Tariff elimination (% change)</th>
<th>Terms-of-trade improvement (% change)</th>
<th>Fixed-cost reduction (% change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>1.0</td>
<td>5.1</td>
<td>2.4</td>
</tr>
<tr>
<td>Wage</td>
<td>3.3</td>
<td>3.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Measure of entrants</td>
<td>3.3</td>
<td>3.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Output</td>
<td>3.3</td>
<td>3.3</td>
<td>1.1</td>
</tr>
<tr>
<td>Tariff revenue</td>
<td>−100.0</td>
<td>84.0</td>
<td>59.7</td>
</tr>
<tr>
<td>Real GDP</td>
<td>1.5</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>Price of composite intermediate</td>
<td>−3.0</td>
<td>−3.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Imports</td>
<td>98.8</td>
<td>98.8</td>
<td>59.7</td>
</tr>
<tr>
<td>Technological efficiency</td>
<td>6.4</td>
<td>6.4</td>
<td>2.2</td>
</tr>
<tr>
<td>Benefit of importing</td>
<td>12.2</td>
<td>12.2</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**Table 9. Entrants’ operating decisions (percentage shares)**

<table>
<thead>
<tr>
<th>Decision</th>
<th>Benchmark</th>
<th>Tariff elimination</th>
<th>Terms-of-trade improvement</th>
<th>Trade-cost reduction</th>
<th>Autarky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not operate</td>
<td>10</td>
<td>21</td>
<td>21</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>Operate technology $N$</td>
<td>78</td>
<td>49</td>
<td>49</td>
<td>56</td>
<td>97</td>
</tr>
<tr>
<td>Operate technology $I$</td>
<td>12</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>

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In addition to reporting how the changes affect our theoretical measure of technological efficiency, we also report how they affect measured real GDP. To calculate real GDP in our model in a manner similar to the way real GDP is calculated in the data, we use base-period prices.\(^7\) If we let \(t\) denote the current period, then real GDP at period-0 prices is

\[
RGDP_t = Y_t - D_t - (1 + \tau_0) p_0 F_i - \phi_i M_{Ni} - \phi_i M_r + \tau_0 p_0 F_i, \tag{46}
\]

where \(M_{Ni}\) is the measure of non-importers and \(M_r\) is the measure of importers. The first term is gross output, the next four terms are expenditure on intermediate goods and expensed costs of operation, and the last term is the rebate of tariff revenue, all valued at period-0 prices. We assume that investment in new firms is a tangible investment rather than an intermediate input, so these expenditures are not subtracted from gross output. The experiments result in a 1.5 percent increase in real GDP. Kehoe and Ruhl (2008) explain why we should not expect this number to be larger.

The first two experiments involved increasing the benefit of importing. The third experiment involves decreasing the fixed cost of importing, \(\phi_i\). To make this experiment somewhat comparable in magnitude to the first two experiments, we select the percentage decrease in \(\phi_i\) so that the share of entrants that choose to operate technology \(I\) is the same as after the previous two experiments (30 percent). This requires a 14.2 percent decrease in \(\phi_i\). The results are presented in Table 8. Since \(B\) does not change in this case, we can isolate the basic effects of reallocation. The increase in aggregate

\(^7\) See Gibson (2010), Kehoe and Ruhl (2008), and Bajona, Gibson, Kehoe, and Ruhl (2010) for further discussion of measured productivity and GDP in trade models.
technological efficiency is not as large as in the previous two experiments, so the changes are mostly smaller in magnitude.

Finally, we consider the extreme case of going from autarky to free trade. In the free trade case, we take the relative price of the imported good from the benchmark calibration. This experiment puts an upper bound on the effects of tariff reduction alone. The welfare gain is 5.2 percent. Table 10 presents the changes in some statistics of interest.

**Table 10. Experiment: Going from autarky to free trade**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Percentage change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>5.2</td>
</tr>
<tr>
<td>Wage</td>
<td>5.2</td>
</tr>
<tr>
<td>Measure of entrants</td>
<td>5.2</td>
</tr>
<tr>
<td>Output</td>
<td>5.2</td>
</tr>
<tr>
<td>Technological efficiency</td>
<td>10.3</td>
</tr>
</tbody>
</table>

**Conclusion**

Most trade models with heterogeneous firms emphasize firms’ export decisions. We have developed a complementary approach that emphasizes firms’ import decisions. Our model has desirable qualitative properties, is straightforward to calibrate, and quantitatively captures many important features of plants’ importing behavior in the data. In terms of policy implications, the welfare gains from unilateral trade liberalization may not be large enough to persuade policymakers in countries that depend on tariffs for revenue. Individuals associated with small non-importing firms are also unlikely to support liberalization. Improvements in the terms of trade generate large welfare gains,
but are external. The simple modeling framework developed here allows for many interesting extensions. These include the roles of dynamics, aggregate uncertainty, multiple countries, monopolistic competition, and joint import-export decisions.
Costs of Starting to Trade
and Costs of Continuing to Trade

Models of international trade increasingly emphasize the trade decisions of individual firms or plants. Following Melitz (2003), these models typically feature heterogeneous firms that face fixed costs of exporting their goods to foreign markets. (See Helpman (2006) for a survey of this literature.) For simplicity, these models are typically static or, if dynamic, hold fixed each firm’s technological efficiency over time. Our modeling approach here also involves heterogeneous firms making trade decisions subject to fixed costs, but we differ from the literature in two important ways: by emphasizing the decision to use imported intermediate goods and by taking firm dynamics seriously.

Modeling the decision to import rather than the decision to export may at first seem to be of little consequence, but the two decisions involve entirely different considerations by firms. In deciding whether to export its good, a firm considers the characteristics of a foreign market and weighs the expected profits from exporting to that market against the costs of entering it. The emphasis is on foreign demand. In deciding whether to use an imported intermediate good, a firm considers how using that input will affect its production process and weighs the additional expected profits against the costs of developing a trade relationship with a foreign input supplier. The emphasis is on technology. Consistent with this, we model a firm’s decision to import as a choice between two technologies: a technology that uses only domestic inputs and a technology that uses both domestic and foreign inputs. In a broader sense, we develop a dynamic general equilibrium model of technology adoption when there are adoption and
continuation costs.

By taking firm dynamics seriously, we can analyze the relative importance of two types of fixed costs: costs of starting to trade and costs of continuing to trade. With respect to importing intermediate goods, we think of these fixed costs as the costs of developing and maintaining relationships with foreign input suppliers. As Melitz (2003) points out, in a stationary equilibrium with no shocks to firms’ technological efficiencies, the distinction between fixed costs of starting to trade and fixed costs of continuing to trade is without consequence: a firm’s trade status is fixed over time and only the expected discounted present value of the fixed costs matters. By contrast, when we allow for idiosyncratic shocks to firms’ efficiencies over time, start-up and continuation costs have different implications, even in a stationary equilibrium. With sunk costs of starting to trade, uncertainty about future shocks discourages firms from investing in trade relationships even when, from a purely static point of view, they would prefer to be engaged in trade. Costs of continuing to trade imply that firms may stop trading even after having paid the start-up costs. Together, these costs allow us to capture the dynamics of firms’ trade status over time.

Our model is motivated by the data. Distinguishing between importers and non-importers perhaps would not be of much importance if the two types of producers appeared similar in the data. But this is not the case. Only a small fraction of plants choose to use imported intermediate inputs and plants that do use imported intermediate inputs are much larger than plants that do not. We document this phenomenon using recent surveys of Chilean manufacturing plants. In the Chilean data, plants that use imported intermediate inputs are 3.6 times larger in terms of gross output and 1.2 times
more productive in terms of value-added per worker than are non-importers. Despite the apparent advantage of importing, most plants do not. Moreover, plants switch import status over time. In our data, 25 percent of plants import at some point, with 12 percent importing for the entire period and 13 percent switching import status at least once.

Our interpretation of the data is that plants would prefer to use imported intermediate inputs but that there are barriers to doing so that take the form of fixed costs. Due to these fixed costs, only the largest, most efficient plants choose to import. Plants start importing if their efficiency increases enough to cover the additional costs and stop importing if their efficiency decreases. Our model formalizes the role imported intermediate inputs play in firm performance by giving each firm a choice between two technologies, one that uses only domestic inputs, and another that uses both domestic and imported inputs.

