SHRINKAGE CRACKING OF SOILS AND CEMENTITIOUSLY-STABILIZED SOILS:
MECHANISMS AND MODELING

By

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SHRINKAGE CRACKING OF SOILS AND CEMENTITIOUSLY-STABILIZED SOILS: MECHANISMS AND MODELING

Abstract

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Shrinkage cracking of soil or cement soil can cause the infiltration or seepage of water into the material and lead to reflective cracking in the structure above it. Shrinkage cracking greatly limits the use of cementitious stabilization. Drying shrinkage is the major problem associated with shrinkage cracking. Although some studies on the drying shrinkage cracking of concrete have been carried out, few research efforts have focused on unsaturated soil and cement soil. Even fewer studies consider the shrinkage stress distribution and evolution of the stress profile during drying and/or cement hydration in shrinkage cracking modeling.

Using two types of soil and two types of cement soil, this study investigates the soil and cement soil properties that are related to drying shrinkage and shrinkage cracking. These material properties, which are considered in this study to be dependent on pore relative humidity (RH) and/or cement hydration and, therefore, are neither uniform nor constant during drying and/or cement hydration, include the humidity isotherm, diffusion coefficient, soil water characteristic curve (SWCC), tensile strength, tensile stress-strain relationship, and shrinkage potential, etc. This study examines the effects of pore RH
and/or cement hydration on these material properties. Models are developed herein to predict the evolution of these material properties during drying and/or cement hydration.

By measuring the humidity isotherm and diffusion coefficient and using the finite element (FE) method, the evolution of the pore RH gradient within the soil or cement soil specimens during drying can be fully captured. A coefficient of moisture shrinkage is proposed to bridge the knowledge gap between pore RH and shrinkage potential, which is defined in this study as the maximum possible drying shrinkage of a material at a given pore RH when there is no restraint. Then, the pore RH and/or cement hydration-dependent tensile strength, tensile stress-strain relationship, and shrinkage potential within the specimens during drying and/or cement hydration can be fully understood using the models developed in this study. Finally, FE modeling case studies were conducted to verify the models and procedure proposed in this study to predict shrinkage stress and shrinkage cracking. The results were compared with results from laboratory experiments and good agreement was found.
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Dedication

This dissertation is dedicated to my father Yukuan Li, my mother Fujing Liu, my wife Liyun Chen, my daughter Siyu Li, and my son Albert Li, who have provided great emotional support and given me so much pleasure.
CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

Roadway infrastructure construction demand large amounts of quality materials such as aggregate. Each year, the United States consumes 1.2 billion tons of sand and gravel (USGS 2009), which requires mining, quarrying, crushing, and transporting natural earth materials for construction. The procurement of natural materials disturbs the environment and depletes natural resources. The manufacturing process also consumes a substantial amount of energy and releases large amounts of greenhouse gases. Sustainable development is imperative and must make use of local materials, recycled materials, and industrial/municipal waste/byproducts, instead of transported materials.

Locally available materials, especially those in situ, offer numerous benefits, such as reducing the need for quarrying and transporting natural aggregate, which saves construction costs and energy consumption and reduces greenhouse gas emissions. However, local or in situ materials often lack the engineering properties needed for sufficient structural integrity. Therefore, mechanical or chemical stabilization of inferior materials is needed to improve such properties. Cementitious binders, such as cement, fly ash, or lime, are frequently used to fulfill this need.

Cementitious stabilization can significantly improve the strength, stiffness, durability, permeability, and stability of host materials to allow them to support the load from the structure above them. For instance, cement-treated expansive soil has significantly higher stability, modulus strength, and resistance to deformation than
untreated soils (Puppala et al. 2003). Such stabilized materials can be used in many engineering practices, such as dams, buildings, airports, parking lots, and pavements.

Although stabilized materials offer many advantages, they have their own engineering problems. One of the biggest challenges is shrinkage cracking, illustrated in Figure 1.1. Shrinkage cracking can cause infiltration or seepage of water into the stabilized materials and thus can lead to reflective cracking in the structure above. It has been reported that shrinkage cracking is the biggest concern that greatly limits the use of cementitious stabilization (Sebesta 2005, Dunlop 2005, Little 1992). Therefore, the mitigation of shrinkage cracking could significantly expand the use of local materials, recycled materials, or waste/byproducts through stabilization.

Figure 1.1: Shrinkage Cracking of Stabilized Materials (after George 2001)

Shrinkage of stabilized materials consists of autogenous shrinkage due to hydration, drying shrinkage due to loss of moisture, and thermal shrinkage due to low temperature contraction (ACI 2008). Drying shrinkage dominates the shrinkage strain (Little 1992). Shrinkage of stabilized material, if restrained (e.g., bonding from underlying layer), causes the development of tensile stress, as shown in Figure 1.2. When the tensile stress exceeds
the tensile strength of the stabilized material, shrinkage cracking occurs (George 1990).

Shrinkage of stabilized materials is affected by many factors, such as moisture, binder content, host material characteristics, and curing after compaction (George 1990). Shrinkage of stabilized materials also is related to relaxation (George 1969). Relaxation of stabilized materials reduces shrinkage stress and mitigates the occurrence of shrinkage cracking.

Drying shrinkage can result from moisture loss, which leads to matric suction and osmotic suction (Kodikara and Chakrabarti 2001). Moisture contents that are greater than the optimum moisture content (OMC) can cause excessive shrinkage cracking (George 1990). Such shrinkage can be reduced by increasing the compaction density and avoiding montorillonite clay (George 1990).
High binder content results in high strength, but also causes high shrinkage cracking potential (Sebesta 2005). Low strength ensures narrow and closely spaced shrinkage cracks, and high strength leads to wide cracks with large spacing (Van Blerk and Scullion 1995).

The characteristics of the host materials directly affect the shrinkage behavior of stabilized materials. Increasing the clay content and plasticity index increases shrinkage. Smectite group clay causes the most shrinkage. The linear shrinkage of the fine fraction of the aggregate is a good indicator of the ultimate drying shrinkage of stabilized materials. Clay that is smaller than 2μm is responsible for shrinking and swelling (Norling 1973).

1.2 PROBLEM STATEMENT

Problems associated with shrinkage cracking have been studied for decades, but with little success. It has been stated that shrinkage cracking is an intrinsic behavior of stabilized materials and cannot be avoided (George 1990). Chakrabarti et al. (2002) reported that drying shrinkage is the major cause of shrinkage cracking associated with the use of cementitiously-stabilized materials (CSM) in pavement construction. Drying shrinkage was believed to be related directly to the moisture loss of cement-stabilized pavement layers (Chakrabarti and Kodikara 2006). It is generally accepted that drying shrinkage is caused by capillary tension, solid surface tension, and the withdrawal of hindered adsorbed water and interlayer water movement from the cement gel (Bazant 2001).
Research into concrete has shown that the correct prediction of the distribution of pore relative humidity (RH) in concrete specimens is critical to the determination of the shrinkage, creep, and thermal dilatation and their effects on the stress state (Bazant and Najjar 1972a). The moisture loss of CSM also is reported to be related directly to drying shrinkage, creep, and strength gain and, therefore, is the primary cause of shrinkage cracking. The modeling of moisture loss during drying is essential for assessing the shrinkage properties and related cracking (Kodikara and Chakrabarti 2005).

During drying, the drop in moisture content occurs first in the surface layers and much later in the core. The shrinkage gradient due to non-uniform moisture loss can result in non-uniform stress within the CSM (Bazant et al. 1992, Grasley and Lange 2004). Therefore, stress gradient is present within the CSM and it keeps changing during the drying process. Although some studies on the drying shrinkage cracking of concrete exist, few of them focus on unsaturated soil and CSM. Even fewer studies consider the shrinkage stress distribution and the evolution of the stress profile during drying and the development of cement hydration in the modeling of shrinkage cracking. If the moisture gradient due to drying is high, the shrinkage stress could localize on the exposed surface and decrease sharply with depth (Sanan and George 1972, Weiss and Shah 2002). Therefore, evaluating shrinkage cracking without considering the stress gradient and its evolution due to drying could be misleading and result in significant errors.