Firms receive a performance increase from using the importing technology but it requires the payment of an additional fixed cost to operate. Most of the literature focuses on selection effects, while we allow for both a selection effect and a technology upgrade effect. Therefore, only the most efficient firms choose to import. Sunk costs ensure that the barrier to start importing is greater than the barrier to stop importing, first by adding additional costs to importing and second by providing firms with an incentive to continuously import to avoid having to repay start-up costs. As a result, trade status becomes history-dependent. That is, there is a band of firm efficiency levels where two firms with the same efficiency draw may be using different technologies.

Quantitatively, the model captures many important features of the data including the dynamics of import status, entry and exit rates, the size distribution, and the large
performance advantage associated with using imported intermediate inputs.

Our paper connects to three strands of the literature: the first concerns the characteristics of producers that engage in trade relative to those that do not, the second emphasizes producers’ decisions regarding importing rather than exporting, and the third seeks to develop better dynamic models of producers’ trade decisions.

A large literature establishes a connection between firm trade status and firm performance. This has been done by Bernard et al. (2003); Bernard, Jensen, and Schott (2005); Eaton, Kortum, and Kramarz (2008); and others. It is a stylized fact that firms that trade are larger and more productive than those that are not. This is particularly true for exporters. For the United States, Bernard and Jensen (1999) find that exceptional performers become exporters, and exporting can further increase plant performance. Bernard et al. (2003) and Bernard, Jensen, and Schott (2005) find similar differences between U.S. exporters and non-exporters. Using Chilean data Pavcnik (2002) finds empirical evidence to support the correlation between export status and plant performance. Eaton, Kortum, and Kramarz (2008) show that, for French firms, efficiency differences account for a large part of export decisions. Empirically, Kasahara and Rodrigue (2008) demonstrate that importers are exceptional performers.

Analysis of firms’ import decisions is much less common than analysis of firms’ export decisions, despite the fact that a large and growing share of trade is trade in intermediate goods (Hummels, Ishii, and Yi, 2001; Helpman, 2006). Chapter 2 analyzes a static version of the model here. Kugler and Verhoogen (2009) use a model where firms with high ability capture larger gains from using high quality inputs. When high quality inputs are imported, the most efficient firms become importers. Amiti and
Konings (2007), examining data from Indonesia, find that lowering tariffs on imported inputs can increase plant productivity through learning, variety, and quality effects. Halpern, Koren, and Szeidl (2009) find using Hungarian data that imported inputs increase plant productivity through complementarity and quality channels. Perhaps the work most closely related to ours is Kasahara and Lapham (2005), who model import choice when firms face stochastic fixed costs.

If firm efficiency is subject to idiosyncratic shocks, then sunk costs cause trade decisions to become history-dependent since it will be less costly for some firms to continuously trade than to start and stop. A similar motivation for history dependence can be found in Alessandria and Choi (2007). Other dynamic models of trade decisions include Arkolakis (2010), Ramanarayanan (2007), Irarrazabal and Opromolla (2009), and Das, Roberts, and Tybout (2007). Luttmer (2007, 2010) uses methods similar to the ones developed here to model firm dynamics in a growth model.

**Data**

In order to motivate our model and, later, to calibrate it, we consider plant-level data from a tradable sector, manufacturing, in a small open economy, Chile. We use data from the annual census of Chilean manufacturing plants (Encuesta Nacional Industrial Anual, or ENIA), collected by Chile’s Instituto Nacional de Estadísticas, from 2001 to 2006. An earlier version of this census was used by Liu (1993), Levinsohn (1999), and Pavcnik (2002), among others. We use a more recent, revised version of the census (Navarro (2008) also uses this version).
The unit of observation in the data is the plant. The census covers a total of 8,014 different plants over the period 2001 to 2006. The data include detailed information on each plant’s inputs, employment, and expenditures. Expenditure on imported raw materials represents a significant outlay for importing plants, on average 7 percent of their gross output, making it a non-trivial part of importing behavior. For each survey, we divide the plants into those that do not report any use of imported raw materials, which we refer to as non-importers, and those that do, which we refer to as importers. Since our model is concerned with long-run effects, when we calculate statistics we average over the sample period, 2001 to 2006. On average, importers are 20 percent of plants. We consider how importers differ from non-importers and how plants change their import status over time.

**Table 11. Importer premia (Dynamic)**

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Ratio of average importer to average non-importer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross output</td>
<td>3.6</td>
</tr>
<tr>
<td>Total materials</td>
<td>3.4</td>
</tr>
<tr>
<td>Employment</td>
<td>3.1</td>
</tr>
<tr>
<td>Value added</td>
<td>3.9</td>
</tr>
<tr>
<td>Value added per worker</td>
<td>1.3</td>
</tr>
</tbody>
</table>

*Differences between importers and non-importers*

First we consider how importers differ from non-importers. Even though most plants do not import, those that do are much larger and more productive than plants that do not. Table 11 summarizes these findings. Differences between importers and non-
importers in this data are robust to a variety of statistical controls (see Chapter 1 for details).

Plant dynamics

The Chilean manufacturing sector is characterized by simultaneous plant entry and exit. The average exit rate over the sample period, 2001 to 2006, is 12.1 percent of active plants. The average entry rate is 12.8 percent. Importers tend to be larger and more efficient than non-importers and are therefore less likely to exit than non-importers, as documented by Lopez (2006). The difference between exit rates for importers and non-importers is about 4 percent. Table 12 summarizes the transition probabilities found in the data.8

Table 12. Transition probabilities (％)

<table>
<thead>
<tr>
<th></th>
<th>Importer</th>
<th>Non-importer</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importer</td>
<td>77</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>Non-importer</td>
<td>3</td>
<td>84</td>
<td>13</td>
</tr>
<tr>
<td>Exit</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

Importers are less likely to exit than non-importers and therefore on average have longer life spans. On average, 11 percent of operating plants switch trade status each year. On average 14 percent of importers stop importing every year. While on average only 3 percent of non-importers start importing each year. The transition probabilities for

---

8 We interpret zero expenditure on imported raw materials to mean that a plant is no longer importing, but we cannot tell whether these plants are drawing from stockpiles. Stockpiling behavior is most likely small due to the annual frequency of our data, and the aggregate variable we use to indicate importing status. Less than 3 percent of reported expenditure on imported raw materials is more than two standard deviations above the mean (which may suggest a particularly large outlay). In the six years of data that we consider, 5 percent of plants switch import status more than once.
importers show a higher level of history dependence than those of non-importers which is consistent with the presence of an entry cost similar to what is found in the model. Plant entry, exit, and import status display no noticeable trend over the sample period.

Model

Consider a small open economy in a stationary competitive equilibrium. (Due to the assumption of stationarity, we omit time from the notation that follows whenever possible.) The economy produces a single output good. The price of the output good is normalized to one. The good may be used in four different ways: for consumption, for export, as an intermediate good, or for payment of fixed costs.

There is no international borrowing and lending, but there is trade in goods. The small open economy exports some of its good and imports a single intermediate good from the rest of the world. Since the economy is small, it takes the relative price of the two goods — the terms of trade — as given. The small open economy may impose an ad valorem tariff on imports.

The single output good is produced by a continuum of single-plant firms. The firms are heterogeneous in technological efficiency. There are decreasing returns to scale in production, so each firm produces output at its optimal level, with more efficient firms producing more output. Each firm, after learning its efficiency level, has a choice of two technologies. One technology uses labor and the domestically produced intermediate good as inputs. The other technology uses labor, the domestically produced intermediate good, and the imported intermediate good as inputs. The choice of technology separates firms into non-importers and importers. Firms’ efficiency levels evolve stochastically.
over time. Firms make endogenous decisions regarding entry, exit, and choice of technology. Each firm also faces an exogenous probability of death each instant.

In this section, we specify the consumer’s problem, the static decisions of firms, the dynamic decisions of firms, firm entry, and the distribution of firms. We then define a stationary competitive equilibrium and provide an algorithm for calculating it.

**Consumer**

In the small open economy, there is a representative consumer who is endowed with quantity of labor \( \bar{L} \) and ownership of the firms. At each instant, the consumer maximizes consumption, \( C \), subject to the budget constraint

\[
C = w\bar{L} + T + \Pi .
\] (47)

The consumer’s three sources of income are labor income, \( w\bar{L} \), where \( w \) is the wage; transfers from the government, \( T \); and the profits of firms, \( \Pi \). The consumer’s subjective discount rate is \( \rho \), \( \rho > 0 \).

**Firms’ static decisions**

Given a firm’s efficiency level and its choice of technology, the firm’s optimal decisions regarding inputs and output are simple static decisions at each instant. Let technology \( N \) (where \( N \) stands for non-importer) be the technology that does not use the imported intermediate good as an input. Let technology \( I \) (where \( I \) stands for importer) be the technology that uses the imported intermediate good as an input. Let \( \eta \) be the efficiency of technology \( I \) relative to technology \( N \).
Consider a firm with efficiency $x$ operating technology $N$. The firm’s output is given by

$$y_N(x) = x^{1-\nu} \Omega_N \left( \ell_N(x), d_N(x) \right)^\nu,$$

where $\Omega_N$ is a standard production function with constant returns to scale, $\ell_N(x)$ is the input of labor, $d_N(x)$ is the input of the domestically produced intermediate good, and $0 < \nu < 1$. The firm’s profits are

$$\pi_N(x) = y_N(x) - w\ell_N(x) - d_N(x) - \phi_N,$$

where $\phi_N$ is the fixed cost of operating. The firm chooses $\ell_N(x)$ and $d_N(x)$ to satisfy the profit-maximization conditions

$$\nu x^{1-\nu} \Omega_N \left( \ell_N(x), d_N(x) \right)^{-1} \Omega_{N_k} \left( \ell_N(x), d_N(x) \right) - w = 0$$

$$\nu x^{1-\nu} \Omega_N \left( \ell_N(x), d_N(x) \right)^{-1} \Omega_{N_d} \left( \ell_N(x), d_N(x) \right) - 1 = 0,$$

where $\Omega_{N_k} = \partial \Omega_N / \partial k$, $k = \ell, d$.

Now consider a firm with efficiency $x$ operating technology $I$. The firm’s output is given by

$$y_I(x) = (x\eta)^{1-\nu} \Omega_I \left( \ell_I(x), d_I(x), f_I(x) \right)^\nu,$$

where $\Omega_I$ is a standard production function with constant returns to scale, $\ell_I(x)$ is the input of labor, $d_I(x)$ is the input of the domestically produced intermediate good, $f_I(x)$ is the input of the imported intermediate good, and $\eta > 0$. The firm’s profits are

$$\pi_I(x) = y_I(x) - w\ell_I(x) - d_I(x) - (1 + \tau)p f_I(x) - \phi_I,$$
where $p$ is the relative price of the imported intermediate good, $\tau$ is the ad valorem tariff on imports, and $\phi_{I}$ is the fixed cost of operating. The firm chooses $\ell_{I}(x), d_{I}(x),$ and $f_{I}(x)$ to satisfy the profit-maximization conditions

$$
\nu(x\eta)^{r-s}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right)^{r-1}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right) - w = 0 \tag{54}
$$

$$
\nu(x\eta)^{r-s}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right)^{r-1}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right) - 1 = 0 \tag{55}
$$

$$
\nu(x\eta)^{r-s}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right)^{r-1}\Omega_{I}\left(\ell_{I}(x), d_{I}(x), f_{I}(x)\right) - (1+\tau)p = 0 \tag{56}
$$

where $\Omega_{I} = \partial\Omega_{I}/\partial k$, $k = \ell, d, f$.

**Firms’ dynamic decisions**

Each firm’s efficiency evolves stochastically according to a continuous Markov process. In addition, at each instant each firm faces exogenous probability of death $\delta$, $0 \leq \delta < 1$. A firm’s operating and technology decisions are dynamic, or forward-looking, decisions in the sense that they take into account the firm’s expectations for the future.

To remain in operation, a firm must continuously operate either technology $N$ or technology $I$ by paying the relevant fixed cost, $\phi_{N}$ or $\phi_{I}$. If a firm ever fails to pay the relevant fixed cost, it exits forever. Because of these fixed costs, a firm will endogenously choose to exit when its efficiency gets sufficiently low. We let $b$ denote the cutoff to operate technology $N$ and let $c$ denote the cutoff to operate technology $I$.

If a firm chooses to operate, it must choose which technology to use. This decision is forward-looking because switching from technology $N$ to technology $I$ involves a sunk cost of $\phi_{Ni}$ units of output. A firm can costlessly switch from technology
I to technology $N$, but if the firm ever wants to switch back to technology $I$ it must pay the sunk cost $\phi_{si}$ again. Given this sunk cost, a non-importer will only switch to using technology $I$ if its efficiency is sufficiently high. We let $B$ denote the cutoff to start importing. Thus non-importers have efficiencies in the range $x \in [b, B)$ and importers have efficiencies in the range $x \in [c, \infty)$.

Before specifying the dynamic programming problems of firms, we define some notation that we use throughout the paper. We specify the dynamic problems of firms using Bellman equations. (Stokey (2009) refers to this as the direct approach. The indirect approach, which is equivalent, uses Hamilton-Jacobi-Bellman equations. See Stokey (2009) and Dixit and Pindyck (1994) for details.) Consider a firm with efficiency $x \in [z, Z)$ that faces lower cutoff $z$ and upper cutoff $Z$. Eventually the firm will face an adjustment, by which we mean that one of the following three events will occur: (i) the firm’s efficiency will reach $z$, (ii) the firm’s efficiency will reach $Z$, or (iii) the firm will exogenously die. We allow $Z = \infty$, in which case event (ii) never occurs. Denote the first time that (i) or (ii) occurs by the random variable $T(x, z, Z)$. Define the probability that adjustment (i) occurs first as

$$
\psi(x, z, Z) = E\left( e^{-\rho T(x, z, Z)} \right) \Pr\left( X(T(x, z, Z)) = z \right).
$$

(57)

Define the probability that adjustment (ii) occurs first as

$$
\Psi(x, z, Z) = E\left( e^{-\rho T(x, z, Z)} \right) \Pr\left( X(T(x, z, Z)) = Z \right).
$$

(58)

Since the consumer owns the firms, we will also need counterparts to (57) and (58) that take into account the consumer’s rate of time preference, $\rho$. Define the expected discounted value of the indicator function for the event of adjustment (i) occurring first as
Define the expected discounted value of the indicator function for the event of adjustment (ii) occurring first as
\[
\tilde{\Psi}(x, z, Z) = E\left(e^{-\rho T(x, z, Z)}\right)\Psi(x, z, Z).
\]  
(60)

Define the expected time until an adjustment occurs as
\[
\tilde{T}(x, z, Z) = E\left(e^{-\beta T(x, z, Z)}T(x, z, Z)\right).
\]  
(61)

Finally, we introduce the concept of local time. For \( \xi \in (z, Z) \), let \( L(\xi; x, z, Z) \) be the expected amount of time that a firm starting from efficiency \( x \) will have efficiency \( \xi \) before adjustment. Let \( \tilde{L}(\xi; x, z, Z) \) be the expected discounted amount of time that a firm starting from efficiency \( x \) will have efficiency \( \xi \) before adjustment, where the discounting reflects the consumer’s rate of time preference, \( \rho \).

The relationship between the above formulas and our particular framework is as follows. A non-importer with efficiency \( x \in [b, B) \) will remain a non-importer until one of three adjustments occurs: (i) its efficiency reaches \( b \) and it chooses to exit, (ii) its efficiency reaches \( B \) and it chooses to become an importer, or (iii) it exogenously dies. For an importer with efficiency \( x \in [c, \infty) \), case (ii) is not relevant, but the other two cases are. The firm will remain an importer until its efficiency reaches \( c \) and it switches back to being a non-importer or it exogenously dies.