On the other hand, unbound geomaterials and CSM properties, such as strength, stiffness, creep, and shrinkage potential (the latter of which is defined in this study as the maximum possible drying shrinkage of a material at a given pore RH when there is no
restraint), are affected significantly by the material’s moisture content and/or cement hydration. As a result of drying and hydration, the tensile strength, stiffness, and shrinkage potential within the specimen are neither uniform nor constant. The evolution of the gradients of these material properties needs to be characterized accurately in order to model and predict shrinkage cracking.

1.3 RESEARCH OBJECTIVES

The objectives of this research are to investigate the effects of drying and/or cement hydration on the material properties that are related to shrinkage and cracking, such as the diffusion coefficient, shrinkage potential, strength, and stiffness, by means of laboratory evaluation and numerical modeling, understand the mechanism of drying shrinkage cracking, and develop models for predicting the drying shrinkage cracking, considering the profile evolution of the moisture and material properties within the soil and cement-stabilized soil specimens during drying and/or cement hydration. To this end, the following tasks were performed in this study.

1. Measure the moisture- and/or hydration-dependent material properties, such as the diffusion coefficient, stress-strain relationship (elastic modulus), shrinkage potential, indirect tensile (IDT) strength, and unconfined compressive strength (UCS), etc., for soil and cement soil specimens.
2. Model and predict the evolution of the pore RH (or moisture content) profile within the soil and cement soil specimens during drying.
3. Establish the relationship between the shrinkage potential and pore RH (or moisture content) and predict the evolution of the shrinkage potential profile within the soil and cement soil specimens during drying and/or hydration.

4. Predict the evolution of the shrinkage stress profile within the soil and cement soil specimens under specific restraint (boundary) conditions during drying and/or cement hydration, using the finite element (FE) method.

5. Validate the models and prediction procedures through laboratory experiments.
CHAPTER 2: LITERATURE REVIEW

This chapter presents a comprehensive literature review of the mechanisms of cementation and shrinkage, as well as current research development and progress in the modeling and prediction of the strain, stress, and cracking that are related to shrinkage in concrete and cementitiously-stabilized materials (CSM).

2.1 MECHANISMS OF STABILIZATION

Mechanisms of stabilization that utilize cement, lime, or fly ash have been summarized by Freeman and Little (1998) as follows:

- Cation exchange: sodium, magnesium, and other cations are replaced by the calcium cations from the available calcium hydroxide.
- Flocculation and agglomeration: flocculation of the clay particles increases the effective grain size and reduces plasticity, thus increasing the strength of the matrix.
- Pozzolanic reaction: the high pH environment created by the available calcium hydroxide solubilizes silicates and aluminates at the clay surface, which in turn react with calcium ions to form cementitious products that are composed primarily of calcium silicate hydrates or calcium aluminate hydrates, or both.
- Carbonate cementation: calcium oxide reacts with carbon dioxide from the atmosphere to form calcium carbonate precipitates, which cement the soil particles.
Cementitious hydration reaction, where calcium silicates and/or calcium aluminates, which are chemically combined in the production of Portland cement clinker or in the coal burning (fly ash) process, hydrate too rapidly (within a few hours) for calcium silicate and/or calcium aluminate hydrates.

The chemical reactions during the hydration procedure include, but are not limited to, some or all of the reactions listed below (Tazawa et al. 1995).

\[
\begin{align*}
2C_2S + 6H_2O & \rightarrow C_3S_2H_3 + 3Ca(OH)_2 \\
2C_2S + 4H_2O & \rightarrow C_3S_2H_3 + Ca(OH)_2 \\
C_3A + 3(CaSO_42H_2O) + 26H_2O & \rightarrow C_3A3CaSO_432H_2O \\
2C_3A + C_3A3CaSO_432H_2O + 4H_2O & \rightarrow 3[C_3ACaSO_412H_2O] \\
C_3A + Ca(OH)_2 + 12H_2O & \rightarrow C_3ACa(OH)_212H_2O \\
C_4AF + 3(CaSO_42H_2O) + 27H_2O & \rightarrow C_3(AF)3CaSO_432H_2O + Ca(OH)_2 \\
2C_4AF + C_3(AF)3CaSO_432H_2O + 6H_2O & \rightarrow 3[C_3(AF)CaSO_412H_2O] + 2Ca(OH)_2 \\
C_4AF + 10H_2O + 2Ca(OH)_2 & \rightarrow C_3AH_6 - C_3FH_6(solid\ solution)
\end{align*}
\]

Two popular theories on the mechanisms of stabilization, namely the crystalline theory by Le Chatelier, which dates to 1882, and the gel theory proposed by Michael in 1893, have been integrated into a combined gel/crystalline theory that describes the different stages of curing (Montgomery 1998). Montgomery (1998) has provided a detailed description of the hydration process of cement concrete: If a proper amount of water is available to ensure complete hydration of the cement, then each cement particle will be coated in water and form a gel-like film. These gel-coated particles of cement themselves
coat the aggregate or soil particles. At this stage, the cement has not begun to set. The reaction between the water and the cement forms small single crystals. With the development of hydration, the single crystals begin to grow into one another and form a crystalline network. The adjoining crystals are attracted to one another by Van de Waal forces instead of a chemical bond. Figure 2.1 illustrates this process.

![Diagram of bonding process between cement and sand particles](image)

Figure 2.1: Bonding Process between Cement Particles and Sand Particles (after Montgomery 1998)

2.2 BENEFITS AND DISTRESSES ASSOCIATED WITH STABILIZED MATERIALS

2.2.1 Current Design Criteria

The design procedure for CSM includes the mix design and the structural design. Current mix design procedures typically require the stabilized mixture to show sufficient compressive strength and durability (George 1990). Standardized mix design procedures,
such as ASTM D559-57 and D560-57, exist (George 1990); however, increasing awareness of the importance of factors such as the resilient modulus, tensile strength, and fatigue resistance of stabilized materials must also be considered (George 1990).

Unconfined compressive strength (UCS) is commonly used as a design parameter for estimating the flexural modulus, which is used in mechanistic pavement design. UCS is also used in pavement material mix design and field quality control (Chakrabarti and Kodikara 2003). Often, the mix design procedures for selecting the optimum binder content are based solely on the UCS. Many state highway agencies and departments of transportation (DOTs) require a minimum UCS value between 2068 kPa (300 psi) and 5516 kPa (800 psi) after seven days (Portland Cement Association 1992). Australian road design guidelines require that the UCS should reach 2500 kPa (363 psi) after 28 days of curing for pavements with medium traffic and 4000 kPa (580 psi) for major highways with heavy traffic (Kodikara and Chakrabarti 2005). Although these levels of strength result in an extremely strong base layer, they do not necessarily guarantee good long-term pavement performance. A typical concern is shrinkage cracking. It is important to minimize the shrinkage, maximize the durability, and at the same time provide adequate strength (Syed 2000). Long-term performance could be improved by reducing the binder content to limit the shrinkage and increase the durability (Little et al. 1995).

Although a substantial amount of research has been conducted on the properties of CSM, the relationship between these properties and their performance in the pavements is not quite clear. The characterization of such materials, the changes of their properties over
time, and their distress models have not been adequately addressed in the AASHTO Interim Mechanistic-Empirical Pavement Design Guide (MEPDG) Manual of Practice.