Consider a non-importer with efficiency \( x \in [b, B) \). The firm’s expected discounted returns from operating technology \( N \) until adjustment are given by
\[
r_N(x, b, B) = E_{X(0)=x} \int_0^{T(x, b, B)} e^{-(\rho+\beta)\Delta t} \pi_N(X(t)) \, dt.
\]  
(62)
Future returns are discounted at rate $\rho + \delta$. This reflects both the consumer’s rate of time preference and the exogenous probability of firm death. Now consider an importer with efficiency $x \in [c, \infty)$. The firm’s expected discounted returns from operating technology $I$ until adjustment are given by

$$r_I(x, c) = E_{X(0) = x} \int_{0}^{T(x,c,\infty)} e^{-(\rho+\delta)t} \pi_I(X(t))dt.$$  \hspace{1cm} (63)

Using the expected discounted local time function, we can eliminate the stochastic integrals in (62) and (63) by expressing the return functions more directly as

$$r_N(x, b, B) = \int_{b}^{B} \tilde{L}(\xi; x, b, B) \pi_N(\xi)d\xi$$ \hspace{1cm} (64)

$$r_I(x, c) = \int_{c}^{\infty} \tilde{L}(\xi; x, c, \infty) \pi_I(\xi)d\xi.$$ \hspace{1cm} (65)

The cutoffs $b$, $c$, and $B$ are endogenous choices of firms, but first we define the value functions of firms taking the cutoffs as given. Given the cutoffs $b$, $c$, and $B$, the expected discounted value of being a non-importer with efficiency $x$, $x \in [b, B)$, is

$$\bar{v}_N(x; b, c, B) = r_N(x, b, B) + \tilde{\Psi}(x, b, B)(\bar{v}_N(B; b, c, B) - \phi_{SI})$$ \hspace{1cm} (66)

and the expected discounted value of being an importer with efficiency $x$, $x \in [c, \infty)$, is

$$\bar{v}_I(x; b, c, B) = r_I(x, c) + \tilde{\psi}(x, c, \infty)\bar{v}_N(c; b, c, B).$$ \hspace{1cm} (67)

The second term on the right side of (66) is the net present value of the option to become an importer. The second term on the right side of (67) is the present value of the option to return to being a non-importer. After substitution, we can express (66) and (67) more directly as

$$\bar{v}_N(x; b, c, B) = r_N(x, b, B) + \tilde{\Psi}(x, b, B) \frac{r_I(B, c) + \tilde{\psi}(B, c, \infty)r_N(c, b, B) - \phi_{SI}}{1 - \tilde{\psi}(B, c, \infty)\Psi(c, b, B)}$$ \hspace{1cm} (68)
Now we can define the value functions with the cutoffs as endogenous choices of each firm. The expected discounted value of a firm with efficiency $x$ operating technology $N$ is

$$v_N(x) = \max \left\{ \max_{b,c,B} \bar{v}_N(x; b, c, B), \bar{v}_f(x) - \phi_{SI}, 0 \right\}$$

and the expected discounted value of a firm with efficiency $x$ operating technology $I$ is

$$v_I(x) = \max \left\{ \max_{b,c,B} \bar{v}_I(x; b, c, B), v_N(x), 0 \right\} .$$

As the outer maximization of (70) shows, a non-importer chooses among remaining a non-importer, paying the sunk cost $\phi_{SI}$ to become an importer, or exiting. As the outer maximization of (71) shows, an importer chooses among remaining an importer, becoming a non-importer, or exiting. In both value functions, the cutoffs $b$, $c$, and $B$ are endogenous decisions of each firm. The optimal cutoffs are independent of a firm’s current efficiency and its import status. Thus they satisfy the first-order conditions from every firm’s dynamic problem:

$$\bar{v}_{jk}(x; b, c, B) = 0 ,$$

for all $x$, where $\bar{v}_{jk} = \partial \bar{v}_j / \partial k$, $j = N, I$, $k = b, c, B$.

**Firm entry**

The cost of firm entry is $\phi_{En}$ units of output. Paying the cost of entry entitles a firm to enter as a non-importer with efficiency $x_0$, $x_0 \in (b, B)$. (It is straightforward to
allow for a probability distribution over initial efficiency draws, but this is not important for our analysis.) Free entry requires that the value of entry equals the cost of entry:

\[ v_N(x_0) = \phi_{E_n}. \]  

(73)

**Firm distributions**

Here we characterize the stationary distributions over firm efficiency. We break up the characterization into two parts. First, we specify the probability distributions over firm efficiency for each type. We let \( g_N(x) \) denote the probability density function over non-importers’ efficiencies with support \( x \in [b, B) \). We let \( g_I(x) \) denote the probability density function over importers’ efficiencies with support \( x \in [c, \infty) \). Second, we specify the measure of each firm type. We denote the measure of non-importers by \( M_N \), the measure of importers by \( M_I \), the measure of entrants by \( M_{En} \), the measure of firms switching from technology \( N \) to technology \( I \) by \( M_{SI} \), the measure of firms switching from technology \( I \) to technology \( N \) by \( M_{SN} \), and the measure of firms that endogenously exit by \( M_{Ex} \). The measure of firms that exogenously exit every instant is \( \delta(M_N + M_I) \). Now we are ready to specify the distributions of firms. We start with the distribution of importers because it is simpler. Entry into the distribution of importers only occurs at efficiency level \( B \). The expected local time function for an importer with efficiency \( B \) is \( L(x; B, c, \infty) \). If we normalize the expected local time function by the expected time to adjustment, \( \bar{T}(B, c, \infty) \), then we obtain the stationary distribution over importers’ efficiencies as the probability density function.
Entry into the distribution of non-importers occurs at two different efficiency levels. New firms enter with efficiency \( x_0 \), while firms that are switching from technology \( I \) to technology \( N \) enter with efficiency \( c \). The total distribution of non-importers is therefore a weighted average of the two types:

\[
g_N(x) = \frac{M_{En}}{M_{En} + M_{SN}} \frac{L(x; x_0, b, B)}{\bar{T}(x_0, b, B)} + \frac{M_{SN}}{M_{En} + M_{SN}} \frac{L(x; c, b, B)}{\bar{T}(c, b, B)}.
\]

(75)

Now we turn to specifying the measures of each firm type. Measure \( M_{En} \) of non-importers enter with efficiency \( x_0 \) every instant. In a stationary equilibrium, the measure of these firms that are adjusting every instant is \( M_{En} / \bar{T}(x_0, b, B) \). Applying the relevant probabilities, the measure of these firms that are endogenously exiting every instant is \( M_{En} \psi(x_0, b, B) / \bar{T}(x_0, b, B) \), the measure that are adopting technology \( I \) every instant is \( M_{En} \Psi(x_0, b, B) / \bar{T}(x_0, b, B) \), and the measure that are exogenously dying every instant is \( M_{En} (1 - \psi(x_0, b, B) - \Psi(x_0, b, B)) / \bar{T}(x_0, b, B) \). Similarly, every instant measure \( M_{SN} \) of firms will switch from being importers to become non-importers with efficiency \( c \). For these firms, the adjustment probabilities are, respectively, \( \psi(c, b, B) \), \( \Psi(c, b, B) \), and \( 1 - \psi(c, b, B) - \Psi(c, b, B) \) and the expected time until adjustment is \( \bar{T}(c, b, B) \). Thus the total measure of firms that choose to exit every instant is

\[
M_{Es} = M_{En} \frac{\psi(x_0, b, B)}{\bar{T}(x_0, b, B)} + M_{SN} \frac{\psi(c, b, B)}{\bar{T}(c, b, B)}
\]

(76)

and the total measure of firms that switch to being importers every instant is
Firms only enter the importer distribution with efficiency $B$. The only forms of adjustment are (i) switching back to being a non-importer and (ii) exogenous exit and the respective probabilities of these events are $\psi(B, c, \infty)$ and $1 - \psi(B, c, \infty)$, with expected time to adjustment being $\overline{T}(B, c, \infty)$. Thus every instant the measure of firms switching from being importers to being non-importers is

$$M_{SN} = M_{SI} \frac{\psi(B, c, \infty)}{\overline{T}(B, c, \infty)}.$$  

(78)

In a stationary equilibrium, the measure of non-importers and the measure of importers must be constant over time. Thus total entry into distribution $N$ must equal total exit from distribution $N$,

$$M_{En} + M_{SN} = \delta M_N + M_{SI} + M_{Ex},$$

(79)

and total entry into distribution $I$ must equal total exit from distribution $I$,

$$M_{SI} = \delta M_I + M_{SN}.$$ 

(80)

This implies that the total measure of each type is the difference between the entry and exit rates divided by $\delta$. We assume that parameter values are such that all of the above measures are strictly positive.

**Market-clearing conditions**

Define aggregate use of the domestic input as

$$D = M_N \int_b^\infty d_N(x) g_N(x) dx + M_I \int_c^\infty d_I(x) g_I(x) dx.$$  

(81)
Define aggregate use of the foreign input as
\[ F = M_I \int_{c}^{\infty} f_I(x)g_I(x)dx. \] (82)

Define aggregate output as
\[ Y = M_N \int_{b}^{\infty} y_N(x)g_N(x)dx + M_I \int_{c}^{\infty} y_I(x)g_I(x)dx. \] (83)

International balance of payments requires that
\[ E = pF, \] (84)

where \( E \) is the quantity of output that is exported. Tariff revenue is rebated to the consumer as a lump-sum transfer, so
\[ T = \tau pF. \] (85)

Aggregate profits are
\[ \Pi = M_N \int_{b}^{\infty} \pi_N(x)g_N(x)dx + M_I \int_{c}^{\infty} \pi_I(x)g_I(x)dx - \phi_{En}M_{En} - \phi_{M}M_{M} . \] (86)

Clearing in the labor market requires that
\[ M_N \int_{b}^{\infty} \ell_N(x)g_N(x)dx + M_I \int_{c}^{\infty} \ell_I(x)g_I(x)dx = \bar{L}. \] (87)

Finally, clearing in the goods market requires that
\[ C + D + E + \phi_N^M M_N + \phi_I M_I + \phi_{En} M_{En} + \phi_{M} M_{M} = Y. \] (88)

**Equilibrium**

A stationary competitive small open economy equilibrium is a list of aggregate measures \( \hat{C}, \hat{E}, \hat{F}, \hat{D}, \hat{Y}, \hat{M}_N, \hat{M}_I, \hat{M}_{En}, \hat{M}_{Es}, \hat{M}_{Si} \), and \( \hat{M}_{Sn} \); a transfer \( \hat{T} \); profits \( \hat{\Pi} \); a wage \( \hat{w} \); firm decision rules \( \hat{y}_N(x), \hat{\ell}_N(x), \hat{d}_N(x), \hat{y}_I(x), \hat{\ell}_I(x), \hat{\pi}_I(x), \hat{\ell}_I(x), \)
\[ \hat{d}_i(x), \hat{f}_i(x), \hat{b}, \hat{c}, \hat{B}, \hat{\nu}_N(x), \hat{\nu}_I(x), \hat{\nu}_N(x,b,B), \hat{\nu}_I(x,c,B), \hat{\nu}_N(x;b,c,B), \hat{\nu}_I(x;b,c,B); \]

and stationary distributions \( \hat{g}_N(x) \) and \( \hat{g}_I(x) \) such that (47)-(63), (66)-(67), and (70)-(88) hold.

The following is an algorithm to calculate the equilibrium. Taking \( w \) as given, solve for \( \hat{\ell}_N(x) \) and \( \hat{d}_N(x) \) using (50) and (51); solve for \( \hat{\ell}_I(x), \hat{d}_I(x), \) and \( \hat{f}_I(x) \) using (54)-(56); calculate \( \hat{y}_N(x), \hat{\nu}_N(x), \hat{\nu}_I(x), \) and \( \hat{\nu}_I(x) \) using (48), (49), (52), and (53);
calculate \( \hat{r}_N(x,b,B) \) and \( \hat{r}_I(x,c) \) using (62) and (63); solve for \( \hat{\nu}_N(x;b,c,B) \) and \( \hat{\nu}_I(x;b,c,B) \) using (66) and (67); solve for \( \hat{b}, \hat{c}, \) and \( \hat{B} \) using (72); and solve for \( \hat{\nu}_N(x) \) and \( \hat{\nu}_I(x) \) using (70) and (71). Then solve for \( \hat{w} \) using (73). Taking \( M_{En} \) as given, solve for \( \hat{M}_{En}, \hat{M}_{SI}, \) and \( \hat{M}_{SN} \) using (76)-(78); solve for \( \hat{M}_N \) and \( \hat{M}_I \) using (79) and (80); and solve for \( \hat{g}_N(x) \) and \( \hat{g}_I(x) \) using (74) and (75). Then solve for \( \hat{M}_{En} \) using (87). Finally, calculate \( \hat{C}, \hat{Y}, \hat{E}, \hat{F}, \hat{D}, \hat{T}, \) and \( \hat{I} \) using (47), (81)-(85), and (86). By Walras’s Law, the remaining market-clearing condition (88) holds.

**Further assumptions**

Here we specify the stochastic process, functional forms for the production technologies, and restrictions on the costs and benefits of importing.

**Stochastic process**

We let firm-level efficiency evolve as a geometric Brownian motion with drift.

That is, firm efficiency evolves according to
\[
\frac{dX(t)}{X(t)} = \mu dt + \sigma dW(t) .
\]  

(89)

Here \( dX(t) / X(t) \) is the percentage change in a firm’s efficiency at time \( t \). It is the sum of a deterministic trend, \( \mu dt \), and a stochastic shock, \( \sigma dW(t) \). The parameter \( \sigma \), \( \sigma \geq 0 \), determines the relative magnitude of the shock and \( W(t) \) is a Weiner process. A Weiner process is

\[
W(t) = \varepsilon \sqrt{t} ,
\]

(90)

where \( \varepsilon \) is a standard normal random variable. The first two moments of a Weiner process are \( E(W(t)) = 0 \) and \( E(W(t)^2) = t \), so \( dX(t) / X(t) \) is normally distributed with mean \( \mu dt \) and variance \( \sigma^2 dt \). For finiteness, we assume that \( \rho + \delta > \mu \).

Geometric Brownian motion is a continuous-time Markov process with independent relative increments. That is, for any times \( t \) and \( s \), \( t > s \),

\[
dX(t) / X(t) - dX(s) / X(s) \]

is an independent random variable that is normally distributed with mean \( \mu(t - s) \) and variance \( \sigma^2(t - s) \). Independent increments imply that one can think of a Brownian motion as the continuous limit of a random walk.

An advantage of using geometric Brownian motion is that it generates simple analytic expressions for the functions \( \psi(x, z, Z) \), \( \Psi(x, z, Z) \), \( \tilde{\psi}(x, z, Z) \), \( \tilde{\Psi}(x, z, Z) \), \( \tilde{T}(x, z, Z) \), \( L(\xi; x, z, Z) \), and \( \tilde{L}(\xi; x, z, Z) \). We provide the formulas in Appendix C.

Functional forms for production technologies

Here we specify functional forms for the constant-returns-to-scale production functions \( \Omega_N \) in (48) and \( \Omega_i \) in (52). Let
\[ \Omega_N(\ell, d) = \ell^\alpha d^{1-\alpha} \]  

(91)

\[ \Omega_I(\ell, d, f) = \ell^\alpha \left( \omega d^\theta + (1-\omega) f^\theta \right)^{1-\alpha}, \]  

(92)

where \( 0 < \alpha < 1 \), \( 0 < \omega < 1 \), and \( \theta < 1 \). With this specification, the elasticity of substitution between labor and intermediate goods is one for all firms, with \( \alpha \) being the share of expenditure on labor.\(^9\) For importers, the elasticity of substitution between domestically produced intermediate goods and imported intermediate goods is \( 1/(1-\theta) \).

We can think of an importer with efficiency \( x \) as using a composite intermediate good, the quantity of which is given by

\[ z_i(x) = \left( \omega d_i(x)^\theta + (1-\omega) f_i(x)^\theta \right)^{1/\theta}. \]  

(93)

The composite intermediate good has price

\[ P = \left( \frac{1}{\omega^\theta} + \frac{1}{(1-\omega)^\theta} \left( (1+\tau) p \right)^\theta \right)^{\frac{1-\theta}{\theta}}. \]  

(94)

Given these functional forms, the static decisions of firms have simple analytic expressions. We provide them in Appendix D.

The size distribution of firms in the model is proportional to the distribution of efficiency draws. As figure 3 shows, the size distribution of plants in the data is upward sloping in the left tail and consistent with a Pareto distribution in the right tail.