2.2.2 Benefits

Cementitious stabilization can improve the stability and strength of pavement materials, which greatly reduces the vertical pressure of the subgrade (Freeman and Little 1998). CSM typically are used to: (1) limit subgrade stresses and deflections to preclude rutting and plastic deformation, (2) provide necessary support to the wearing course to limit transient deflections and retard fatigue cracking, (3) provide a working platform to expedite construction, and (4) prevent moisture changes in the subgrade (Little et al. 1987). The following list presents some of the engineering advantages of soil stabilization (Little et al. 1987):

- Functions as a working platform (expedites construction).
- Reduces the need for dusting.
- Waterproofs the soil.
- Upgrades marginal aggregates or soils.
- Improves strength.
- Improves durability.
- Controls volume changes of soils.
- Improves soil workability.
- Dries wet soils.
- Reduces pavement thickness requirements.
- Conserves aggregate.
- Reduces construction and haul costs.
- Conserves energy.
- Provides a temporary or permanent wearing surface.

### 2.2.3 Distresses

The performance of pavements with cementitiously-stabilized layers frequently is associated with the fact that such layers often are designed solely for strength, and the properties that characterize shrinkage and durability potential are not necessarily required (Syed 2000). The primary distresses that affect the performance of pavements with cementitiously-stabilized layers are load-induced fatigue cracking and shrinkage cracking (George 1990). Fatigue cracks typically initiate from the bottom of the cementitiously-stabilized layers and are related to the traffic load and material strength. Shrinkage cracks typically start from the surface of the stabilized layers and are related to the volume change in the cementitiously-stabilized layers (George 1990). Originally, a common practice for using stabilized materials was to provide enough stabilizers to ensure high strength and high stiffness. However, Little et al. (1995) showed that rigidly stabilized bases often lead to premature failure in the form of severe shrinkage cracking followed by accelerated fatigue cracking. Therefore, the shrinkage property should be a major concern as well as strength and stiffness.

At the time of its occurrence, shrinkage cracking in cementitiously-stabilized layers has a relatively minimal effect on the ride quality of the pavement. However, its
‘secondary deterioration’ effects, such as the deflection and weakening of the subgrade, can be highly detrimental to the pavement’s performance (Freeman and Little 1998). Shrinkage cracks start as single isolated cracks, but they can branch out to form connected multiple cracks with the development of the shrinkage and the application of traffic loads. These cracks can reduce the overall stiffness of the pavement, allow water infiltration and capillary rise into the base and subgrade, and provide pathways for erosion of the CSM (Chakrabarti and Kodikara 2003).

Furthermore, the size and spacing of the shrinkage cracks greatly affect the performance of pavements with cementitiously-stabilized layers. Closely-spaced and small cracks have less opportunity to reflect through the surface than widely-spaced and large cracks. The cracks that penetrate through the surface have the most serious effect on the performance of the road because they permit the ingress of the surface water to the sub-base and subgrade (Bofinger and Sullivan 1971).

2.3 FACTORS THAT AFFECT PROPERTIES OF CSM

A comprehensive literature review of the factors that affect the properties of CSM was provided by White (2006) and incorporated in Laboratory Characterization of Cementitiously Stabilized Pavement Materials. The information in this section is mainly based on this article, unless indicated otherwise. The factors known to have an effect on stabilized material properties include the host materials, binder type and content, moisture content, compaction, mix uniformity, curing conditions, and the age of the compacted mixture.
2.3.1 Host Materials

The properties of the host materials have a significant effect on the performance of stabilized materials. The traditional concept, which considers the soil itself as inert, has been overridden by the reality that the soil components actively participate in the hydration process, and the strength is due to the development of cementitious linkages between the hydration products and soil particles (Syed 2000). Therefore, the binder type should be chosen according to the properties of the host material. Lime is suitable for stabilizing medium, moderately fine, and fine-grained soils in order to decrease plasticity, increase workability, reduce swell, and increase strength. The Portland Cement Association (PCA) reports that cement is suitable for all types of soils. However, well-graded granular materials with sufficient fines to form a floating aggregate matrix produce the best results. Furthermore, fly ash can act as pozzolans and/or fillers to reduce the air void content. The filler role is not appropriate for fine-grained soils. Most clays are pozzolanic and do not need additional pozzolans. Therefore, silts generally are considered to be the most suitable fine-grained soil type for fly ash or cement-fly ash stabilization (Little et al. 1987).

Well-graded granular soils can be adequately stabilized with low binder content and rarely exhibit shrinkage cracking. However, for fine-grained materials, especially those with a tendency towards uniform gradation that requires more binder to achieve a specified strength, shrinkage cracking is not unusual (Bofinger and Sullivan 1971). The inclusion of fine-grained soil has a significantly negative effect on the shrinkage property of CSM and increases the possibility of cracking (Chakrabarti and Kodikara 2005).
Few documents about the effect of granular materials on the properties of CSM are available. Most of the quarried materials used in the base and sub-base layers were thought to be suitable for in situ stabilization in pavement rehabilitation projects. When the depth of the stabilization was to exceed the depth of the existing pavement and therefore incorporate some subgrade materials into the stabilized layer, a reduction in the quality of the pavement was anticipated. For in situ recycled road pavements using cement stabilization, Hodgkinson (1991) believed that the granular material should have a maximum aggregate size less than 40 mm, uniformly graded with at least 50% (by weight) granular material retained on a 0.425-mm sieve, and a plasticity index less than 20.

2.3.2 Binder Types

The type of binder used in CSM also has a substantial effect on pavement performance. In comparison with the use of lime, fly ash, or blended binder (cement blended with lime and/or fly ash), CSM that use cement as their binder provide higher stiffness values, more shrinkage, and more susceptibility to fatigue. When compared to stabilization methods that use a single binder, stabilization with blended cement shows less severe shrinkage but similar long-term strength and stiffness, even though more time is needed to attain those qualities. Blended cements are being used increasingly in Australia due to their cost-effectiveness and the reality that fly ash and lime allow longer working times (Hodgkinson 1991). Lime- and/or fly ash-stabilized materials have similar properties to those stabilized by blended cement but with much lower strength, stiffness, and shrinkage values. The reduction in shrinkage, which is attributed to the reduced heat that is
generated during hydration, is believed to be the reason for the much better performance of
the lime- and/or fly ash-stabilized materials than those materials stabilized by cement or
blended cement. However, for essentially granular materials, lime stabilization could
provide only short-term, not long-term, benefits and generally is not used in Australia
(Hodgkinson 1991).

2.3.3 Binder Content

The binder content is another critical factor that affects the properties of CSM. In
Australia, the binder contents that typically are used in CSM range from 0% to 8% by dry
weight, which is much less than the percentages used in concrete, which are over 15
percent (Chakrabarti et al. 2002). Increasing the amount of binder can increase the strength,
stiffness, and shrinkage values of CSM. Increasing the binder content is also known to
increase resistance to erosion. Vorobieff (1998) recommended 4% minimum binder
content. Hodgkinson (1991) recommended a cement content of 3.4% for the stabilization
of in situ recycled pavement materials, which are essentially granular. In cases where high
clay/fines content was present, the cement content could be increased to 4.6 percent.
Chakrabarti and Kodikara (2003) reported that for crushed rock stabilized by blended
cement, the shrinkage decreased with the increase of binder content up to 4%, and then
appeared to increase again. Bofinger and Sullivan (1971) observed that for a given soil, the
higher the cement content, the greater the crack spacing.
2.3.4 Moisture Content

Proper moisture content during the mixing and forming stages of a cement mixture is of great importance, because insufficient water will cause inadequate cement hydration, and excessive water will reduce the final setting strength (Montgomery 1998). Moisture loss is related directly to drying shrinkage, creep, and strength gain (Chakrabarti Kodikara 2006). It is commonly accepted that the moisture content of CSM during compaction affects the density achieved for a given compactive effort. The density of a soil mixture increases with the increase in moisture, reaches a maximum value, and there after starts to decrease upon further increase of the moisture content because the excess moisture restricts the soil particles from being closer to each other (Holtz and Kovacs 1981). The moisture content that corresponds to the highest dry density achieved is referred to as the optimum moisture content (OMC), which is an important parameter because density can significantly affect the fatigue performance of CSM.