\(^9\) In the manufacturing plant data used in this paper, the share of expenditure devoted to labor is roughly constant over the sample period, 2001 to 2006. This is consistent with our choice of Cobb-Douglas production functions.
Costs and benefits of importing

Here we discuss the costs and benefits of importing and place further restrictions on parameters. In the model, the costs of importing are fixed costs, while the benefits of importing depend on a firm’s scale of operation.

First consider the benefits of importing. We suppose that, in the absence of any additional fixed costs of importing, every firm would choose to be an importer. With functional forms (91) and (92), this condition can be expressed as $\eta P^{-(1-\alpha)\nu/(1-\nu)} > 1$. The marginal benefit of importing is thus given by a combination of the relative efficiency of technology $I$, as given by $\eta$, and the relative cost of intermediate inputs, as given by $P$. 
Now consider the costs of importing. We assume that parameter values are such that the ordering of the cutoffs is \( b < c < B \). The value functions then imply that

\[
\begin{align*}
  v_N(b) &= 0 \\
  v_N(c) &= v_I(c) \\
  v_N(B) &= v_I(B) - \phi_{Sf}.
\end{align*}
\]

In this case, for a firm with efficiency \( x \in (c, B) \), the optimal choice of technology depends on the firm’s past trade status. This implies that \( \phi_I > \phi_N \) and \( \phi_{Sf} > 0 \) (though \( \phi_I > \phi_N \) and \( \phi_{Sf} > 0 \) are not sufficient to obtain \( b < c < B \)).

The resulting firm size distribution in the model displays many of the qualitative properties of the data. Both distributions are upward sloping in the left tail, and downward sloping in the right tail. The size distribution of importers is Pareto in the right tail. The size distributions overlap — as a result of the presence of a sunk cost — which is consistent with the data. This is an improvement on the static model where every importer is larger than every non-importer. Figure 4 shows a drawing of the size distributions in the model.
Quantitative analysis

In this section we examine the extent to which the model can quantitatively capture important features of the data, particularly the data on Chilean manufacturing plants discussed above. We calibrate the model and use it to quantitatively analyze the effects of unilateral trade liberalization.

Calibration

We calibrate the model to closely match important facts from the Chilean manufacturing data discussed above. Some parameters are simply normalized or are taken from the literature, while others we specifically choose to match particular facts. We set the length of one unit of time as one year.

Since we are primarily interested in relative comparisons, we normalize a number of parameters. As normalizations we set the labor endowment, $L$, the initial efficiency $g_N(x)$, and $g_I(x)$. We also normalize the efficiency $c_i(x,0)$.
draw, \( x_0 \), and the cost of entry, \( \phi_{En} \), to one. Given this normalization all other fixed costs can be interpreted as relative to the cost of entry. We choose the terms of trade, \( p \), so that \( P = 1 \).

Table 13. Summary of the calibration (Dynamic)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>( \phi_{En} )</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>( p )</td>
<td>0.1</td>
<td>Normalization to get ( P = 1 )</td>
</tr>
<tr>
<td>( x_0 )</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.04</td>
<td>Real interest rate of 4%</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.85</td>
<td>Atkeson and Kehoe (2005)</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.5</td>
<td>Ruhl (2004)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.08</td>
<td>World Bank’s World dataBank</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.09</td>
<td>Rate of exit of importers</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.35</td>
<td>Expenditure on labor as a share of total expenditure on labor and intermediate goods</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.81</td>
<td>Importers’ expenditure on imported intermediate goods as a share of total expenditure on intermediate goods of 0.34</td>
</tr>
<tr>
<td>( \mu, \sigma, \eta )</td>
<td>– 0.1, 0.54, 1.15</td>
<td>Jointly chosen to minimize discrepancy with the set of facts in Table 11 and 12</td>
</tr>
<tr>
<td>( \phi_N, \phi_I, \phi_{SI} )</td>
<td>0.25, 0.33, 0.01</td>
<td>facts in Table 11 and 12</td>
</tr>
</tbody>
</table>

Some parameters we take from the literature. Following Atkeson and Kehoe (2005), we set the degree of diminishing returns faced by both technologies, \( \nu \), equal to 0.85. They find that this value closely matches plant-level data on U.S. manufacturing. We choose \( \theta \) such that the elasticity of substitution between domestic and foreign intermediate
inputs is 2 (Ruhl, 2004). We set the consumer’s rate of time preference at 4 percent, a standard value for the real interest rate.

The World Bank estimates that Chile’s average tariff rate in 2001 was 8 percent. We set $\tau$ accordingly. Among plants that import, expenditure on imported intermediate goods as a share of total expenditure on intermediate goods is 34 percent. We set the parameter $\omega$ to match this. In the model, $\alpha$ is expenditure on labor as a share of expenditure on both labor and intermediate goods. In the data this share is 0.35. In the data, the rate of exit for importers is 9 percent. In our model, this corresponds to the rate of exogenous exit, so we set $\delta = 0.09$.

Then, given a value for $\eta$, we can simultaneously choose the values of $\mu$, $\sigma$, $b$, $c$, and $B$ to match the following five facts: (i) the average rate of entry and exit is 12.45 percent, (ii) the total rate at which firms switch import status is 5.2 percent; (iii) the share of firms that import is 20 percent, (iv) the average importer has output of 3.6 times that of the average non-importer, and (v) the coefficient of variation for gross output is 6.0. Since $b$, $c$, and $B$ are equilibrium objects, we use the first-order conditions from a firm’s problem to find the corresponding parameter values for $\phi_N$, $\phi_I$, and $\phi_{SI}$. To obtain a value for $\eta$, we iterate using the above procedure until importers have value added per worker of 1.2 times that of non-importers. Table 13 summarizes the entire calibration, while table 14 shows the extent to which the model captures the above six targets. Tables 15 and 16 revisit the data discussed earlier and make comparisons with the model.
Table 14. Calibration targets

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total rate of exit (%)</td>
<td>12.45</td>
<td>12.45</td>
</tr>
<tr>
<td>Share of plants that import (%)</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Gross output of average importer relative to gross output of average non-importer</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Total rate at which plants switch import status (%)</td>
<td>5.2</td>
<td>5.2</td>
</tr>
<tr>
<td>Coefficient of variation for gross output</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Value-added per worker of average importer relative to value-added per worker of average non-importer</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Experiments using the benchmark calibration

We consider two forms of trade liberalization: a decrease in the *ad valorem* tariff rate and an improvement in the terms of trade. Comparing stationary equilibria allows us to analyze the long-term effects of trade liberalization. Decreases in $\tau$ and $p$ have similar qualitative effects. The main difference is in how they affect government revenue. A decrease in $\tau$ or $p$ (i) increases the cutoff to operate, $b$; (ii) decreases the cutoff to start importing, $B$; (iii) decreases the cutoff to stop importing, $c$; (iv) decreases the measure of entrants, $M_{En}$; (v) increases the wage, $w$; (vi) increases social welfare, $C$; and (vii) increases output, $Y$. 
Table 15. Data vs. Model (Importer premia)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross output</td>
<td>3.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Total materials</td>
<td>3.4</td>
<td>3.6</td>
</tr>
<tr>
<td>Employment</td>
<td>3.1</td>
<td>3.6</td>
</tr>
<tr>
<td>Value added</td>
<td>3.9</td>
<td>4.3</td>
</tr>
<tr>
<td>Value added per worker</td>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 16. Data vs. Model (Transition probabilities)

<table>
<thead>
<tr>
<th>Data</th>
<th>Importer</th>
<th>Non-importer</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importer</td>
<td>77</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>Non-importer</td>
<td>3</td>
<td>84</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>Importer</th>
<th>Non-importer</th>
<th>Exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importer</td>
<td>82.5</td>
<td>8.5</td>
<td>9</td>
</tr>
<tr>
<td>Non-importer</td>
<td>4</td>
<td>83</td>
<td>13</td>
</tr>
</tbody>
</table>

Trade liberalization reallocates resources from non-importers to importers. As a result, importers become larger on average and the least efficient firms are forced to exit. Greater output per unit of labor leads to an increase in average efficiency. Trade liberalization drives out the least efficient firms because they can no longer profitably operate at the higher wage, while the most efficient non-importers upgrade to the superior technology, since the benefit of using it has increased.
The effects of trade liberalizations which increase the benefit of importing (versus decreasing the cost) are magnified by the technology upgrading effect. In the absence of any selection effect, technology upgrading would result in importers being $\beta$ times larger than non-importers. The selection effect determines the extent to which the size ratio is greater than $\beta$. Let $S$ be size of the average importer relative to the size of the average non-importer. Using a logarithmic decomposition to account for this ratio, the share due to the technology upgrading effect is given by $\log \beta / \log S$; the remaining share is due to the selection effect. Our base calibration implies that technology upgrading is responsible for 18 percent the size difference between the average importer and average non-importer.

**Conclusion**

We have developed a dynamic general equilibrium model of importing by firms. More broadly, we have developed a model of the dynamics of technology adoption when there are start-up and continuation costs. The model is analytically tractable and easily generalizable. The model qualitatively and quantitatively captures the dynamics of importing by firms and the effects of trade liberalization.

The framework developed here could be extended in many ways. We only consider a stationary equilibrium, but the role of sunk costs in the presence of aggregate shocks is a fruitful area for future research. We consider only a firm’s decision to import, but adding the decision to export would be an interesting extension. Given that firms use imported intermediate goods with varying intensities, allowing for more than one import technology would also be a worthwhile extension.
References


Appendix A. Analytic solution with trade

Let

\[ P = \left( \mu^{1-\rho} + (1-\mu)^{1-\rho} \left( (1+\tau)p \right)^{\frac{\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}} \]  

(98)

\[ B = \eta P^{\frac{(1-\alpha)\nu}{1-\nu}} \]  

(99)

\[ A = \theta \left( \frac{\phi_N^{1-\gamma} + (B-1)^\gamma (\phi_i - \phi_N)^{1-\gamma}}{(\gamma - 1)\phi_M} \right)^{\frac{1}{\gamma}} \]  

(100)

\[ K = \nu^{\alpha} (1-\nu)\omega \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \]  

(101)

Then

\[ d_N(a) = \frac{a(v(1-\alpha))}{A(1-v)} \]  

(102)

\[ \ell_N(a) = \frac{a(v(1-\alpha))^{\frac{1-\alpha}{\alpha}}}{(A(1-v))^{\frac{1-\nu(1-\alpha)}{\alpha v}}} \]  

(103)

\[ y_N(a) = \frac{a}{A(1-v)} \]  

(104)

\[ \pi_N(a) = \frac{a}{A} - \phi_N \]  

(105)

\[ d_i(a) = \frac{abv(1-\alpha)\mu^{1-\rho} P^{\frac{\rho}{1-\rho}}}{A(1-v)} \]  

(106)

\[ \ell_i(a) = \frac{ab\left( v(1-\alpha) \right)^{\frac{1-\alpha}{\alpha}}}{(A(1-v))^{\frac{1-\nu(1-\alpha)}{\alpha v}}} \]  

(107)

\[ f_i(a) = \frac{abv(1-\alpha)(1-\mu)^{1-\rho} \left( (1+\tau)p \right)^{\frac{1}{1-\rho}} P^{\frac{\rho}{1-\rho}}}{A(1-v)} \]  

(108)

\[ y_i(a) = \frac{ab}{A(1-v)} \]  

(109)
\[ \pi_t(a) = \frac{aB}{A} - \phi_t \] (110)

\[ w = A^{\alpha v} K \] (111)

\[ a_N = A\phi_N \] (112)

\[ a_j = \frac{A(\phi_j - \phi_N)}{B-1} \] (113)

\[ M = \frac{A^{\alpha v} KL(1-\nu)}{\alpha v \gamma \phi_M} \] (114)

\[ Y = \frac{A^{\alpha v} KL}{\alpha v} \] (115)

\[ D = \frac{A^{\alpha v} KL(1-\alpha)\left(\phi_N^{1-\gamma} + \left(\frac{1}{\mu^{1-\rho}} P^{1-\rho} B - 1\right)(B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)}{\alpha \left(\phi_N^{1-\gamma} + (B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)} \] (116)

\[ F = \frac{A^{\alpha v} KL(1-\alpha)(1-\mu)\left((1+\tau) p\right)^{-\frac{1}{\gamma}} P^{\frac{\rho}{\gamma}} B(B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}}{\alpha \left(\phi_N^{1-\gamma} + (B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)} \] (117)

\[ X = \frac{A^{\alpha v} KLp(1-\alpha)(1-\mu)\left((1+\tau) p\right)^{-\frac{1}{\gamma}} P^{\frac{\rho}{\gamma}} B(B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}}{\alpha \left(\phi_N^{1-\gamma} + (B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)} \] (118)

\[ T = \frac{A^{\alpha v} KL \tau p(1-\alpha)(1-\mu)\left((1+\tau) p\right)^{-\frac{1}{\gamma}} P^{\frac{\rho}{\gamma}} B(B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}}{\alpha \left(\phi_N^{1-\gamma} + (B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)} \] (119)

\[ C = A^{\alpha v} KL \left\{ 1 + \frac{\tau p(1-\alpha)(1-\mu)\left((1+\tau) p\right)^{-\frac{1}{\gamma}} P^{\frac{\rho}{\gamma}} B(B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}}{\alpha \left(\phi_N^{1-\gamma} + (B-1)^{-\gamma} (\phi_t - \phi_N)^{1-\gamma}\right)} \right\} \] (120)
Appendix B. Analytic solution under autarky

Let

\[
A = \theta \left( \frac{\phi_N^{1-\gamma}}{(\gamma - 1) \phi_M} \right)^{\frac{1}{\gamma}} \tag{121}
\]

\[
K = v^{\alpha} (1-v) \frac{1}{1-\alpha} \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} . \tag{122}
\]

Then

\[
d_N(a) = \frac{a\nu(1-\alpha)}{A(1-\nu)} \tag{123}
\]

\[
\ell_N(a) = \frac{a\left(\nu(1-\alpha)\right)^{\frac{1-\alpha}{\alpha}}}{(A(1-\nu))^{\frac{1-v(1-\alpha)}{\alpha\nu}}} \tag{124}
\]

\[
y_N(a) = \frac{a}{A(1-\nu)} \tag{125}
\]

\[
\pi_N(a) = \frac{a}{A} - \phi_N \tag{126}
\]

\[
w = A^{\frac{1-\nu}{\alpha\nu}} K \tag{127}
\]

\[
a_N = A\phi_N \tag{128}
\]

\[
M = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\nu)}{\alpha\nu\phi_M} \tag{129}
\]

\[
Y = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL}{\alpha\nu} \tag{130}
\]

\[
D = \frac{A^{\frac{1-\nu}{\alpha\nu}} KL(1-\alpha)}{\alpha} \tag{131}
\]

\[
C = A^{\frac{1-\nu}{\alpha\nu}} KL . \tag{132}
\]
Appendix C. Formulas with geometric Brownian motion

Consider a firm with efficiency $x \in (z, Z)$. Suppose that the firm’s efficiency evolves according to a geometric Brownian motion $X(t)$, where

$$\frac{dX(t)}{X(t)} = \mu dt + \sigma dW(t). \quad (133)$$

Here we assume that $\sigma > 0$ and $\mu \neq \sigma^2 / 2$ (analytic formulas are available for the other cases, though we do not provide them here (see Stokey (2009))). Eventually the firm will face an adjustment, meaning that one of three events will occur: (i) the firm’s efficiency will reach $z$, (ii) the firm’s efficiency will reach $Z$, or (iii) the firm will exogenously die. Denote the first time that (i) or (ii) occurs by $T(x, z, Z)$. Let

$$J = \sqrt{(\mu - \sigma^2 / 2)^2 + 2\sigma^2} \quad (134)$$

$$\lambda_1 = -\left(\frac{\mu - \sigma^2 / 2}{\sigma^2}\right) + J \quad (135)$$

$$\lambda_2 = -\left(\frac{\mu - \sigma^2 / 2}{\sigma^2}\right) + J \quad (136)$$

$$\tilde{J} = \sqrt{(\mu - \sigma^2 / 2)^2 + 2(\rho + \delta)\sigma^2} \quad (137)$$

$$\tilde{\lambda}_1 = -\left(\frac{\mu - \sigma^2 / 2}{\sigma^2}\right) + \tilde{J} \quad (138)$$

$$\tilde{\lambda}_2 = -\left(\frac{\mu - \sigma^2 / 2}{\sigma^2}\right) + \tilde{J} \quad (139)$$