2.3.5 Curing Conditions

The curing conditions for CSM have a significant effect on the final performance of the materials. The two keys to curing are temperature and moisture. High temperatures are known to accelerate the curing process of CSM. Little et al. (2002) reported that accelerated curing at 45°C (113°F) for 28 days is an approximate simulation of curing at 23°C (73°F) for 100 days, and curing at 45°C (113°F) for about 40 to 45 days corresponds roughly to curing at 23°C (73°F) for six months. Bullen (1994) reported that the curing
temperature of 100°C (212°F) could cause microcracking and a significant reduction of the elastic modulus.

A sufficient moisture supply is important for the hydration of CSM during the curing process. If enough moisture is provided, a material with high strength and stiffness values but less ductility will be achieved. However, excessive amounts of water will result in a reduction in density, which will adversely affect the properties of CSM in terms of fatigue. If CSM are cured under dry conditions, the increase in shrinkage due to drying can cause microcracks. In the laboratory, samples of CSM can be cured in a fog room with about 100% RH or be wrapped in plastic to avoid moisture loss (White 2006).

To ensure the complete hydration process of the cement and maximize the final strength of the pavement, the moisture content needs to be optimized. At the same time, the existing moisture needs to be prevented from escaping. Montgomery (1998) explains this concept as follows:

In practice this has been done in two ways. Once the initial cement has set, the formed mixture can be submersed in water during the hardening process. This guarantees that there is sufficient water present for the cement to hydrate, but since the immersion occurs after initial setting, the extra water present will not affect particle intimacy and jeopardize the final strength of the formed mixture. The other, simpler, method is to keep the formed mixture in an environment with a 100% humidity. This prevents water within the block from escaping to the surface too quickly as the surface evaporation will be almost nonexistent in an environment with a 100% humidity (Montgomery 1998).
2.3.6 Age

The hydration of CSM is a long process. The strength and stiffness of the CSM continue to increase for a long time after mixing and compaction. Over 80% of the strength gained within the first 30 days and the subsequent increase in strength can continue even after 50 days for CSM. This process is even longer for blended cement, lime- and/or fly ash-stabilized materials (TSA 1998).

Little et al. (2002) reported long-term strength gain for lime-cement-fly ash (LCF) -stabilized base layers. The fact that the strength more than doubled between the six-month period and the ten-year period indicates that the CSM experienced long-term strength gain. This gain in strength was attributed to the pozzolanic reactions and resultant autogenous healing of the microcracks caused by these reactions.

Bofinger and Sullivan (1971) investigated the tensile strength development of cement-stabilized Harmondsworth brickearth with curing times from 15 minutes to 14 days. The specimens were cured and sealed with paraffin wax at 20°C prior to the strength tests. The relationship between the tensile strength and the curing time is shown in Figure 2.2. The figure indicates that the rate of the tensile strength gain is very high during the first few hours of curing (noting that the abscissa is in minutes in Figure 2.2).
Figure 2.2: Relationship between Tensile Strength and Curing Time of Cement-Stabilized Harmondsworth Brickearth (after Bofinger and Sullivan 1971)

2.4 MECHANISMS AND TYPES OF SHRINKAGE

The causes of shrinkage cracking for concrete were studied extensively in the 1960s and 1970s (Syed 2000). During more recent decades, the topic of shrinkage has gained more attention. For example, shrinkage has been found to be related to the deformation of fine aggregate, the amount of water used during placement, and the lack of adequate curing (George 1972). Most examples in the literature categorize shrinkage into four types: (a) autogenous shrinkage, (b) drying shrinkage, (c) thermal shrinkage, and (d) carbonation shrinkage. The mechanisms of shrinkage that pertain to CSM are discussed in the following sections for each type of shrinkage.

2.4.1 Autogenous Shrinkage

Hydration is the source of autogenous shrinkage. During hydration the volume of the compound produced by the reaction between the binder material and the water is
smaller than the original total volume. This phenomenon also is called *Le Chatelier contraction* (Tazawa et al. 1995). At the same time, the chemical reaction consumes moisture, which results in the drying of the specimen. Autogenous shrinkage refers to the macroscopic volume reduction without any moisture exchange to and from the specimen under a constant temperature (Tazawa et al. 1995). Neithalath et al. (2005) defined autogenous shrinkage as “shrinkage that occurs in sealed concrete when water is consumed by the hydration reactions causing both a reduction in the volume of the reacted products and a reduction in the internal pore pressure.” Therefore, autogenous shrinkage generally is related to the extent of hydration that has taken place (Neithalath et al. 2005).

The chemical reactions between the water and the binder used in soil stabilization materials, such as Portland cement, lime, and/or fly ash, are very complicated. Tazawa et al. (1995) reported that the volume reduction of the compound of the chemical reaction from the volume of the original input material could reach 8% to 10%, based on the major chemical reaction during hydration. Most of the chemical shrinkage leads to internal air voids. Therefore, there is little macroscopic volume reduction of the specimen, as shown in Figure 2.3 for cement paste. However, no studies found in the literature indicate how this microstructural change of the internal voids affects material properties such as moisture diffusion and strength. The effect of drying that is due to interior chemical reactions and the resultant reduction of pore water is similar to the effect of external drying, which is discussed in Section 2.4.2 on drying shrinkage.
Figure 2.3: Chemical Shrinkage and Intrinsic Void Formation (after Tazawa et al. 1995)

Figure 2.4 shows the relationship between autogenous shrinkage and the Le Chatelier contraction for cement paste. The figure indicates that autogenous shrinkage is not proportional to the volume change of the chemical reaction and that no simple relationship exists between them (Tazawa et al. 1995). However, because autogenous shrinkage occurs when there is no moisture exchange to and from the specimen, autogenous shrinkage is actually the macro effect from both the Le Chatelier contraction and the movement or loss of water due to the internal chemical reaction. Other researchers, such as Wittmann (2004), define autogenous shrinkage as the “volume change as function of reduced water content due to hydration.” However, the author of this dissertation agrees with the concept that autogenous shrinkage also includes the macro effect of the Le Chatelier contraction, even though this effect could be small compared with the shrinkage due to the internal moisture loss that results from hydration.
Figure 2.4: Relationship between Autogenous Shrinkage and Le Chatelier Contraction (after Tazawa et al. 1995)

Bofinger and Sullivan (1971) investigated the shrinkage development of cement-stabilized Harmondsworth brickearth for 14 days of curing. The specimens were cured and sealed in paraffin wax at 20°C. Therefore, no moisture exchange occurred between the specimens and the environment, so the shrinkage type was autogenous shrinkage only. Figure 2.5 presents the relationship between shrinkage and curing time. It is obvious that the autogenous shrinkage rate was very high during the first few hours of curing. There was no measurement of autogenous shrinkage within 15 minutes. After 15 minutes, the autogenous shrinkage rate decreased with the increase in curing time. Bofinger and Sullivan (1971) also reported that shrinkage strain in a continuous soil-cement slab that showed no cracks after 3.5 hours of curing was estimated to be 9.80x 10^{-4}, of which 5.81 x 10^{-4}, or 59.29%, occurred within the first 30 minutes.
Wittmann (2004) presented the development of autogenous shrinkage along with thermal shrinkage due to hydration heat with respect to curing time, as shown in Figure 2.6. The figure shows that autogenous shrinkage occurred mainly within the first day and was coupled with thermal shrinkage.

Figure 2.6: Autogenous Shrinkage of Concrete at Early Age (after Wittmann 2004)
2.4.2 Drying Shrinkage

A survey carried out by Chakrabarti et al. (2002) revealed that cracking due to drying shrinkage was a major problem associated with the use of CSM in pavement construction. External drying is referred to as the process by which the moisture content inside the specimen tries to equilibrate with the surrounding environment by evaporation and diffusion processes, which results in a moisture gradient in the specimen (Neithalath et al. 2005). Drying shrinkage was thought to be related directly to moisture loss of the cement-stabilized pavement layer (Chakrabarti and Kodikara 2006). Herein, the moisture loss refers to the desiccation by evaporation from unprotected surfaces, excluding the loss of internal moisture due to hydration. The moisture loss depends on the surface area, the lengths of the moisture migration pathways, and the drying environment (Chakrabarti and Kodikara 2006). Figure 2.7 shows the drying shrinkage development with respect to drying time. For specimens with the same binder content, the ultimate drying shrinkage under one-dimensional (1-D) drying conditions is consistently higher than that under three-dimensional (3-D) drying conditions (Chakrabarti and Kodikara 2006).

![Figure 2.7: Drying Shrinkage against Drying Time (after Chakrabarti and Kodikara 2006)]
The moisture gradient can cause high matric suction to develop in the pore water in the voids adjacent to the surface. The osmotic suction that is caused by dissolving salts further increases the total suction. The combination of the suction and the restraints of deformation is the driving force behind shrinkage cracking (Dao et al. 2008). The moisture content and shrinkage vary at different locations, and their profiles change with time (i.e., they are time- and space-dependent). Therefore, nonlinear moisture diffusion theory is commonly used in predicting the drying process (Chakrabarti and Kodikara 2006).

2.4.3 Thermal Shrinkage

Williams (1986) reported that although cement hydration could be an important factor of shrinkage, temperature was of equal if not greater significance. For cement-stabilized granular material with a thermal coefficient of 10μs (microstrains) per degree Celsius, which is typical for such material, a temperature fall of only a few degrees would induce a strain sufficient to cause fracture in a brittle material when its strength was low at the initial stage of curing (Williams 1986). Fortunately, during the early age of CSM, hydration heat tends to increase the temperature. For concrete, as a general rule, the early heat of hydration is about 5°C to 8°C (10°F - 15°F) of the adiabatic temperature rise per 45 kg of cement. The heat rise typically occurs within the first 12 hours of mixing. The heat generated by hydration in CSM depends on the binder content and hydration rate. During the heating process, the CSM will expand. Some of the thermal expansion is elastic, so the material can return to its original dimensions when it is cooling. However, any non-elastic portion will result in permanent thermal shrinkage (Holt 2001). The risk of shrinkage
cracking increases with an increase of binder content due to the increased heat during hydration (White 2006). Considering overnight temperature drops, the cooling of the heat that is generated by hydration, and the relative low strength or strain capacity, the temperature fluctuation during the early age of CSM in the field could have a significant effect on shrinkage cracking. At a later age, the thermal shrinkage is primarily a result of the environmental temperature fluctuation. In the case of a seasonal temperature fall of 30°C for a gravel lean concrete, cracking is totally unavoidable under conditions of full restraint (Williams 1986). However, George (1969) reported that the contraction that results from drying is more important in stabilized soils than that caused by temperature, probably due to the low binder content in stabilized soils.

2.4.4 Carbonation Shrinkage

Most research studies of carbonation shrinkage are based on concrete material. However, the mechanisms are applicable to CSM, although the amount of carbonation shrinkage in CSM may be less than that in concrete.

Carbonation shrinkage occurs when the concrete is exposed to air that contains carbon dioxide. The compounds that were formed during hydration can react chemically with carbonic acid, which is a solution that is formed when atmospheric carbon dioxide dissolves in water. When the calcium compounds react with the carbonic acid, the calcium forms calcium carbonate, the combined water is released as ‘free water’, and the rest of the compounds become silica and alumina gels (Erlin and Hime 2004). These carbonation
reactions can reduce the pH of the concrete from 12.4 and higher to about 8.5 to 9 (Erlin and Hime 2004).

During the process of concrete carbonation, the weight of the concrete increases and the concrete undergoes irreversible carbonation shrinkage. Houst (1997) proved by experiments that when the RH is within the range of 50% to 80%, carbonation develops much faster than when the RH is out of this range. The carbonation process proceeds slowly and produces small shrinkage at RH values that are below 25% or near 100 percent.

Netterberg and Paige-Green (1984) carried out a literature review and research that focused on the carbonation of lime- and cement-stabilized layers in pavements. The carbonation was thought to occur primarily during curing and under the subsequent exposure to the atmosphere prior to sealing, even under a prime coat. The carbonation during curing can cause the lime to revert to the limestone from which it was made and thus becomes no longer available to complete the stabilization process and to preserve the cementing compounds.

Although several hypotheses have been proposed to explain carbonation shrinkage, it is still not clear how it happens. Further research is needed on carbonation shrinkage.

2.5 MODELING OF DRYING SHRINKAGE STRAIN

Among the four types of shrinkage, drying shrinkage is the major cause of shrinkage cracking that is associated with the use of CSM in pavement construction (Chakrabarti et al. 2002). Autogenous shrinkage is relatively insignificant. Temperature changes that are due to hydration heat and the environment can be superposed
conveniently with drying shrinkage. Carbonation shrinkage is a long-term reaction, especially when the CSM have low RH values and are covered by an upper layer. Therefore, the study of drying shrinkage is the major concern of the remainder of this literature review.

Drying shrinkage is believed to be related directly to moisture loss of cement-stabilized pavement layers (Chakrabarti and Kodikara 2006). It is generally accepted that drying shrinkage is caused by capillary tension, solid surface tension, and the withdrawal of hindered adsorbed water and interlayer water movement from the cement gel (Bazant 2001).

Recent research in concrete has shown that the correct prediction of the distribution of the pore humidity in concrete specimens is needed to determine the shrinkage, creep, thermal dilatation, and their effects on the stress state (Bazant and Najjar 1972a). The moisture loss of CSM also is reported to be related directly to drying shrinkage, creep, and strength gain and, therefore, is the primary cause of shrinkage cracking. The modeling of the moisture loss during drying is essential for assessing the shrinkage properties and related cracking (Kodikara and Chakrabarti 2005).

Moisture loss from any porous material depends mainly on the surface area, the lengths of the moisture migration pathways, and the drying environment (Chakrabarti and Kodikara 2006). Nonlinear moisture diffusion theory is commonly used to describe the drying process of CSM. Unsaturated soil mechanics and micromechanics typically are used to relate the moisture loss to shrinkage strain and stress.
Based on the literature review, the procedures required to predict drying shrinkage stress are: (1) predict the moisture loss gradient or the pore RH gradient at any time, based on the theory of diffusion; (2) construct the relationship between moisture loss or pore RH and shrinkage strain; and (3) establish the relationship between shrinkage stress and the shrinkage strain.

2.5.1 Moisture or Relative Humidity Distribution

Based on the work of Bazant and Najjar (1972a), the rate of the diffusion of water can be characterized by moisture flux, J, which denotes the mass of the water that passes through a unit area perpendicular to J per unit time. Moisture flux, J, can be expressed two different ways. It can be defined in terms of the pore RH gradient or in terms of the free water content gradient, as Equation (2.9) (Abbasnia et al. 2009).

\[ J = -D \text{grad}(\text{RH}) \quad \& \quad J = -D\text{grad}(w_e) \]  \hspace{1cm} (2.9)

where J is the moisture flux; D is the diffusion coefficient corresponding to either pore RH or evaporable water content; RH is the pore RH; and we is the evaporable water content.