The probability that adjustment (i) occurs first is

$$\psi(x, z, Z) = E\left(e^{-\delta T(x, z, Z)}\right) \Pr\left(X(T(x, z, Z)) = z\right)$$

$$= \frac{x^x Z^z - Z^x x^z}{x^x Z^z - Z^x Z^z} \quad (140)$$

The probability that adjustment (ii) occurs first is

$$\Psi(x, z, Z) = E\left(e^{-\delta T(x, z, Z)}\right) \Pr\left(X(T(x, z, Z)) = Z\right)$$

$$= \frac{z^x x^z - Z^x z^z}{z^x Z^z - Z^x z^z} \quad (141)$$
Taking into account the consumer’s subjective discount factor, the expected discounted value of the indicator function for the event of adjustment (i) occurring first is

$$\tilde{\psi}(x,z,Z) = E\left(e^{-\rho T(x,z,Z)}\right)\psi(x,z,Z)$$

$$= \frac{x^h_{\lambda} Z^h_{\lambda} - Z^h_{\lambda} x^h_{\lambda}}{z^h_{\lambda} Z^h_{\lambda} - Z^h_{\lambda} z^h_{\lambda}}.$$ \hspace{1cm} (142)

The expected discounted value of the indicator function for the event of adjustment (ii) occurring first is

$$\tilde{\Psi}(x,z,Z) = E\left(e^{-\rho T(x,z,Z)}\right)\Psi(x,z,Z)$$

$$= \frac{z^h_{\lambda} x^h_{\lambda} - x^h_{\lambda} z^h_{\lambda}}{z^h_{\lambda} Z^h_{\lambda} - Z^h_{\lambda} z^h_{\lambda}}.$$ \hspace{1cm} (143)

The expected time until adjustment is

$$\bar{T}(x,z,Z) = E\left(e^{-\delta T(x,z,Z)}T(x,z,Z)\right)$$

$$= \frac{1}{\delta} \left(1 + \frac{x^h_{\lambda} z^h_{\lambda} - x^h_{\lambda} Z^h_{\lambda} + Z^h_{\lambda} x^h_{\lambda} - z^h_{\lambda} x^h_{\lambda}}{z^h_{\lambda} Z^h_{\lambda} - Z^h_{\lambda} z^h_{\lambda}}\right).$$ \hspace{1cm} (144)

For $\xi \in (z,Z)$, the expected local time function is

$$L(\xi; x,z,Z) = \begin{cases} \frac{1}{\lambda J} \left(\frac{x}{\xi} \right)^{\lambda} - \Psi(x,z,Z) \left(\frac{Z}{\xi} \right)^{\lambda} - \psi(x,z,Z) \left(\frac{z}{\xi} \right)^{\lambda} & \text{if } z \leq \xi \leq x \\ \frac{1}{\lambda J} \left(\frac{x}{\xi} \right)^{\lambda} - \Psi(x,z,Z) \left(\frac{Z}{\xi} \right)^{\lambda} - \psi(x,z,Z) \left(\frac{z}{\xi} \right)^{\lambda} & \text{if } x \leq \xi \leq Z \end{cases}.$$ \hspace{1cm} (145)

The expected discounted local time function is the counterpart to (145) after taking into account the consumer’s rate of time preference. It is given by

$$\bar{L}(\xi; x,z,Z) = \begin{cases} \frac{1}{\lambda J} \left(\frac{x}{\xi} \right)^{\lambda} - \tilde{\Psi}(x,z,Z) \left(\frac{Z}{\xi} \right)^{\lambda} - \tilde{\psi}(x,z,Z) \left(\frac{z}{\xi} \right)^{\lambda} & \text{if } z \leq \xi \leq x \\ \frac{1}{\lambda J} \left(\frac{x}{\xi} \right)^{\lambda} - \tilde{\Psi}(x,z,Z) \left(\frac{Z}{\xi} \right)^{\lambda} - \tilde{\psi}(x,z,Z) \left(\frac{z}{\xi} \right)^{\lambda} & \text{if } x \leq \xi \leq Z \end{cases}.$$ \hspace{1cm} (146)
Appendix D. Analytic expressions for static decisions of firms

Taking as given the cutoffs, $b$, $c$, and $B$; the wage, $w$; and the measure of entrants, $M_{En}$; and using the formulas from Appendix C, we analytically solve for the rest of the equilibrium objects. Let

$$ P = \left( \frac{1}{\omega^{1-\theta}} + (1-\omega)^{1-\theta} \left((1+\tau)p\right)^{1-\theta} \right)^{1-\theta} \theta $$

(147)

$$ \beta = \eta P^{\frac{(1-\alpha)\nu}{1-\nu}} $$

(148)

$$ A = \frac{\alpha^\nu}{\nu^\nu (1-\alpha)^{1-\nu} \nu^\nu (1-\nu)}.$$  

(149)

Then firms’ static decisions are given by

$$ \ell_N(x) = \frac{x \alpha \nu}{A(1-\nu)w} $$

(150)

$$ d_N(x) = \frac{x(1-\alpha)\nu}{A(1-\nu)} $$

(151)

$$ y_N(x) = \frac{x}{A(1-\nu)} $$

(152)

$$ \pi_N(x) = \frac{x}{A} - \phi_N $$

(153)

$$ \ell_1(x) = \frac{x \beta \alpha \nu}{A(1-\nu)w} $$

(154)

$$ d_1(x) = \frac{x \beta (1-\alpha)\nu \omega^{1-\theta} P^{1-\theta}}{A(1-\nu)} $$

(155)

$$ f_1(x) = \frac{x \beta (1-\alpha)\nu (1-\omega)^{1-\theta} \left((1+\tau)p\right)^{1-\theta} P^{1-\theta}}{A(1-\nu)} $$

(156)

$$ y_1(x) = \frac{x \beta}{A(1-\nu)} $$

(157)

$$ \pi_1(x) = \frac{x \beta}{A} - \phi_1. $$

(158)
Using the formulas in Appendix C, it is straightforward to calculate the rest of the equilibrium objects:

\[
\begin{align*}
    r_N(x, b, B) &= \int_b^\infty \tilde{L}(\xi; x, b, B) \pi_N(\xi) d\xi \\
    r_I(x) &= \int_c^\infty \tilde{L}(\xi; x, c, \infty) \pi_I(\xi) d\xi \\
    \bar{v}_N(x; b, c, B) &= r_N(x, b, B) + \bar{\psi}(x, b, B) \frac{r_I(B, c) + \bar{\psi}(B, c, \infty) r_N(c, b, B) - \phi_{SI}}{1 - \bar{\psi}(B, c, \infty) \bar{\psi}(c, b, B)} \\
    \bar{v}_I(x; b, c, B) &= r_I(x, c) + \bar{\psi}(x, c, \infty) \frac{r_N(c, b, B) + \bar{\psi}(c, b, B) (r_I(B, c) - \phi_{SI})}{1 - \bar{\psi}(B, c, \infty) \bar{\psi}(c, b, B)} \\
    M_{SI} &= \frac{\Psi(x_0, b, B)}{\bar{T}(x_0, b, B)} \frac{M_{En}}{1 - \bar{\psi}(B, c, \infty) \Psi(c, b, B)} \\
    M_{SN} &= M_{SI} \frac{\psi(B, c, \infty)}{\bar{T}(B, c, \infty)} \\
    M_{ES} &= M_{En} \frac{\psi(x_0, b, B)}{\bar{T}(x_0, b, B)} + M_{SN} \frac{\psi(c, b, B)}{\bar{T}(c, b, B)} \\
    M_N &= \frac{M_{En} + M_{SN} - M_{SI} - M_{ES}}{\delta} \\
    M_I &= \frac{M_{SI} - M_{SN}}{\delta} \\
    g_N(x) &= \frac{M_{En}}{M_{En} + M_{SN}} \frac{L(x; x_0, b, B)}{\bar{T}(x_0, b, B)} + \frac{M_{SN}}{M_{En} + M_{SN}} \frac{L(x; c, b, B)}{\bar{T}(c, b, B)} \\
    g_I(x) &= \frac{L(x; B, c, \infty)}{\bar{T}(B, c, \infty)}
\end{align*}
\]