When the temperature, T, is variable, but not rapidly changing and almost uniform, T and the progression of hydration (aging, maturing) are taken into account, and the diffusion equation that governs the drying of porous material can be expressed as Equation (2.10). This equation is based on the assumption that the slope of the moisture isotherm, k = \( \left( \frac{\partial \text{RH}}{\partial w} \right)_T \), usually is almost constant over a wide range of humidity values, such as from 0.15 to 0.95, especially for dense materials.

\[ \frac{\partial \text{RH}}{\partial t} = \text{div}(D \text{gradRH}) + \frac{\partial \text{RH}_s}{\partial t} + K \frac{\partial T}{\partial t} \]  \hspace{1cm} (2.10)
where RH is the pore RH, which is the ratio of the vapor pressure over the vapor pressure at saturation; D is the diffusion coefficient, which is a function of RH; \( \frac{\partial \text{RH}_S}{\partial t} \) is the change in RH caused by hydration during unit time intervals at a constant water content and temperature; RHs(t) is the pore relative humidity, which would occur in time t in a sealed, initially wet specimen; T is temperature; and K = \( \left( \frac{\partial \text{RH}}{\partial T} \right)_w \) = is the hygrothermic coefficient, which is a function of RH and reflects the change in RH due to a one-degree change in T at a constant water content and a fixed degree of hydration.

If T is constant and the rate of hydration is negligible (later age of the CSM), Equation (2.10) can be expressed as Equation (2.11).

\[
\frac{\partial \text{RH}}{\partial t} = \text{div}(D \text{gradRH}) \quad (2.11)
\]

The diffusion equation in terms of evaporable water content is based on Fich’s 2\(^{nd}\) law, which expresses the mass flux as a function of the concentration gradient, as shown by Equation (2.12).

\[
J = -D \text{grad} (w_e) \quad (2.12)
\]

The general expression that considers the effect of variable temperature and progressing hydration in terms of evaporable water content can be expressed as Equation (2.13) (Bazant and Najjar 1972a).

\[
\frac{\partial w_e}{\partial t} = \text{div}(D \text{grad} w_e) - \frac{\partial w_n}{\partial t} \quad (2.13)
\]

where \( w_e \) is the evaporable water content; D is the diffusion coefficient corresponding to evaporable water content; and \( \frac{\partial w_n}{\partial t} \) represents the rate of loss in the evaporable water content due to hydration.
Bazant and Najjar (1972a) reported that the formulation of moisture diffusion in terms of pore RH is preferable to the evaporable water content for several reasons. First, the use of Fich’s law is not justified when hydration proceeds, because the distribution of that pore volume that is available to evaporable water becomes non-uniform. Therefore, equal evaporable water content values at various points do not correspond to equal values of pore RH and Gibbs’ free energy. Also, the use of pore RH is more practical for the rational analysis of creep and shrinkage effects. Kodikara and Chakrabarti (2005) also suggest that the formulation of moisture diffusion in terms of pore RH can remain continuity at layer boundaries.

The governing equation of moisture movement can be solved mathematically or numerically, depending on some assumptions and the application of suitable boundary conditions.

1. Closed-form Method

Closed-form solutions generally are based on the linear diffusion equation, which is known to give a very poor fit of experimental data (Bazant and Najjar 1972a). In this case, the diffusion coefficient is assumed as a constant, and the governing differential equation can be solved mathematically. Some early studies on drying shrinkage (Pickett 1946, Kawamura 1969, George 1971, Sanan and George 1972) were based on this assumption. However, Neithalath et al. (2005) reported that for short drying times, a linear approach could greatly simplify the calculations while still providing a reasonable estimate. His solution of the governing differential equation can be expressed as Equation (2.14).

\[
\text{RH}(x,t) = \text{RH}_I - (\text{RH}_I - \text{RH}_S) \left[ \text{erfc} \left( \frac{x}{2\sqrt{Dt}} \right) \right]
\]  \hspace{1cm} (2.14)
where $\text{RH}(x, t)$ is the relative humidity at a depth $x$ from the drying surface at a time from the start of drying $t$; $\text{erfc}$ is the complementary error function; $\text{RH}_S$ is the relative humidity at the surface of the specimen; $\text{RH}_I$ is the relative humidity in the interior of a sealed specimen; and $D$ is the diffusion coefficient.

2. Numerical Method

With the development of the finite difference method, numerical methods could be used in the analysis of moisture diffusion. The assumption that the diffusion coefficient is constant, which was used to simplify the calculation, is not necessary.

The dependence of the diffusion coefficient on the pore water content was first considered by Pihlajavaara and Vaisanen (1965). Although they did not report any attempts to fit actual data, they observed that the diffusion coefficient decreased several times when the RH dropped from 100% to 70 percent. Since then, different forms of relationships between the diffusion coefficient and moisture or pore RH have been established by different researchers.

Bazant and Najjar (1972b) carried out an analytical study of the dependence of diffusivity on pore RH, temperature, and other variables (such as hydration). They concluded that the diffusion coefficient strongly depends on the pore RH, and thus, the diffusion problem is strongly nonlinear, based on the well-known Fich’s law. The diffusion coefficient is expressed as a function of pore RH, as shown in Equation (2.15). The predicted results based on nonlinear diffusion theory fit the test data better than those based on linear diffusion theory.

$$D(\text{RH}) = D_1 \left[ \alpha + \frac{1-\alpha}{1+\left(\frac{1-\text{RH}}{1-\text{RH}_c}\right)^n} \right]$$  \hspace{1cm} (2.15)
where $D$ is the diffusion coefficient; $RH$ is the pore relative humidity; $D_1$ is the maximum of $D(RH)$ for $RH=1.0$; $\alpha=D_0/D_1$ with $D_0$ as the minimum of $D(RH)$ for $RH=0.0$; and $n$ is constant.

Sakata (1983) studied moisture diffusion and distribution in concrete by assuming that the behavior of moisture during drying is governed by the nonlinear diffusion equation. The moisture diffusion coefficient is expressed as a function of the specific moisture content (not the pore RH), based on experiment results, as shown in Equation (2.16).

$$D(\theta) = D_0 + \frac{D_s-D_0}{1+\left(\frac{100-\theta}{100-\theta_c}\right)^n} + \frac{D_s-D_0}{1+\left(\frac{100-\theta}{100-\theta_c}\right)^n} \times \frac{\theta-100}{100}$$

Equation (2.16)

where $D$ is the diffusion coefficient; $D_0$ is the diffusion coefficient for oven-dry conditions or $\theta=0\%$; $D_s$ is the diffusion coefficient in the saturated condition or $\theta=100\%$; $\theta$ is the relative moisture content with the saturated condition as 100% and the oven-dry condition as 0%; $\theta_c$ is the relative moisture content at the inflection point of the curve; and $n$ is constant.

Kodikara and Chakrabarti (2005) developed Equation (2.17) as the relationship between the diffusion coefficient and relative moisture content for cement-stabilized crushed rock. This relationship is plotted in Figure 2.8 for different contents of ‘general’ Portland cement (GP%).

$$D(\theta) = D_0 + c \left(\frac{\theta}{1-\theta}\right)^d$$

Equation (2.17)

where $D$ is the diffusion coefficient; $D_0$ is the diffusion coefficient under oven-dry conditions or $\theta=0\%$; $\theta$ is the relative moisture content, which is the moisture content normalized by the initial moisture content, $w_0$; and $c$ and $d$ are constants.
Abbasnia et al. (2009) compared the linear diffusion equation, the Bazant and Najjar equation, and the Sakata equation. They reported that both the Bazant and Najjar equation and Sakata equation reflect the experiment data fairly well and much better than the linear equation, as shown in Figure 2.9.

Figure 2.8: Diffusion Coefficient versus Relative Moisture Content for Different Percentages of General Portland (GP) Cement Binder (after Chakrabarti and Kodikara 2006)

Figure 2.9: Measured and Predicted Moisture Loss for Linear, Bazant (and Najjar), and Sakata Diffusion Equations (after Abbasnia et al. 2009)
In the numerical procedure, the initial condition and environmental flux boundary condition must be applied. According to Kodikara and Chakrabarti (2005), the initial condition is the initial moisture content at the beginning of the drying process. These moisture contents can be converted to equivalent RH values using the humidity isotherm. The environmental flux boundary condition, which defines the moisture evaporation (moisture loss) from the material surface, is determined by the environmental flux boundary condition at the surface, as shown by Equation (2.18).

\[
E_s = -D \left( \frac{\partial w}{\partial n} \right)_s = f(RH_s - RH_{en}) \tag{2.18}
\]

where \( f \) is a surface factor that takes into account the wind velocity, surface temperature and surface roughness; \( n \) is the unit vector normal to the drying surface; and \( RH_{en} \) and \( RH_s \) are respectively the environmental RH and the pore RH at the material surface.

### 2.5.2 Shrinkage Strain

Typically, the free shrinkage strain distribution in specimens is not measured directly because such measurements are difficult to take; however, it can be properly modeled as a function of pore RH. Therefore, the relationship between shrinkage and pore RH at each location needs to be established (Kim and Lee 1998). The shrinkage coefficient can be used to express the change in strain as a function of moisture or change in RH (Ayano and Wittmann 2002). As is well known, drying shrinkage may be approximately proportional to the loss of water from the specimen (Pickett 1946, George 1968a, Sanan and George 1972, Kawamura 1969), whereas other investigators have established a nonlinear relationship between drying shrinkage and pore RH change or moisture loss.
1. Linear shrinkage strain and moisture loss or relative humidity change relationship

Similar to modeling moisture diffusion, first the relationship between the free shrinkage strain and the moisture loss or RH change is assumed as linear. Pickett (1946) reported that the shrinkage of concrete is a linear function of moisture loss and used shrinkage strain directly in the governing diffusion equation. George (1968a) experimentally established a linear relationship between shrinkage and moisture content loss, except during the final stages of drying. Sakata (1983) showed experimentally that the relationship between moisture loss and shrinkage strain near the drying surface could be expressed approximately by a straight line. Sanan and George (1972) assumed that shrinkage due to drying is proportional to moisture loss in their research to predict the shrinkage stress in a soil cement base. Kawamura (1969) studied the shrinkage stress in concrete based on the same assumption as that of Sanan and George.

Neithalath et al. (2005) reported that free shrinkage strain relates approximately to the change in RH linearly if the RH change is small (i.e., RH>50%), as shown by Equation (2.19). However, for CSM used in pavement, the RH is typically low, and thus it may not be appropriate to assume a linear relationship.

\[ \varepsilon_{sh}(t) = \varepsilon_{sh-\infty}[100\% - \text{RH}(x, t)] \]  

(2.19)

where \( \varepsilon_{sh}(t) \) is the shrinkage strain; \( \varepsilon_{sh-\infty} \) is a constant that is called the free shrinkage coefficient; and RH is relative humidity.
2. Nonlinear shrinkage strain and moisture loss or relative humidity change relationship

Bazant and Xi (1994) studied the relationship between the free shrinkage strain rate and pore RH rate. They reported that in the case of ideal uniform free shrinkage without any restraint, no stress will develop. The local shrinkage strain rate can be expressed as Equation (2.20). The shrinkage coefficient is expressed by Equation (2.21). It should be noted that these equations were derived for concrete.

\[ \varepsilon_{sh} = K_{sh} \left( R\dot{H} \right) \]  
(2.20)

\[ k_{sh} = -\epsilon_s^0 \ g_s(t) \frac{df_s(RH)}{d(RH)} \]  
(2.21)

where \( k_{sh} \) is the shrinkage coefficient; \( \epsilon_s^0 \) is the ultimate shrinkage; \( g_s(t) = E(t_0)/E(t) \); and \( E(t) \) can be estimated by:

\[ E(t) = E(28) \sqrt{\frac{t}{4 + 0.85t}} \]

with: \( E(28) = 33 \ Y^{1.5} \sqrt{f_c} \)

where \( Y \) is the unit weight of concrete in lb/ft\(^3\); and \( f_c \) and \( E \) are in psi for \( t \rightarrow \infty \), \( g_s(t) \rightarrow 1 \).

However, the expression \( f_s(RH) \) has not yet been proved by experiments. It was assumed approximately as Equation (2.22) (Bazant and Xi 1994).

\[ f_s(RH) = 1 - RH^3 \]  
(2.22)

Bazant et al. (1992) also used an approximate linear equation to predict free shrinkage strain, as shown by Equation (2.23).

\[ \varepsilon_{sh} = k[1 - (RH)^3] \]  
(2.23)
where $\varepsilon_{sh}$ is free shrinkage; $k$ is an empirical constant, generally between $-0.0002$ and $-0.0010$ for concrete; and RH is relative humidity.

Bazant and Xi’s model was used also by Kim and Lee (1998). They considered a nonlinear diffusion coefficient and the effect of creep in concrete. Their analytical results were in good agreement with the test results, as shown in Figure 2.10.

Abbasnia et al. (2009) reported that free shrinkage strain as a material property could be taken as proportional to the variation in pore RH, as shown by Equation (2.24) in which the shrinkage coefficient is not constant, but a function of RH. However, an expression for the shrinkage coefficient, $k_{sh}(RH)$, was not provided.

$$
\varepsilon_{sh}(t) = k_{sh} f(RH) = k_{sh}(RH)(1 - RH)
$$

(2.24)

where $\varepsilon_{sh}$ is free shrinkage strain; and $k_{sh}$ is the shrinkage coefficient, which is not a constant but a function of pore RH.
The Chakrabarti and Kodikara model is based on the assumption that drying shrinkage is a result of matric suction within the pore structure. The matric suction can be related to the pore RH using the Kelvin-Laplace equation (2.25) (Chakrabarti and Kodikara 2006).

$$\psi = -\frac{RT}{w_v\left(\frac{1}{\rho_w}\right)} \ln(RH)$$  \hspace{1cm} (2.25)

where $\psi$ is the matric suction (MPa); R is the universal gas constant (8.31 MPa·cm³/(mol·K) or J/(mol·K)); T is the temperature of the sample in Kelvin (K); $w_v$ is the molecular mass of water (18 g/mol); $\rho_w$ is the unit weight of water (1 g/cm³); and RH is the pore relative humidity.

Chakrabarti and Kodikara (2006) assumed that the material exhibits isotropic stress generation and nonlinear elastic behavior during drying. Therefore, the shrinkage stress increment $\Delta\sigma_{sh}$ could be related to the shrinkage strain increment $\Delta\varepsilon_{sh}$ (under specific restraint) using Equation (2.26).

$$\Delta\varepsilon_{sh} = \frac{(1-2v)}{E} \Delta\sigma_{sh} + \frac{\Delta\psi}{H}$$  \hspace{1cm} (2.26)

where E is the tangent elastic modulus; v is Poisson’s ratio; $\Delta\psi$ is the increments of moisture suction; and H is a tangent modulus with respect to suction.

For the free shrinkage condition, the shrinkage stress should be zero. The total shrinkage strain can be expressed as Equation (2.27).

$$\Delta\varepsilon_{sh} = \frac{\Delta\psi}{H}$$  \hspace{1cm} (2.27)

Combining Equations (2.25) and (2.27), the free shrinkage can be expressed as Equation (2.28):
\[ \Delta \varepsilon_{sh} = \Delta \left( \frac{RT}{W_c \ln RH} \right) / H \]  

(2.28)

Other methods can be used to determine the shrinkage strain, such as regressive equations obtained from test results (Asad et al. 1997) and equations based on matric suction that consider the degree of saturation and volume fraction of the aggregate (Grasley and Lange 2004).

### 2.6 MODELING OF SHRINKAGE CRACKING

#### 2.6.1 Shrinkage Stress

Shrinkage stress is not caused by shrinkage alone. It results from the restraint of shrinkage, either by an internal restraint in the free shrinkage case or a combination of external and internal restraints in the externally restrained shrinkage case. In other words, shrinkage stress is not related directly to the material’s shrinkage strain, but to the shrinkage strain that is restrained from occurring (Grasley 2003). Cement-based materials can be restrained externally by forces applied on the surface of the specimen, whereas the internal restraint is an intrinsic property of cement-based materials. Therefore, there are two types of internal restraint: self-restraint and aggregate restraint. The shrinkage stress that is due to internal restraint can be coupled with that which is due to external restraint (Bisschop and Mier 2002). This literature review focuses on shrinkage stress that is due to external restraint, because the internal restraint is relative small compared to external restraint in pavement layers.
When a material’s shrinkage strain is restrained, a total stress increment $d\sigma_{ij}$ arises in an element of the specimen, which can be expressed as Equation (2.29) (Peron et al. 2009).

$$d\sigma_{ij} = D_{ijkl} d\varepsilon_{kl}^{m} = D_{ijkl} \left( d\varepsilon_{kl} - d\varepsilon_{kl}^{h} \right)$$

where $d\sigma_{ij}$ is the total stress increment; $D_{ijkl}$ is a tangent stiffness matrix; $d\varepsilon_{kl}^{m}$ is the mechanical strain increment responsible for stress generation; $d\varepsilon_{kl}$ is the observed strain increment with restrained shrinkage; and $d\varepsilon_{kl}^{h}$ is the shrinkage strain increment (free shrinkage).

During the shrinkage process in which drying and hydration develop, the material properties, such as the elastic modulus and creep compliance, could change significantly, which complicates the stress due to shrinkage and restraint (Peron et al. 2009).

Several studies have been conducted on shrinkage stress that is based on various shrinkage distributions, restraint conditions, and material properties (elastic or viscoelastic). George (1971) summarized previous studies of shrinkage stress under different restraint conditions and reported that, in an unrestrained plate, the stress due to uniform shrinkage is zero. In a completely restrained pavement slab, the uniform shrinkage leads to uniform stress throughout the depth of the slab without any distortion. However, if the slab is partially restrained, or subjected to a strain gradient throughout the depth, it tends to warp.

In a slab, if shrinkage stress is linearly distributed but not symmetrical to the central plane, the stress can be expressed by Equation (2.30) (Timoshenko and Goodier 1971).

$$\sigma_{xx} = \sigma_{yy} = -\frac{E S_{x}}{1 - \nu} + \frac{1}{h(1 - \nu)} \int_{0}^{h} E S_{z} d z + \frac{12z}{h^{3}(1 - \nu)} \int_{0}^{h} E S_{x} z d z$$

(2.30)
where $\sigma_{xx}$ and $\sigma_{yy}$ are the normal components of stress parallel to the x and y axes in the plane of the slab; E is the modulus of elasticity of the slab material; $\nu$ is the Poisson’s ratio of the slab material; h is the thickness of the slab; and $S$ is the free shrinkage.

Westergaard (1926) solved for the stress of linear distribution for a slab with a finite width and infinite length supported on a Winkler foundation (homogeneous foundation whose reaction against the slab is proportional to the deflection).

For nonlinear strain distribution, Reddy et al. (1963) presented solutions for stress and deflections in a finite circular slab by considering that warping could result in partial support of the slab by the ground.

For the shrinkage stress in a slab with one surface perfectly bonded to a rigid foundation, the tensile shrinkage stress at the bonding surface of the slab can be simply expressed as Equation (2.31) (George 1971).

$$\sigma_{xx} = \sigma_{yy} = \frac{ES}{1-\nu} \quad (2.31)$$

A theoretical expression of the stress in the top of the slab was obtained by Durelli and Parks (1967) and is presented as Equation (2.32).

$$\sigma_{xx} = \sigma_{yy} \cong 2ES \quad (2.32)$$

where $\sigma_{xx}$ and $\sigma_{yy}$ are the normal components of stress along the x, y directions in the plane parallel to the bonding surface; E is the modulus of elasticity of the slab material; $S$ is the shrinkage strain of the slab material; and $\nu$ is the Poisson’s ratio of the slab material.

Sanan and George (1972) obtained shrinkage stress solutions for soil cement base layers, based on both elastic and viscoelastic material assumptions. The solutions are for an infinitely long slab with width B resting on a Winkler foundation, with three restraint
conditions: complete restraint, restraint against warping only, and restraint against longitudinal displacement only. These three restraint conditions may be suitable also for other idealized pavement conditions. For example, complete restraint is for an infinite pavement layer (such as a base made with CSM), which is fully bounded with the underlying layer. Restraint against warping may be suitable for the pavement layer, which has already cracked (and, therefore, has a finite dimension) but is still bounded with the underlying layer. However, after cracking, the actual conditions could be much more complicated. For example, there could be partial restraint against warping and longitudinal displacement.

Zhang and Leng (2004) derived a solution based on the restraint that comes from bilinear underlying friction stress. They reported that with the development of slippage between a jointed concrete slab and the underlying support layer, the shear stress between these two layers initially increases almost linearly to a certain steady value, the so-called steady-state frictional stress, $\tau_0$, as shown in Figure 2.11. $\tau_0$ and $\delta_0$ in Figure 2.11 normally are governed by the type of underlying layer and can be determined using sliding tests. After the restraint is determined, the stress due to shrinkage can be solved using equilibrium and compatibility equations. The limitation of Zhang and Leng’s solution is that it does not consider the shrinkage strain gradient and viscous properties of the material.
George (1969) reported that shrinkage stress relates strongly to the shrinkage rate, if the materials are treated as viscoelastic.

2.6.2 Relaxation

Although shrinkage can result in stress in cementitiously-stabilized layers, relaxation in the layer could relieve this stress significantly. Shrinkage stress and relaxation stress are similar in their orders of magnitude, as shown in Figure 2.12. Therefore, ignoring the relaxation effect could introduce significant errors in stress estimations (Weiss et al. 1998).
Before the effect of stress relaxation is considered, the true stress state within the specimen is still unknown. However, the potential stress estimated by previous section could be used to analyze the effect of material creep with reasonable accuracy, as shown in the Grasley and Lange study (2004) where the measured shrinkage strain under restraint is expressed here as Equation (2.33).

\[ \varepsilon_T = \varepsilon_{sh} + \varepsilon_{cre} + \varepsilon_{\sigma} \]  

(2.33)

where \( \varepsilon_T \) is the measured shrinkage strain under restraint; \( \varepsilon_{sh} \) is the true unrestrained free shrinkage; \( \varepsilon_{cre} \) is the strain relaxed by creep; and \( \varepsilon_{\sigma} \) is the remaining strain associated with the stress due to restraint.

The estimated stress before considering the creep can be used as the potential stress under which the creep develops. The creep strain under this potential stress is calculated using creep compliance. The stress reduction that corresponds to this creep strain can be

Figure 2.12: Stress Analysis Showing Creep Relaxation (after Weiss et al. 1998)
calculated under the assumption that the material is elastic. Finally, the true estimated stress is obtained by subtracting the stress reduction due to creep from the estimated potential stress. This process must be carried out step by step for each small time interval.

By taking into consideration the fact that stress relaxation reduces the actual shrinkage stress level, Weiss et al. (1998) calculated stress by coupling the effects of shrinkage strain and stress relaxation. Because both shrinkage and creep are time-dependent processes, they considered the strain components incrementally. The total differential strain of a specimen element at time t loaded at time $\xi$ (0<$\xi$<t) with an infinitesimal stress increment $d\sigma$ is expressed as Equation (2.34).

$$d\varepsilon(t, \xi) = d\varepsilon_{\sigma}(\xi) + d\varepsilon_{\text{cre}}(t, \xi) + d\varepsilon_{\text{sh}}(\xi)$$  \hspace{1cm} (2.34)

where $d\varepsilon(t, \xi)$ is the total differential strain; $d\varepsilon_{\sigma}(\xi)$ is the elastic strain at loading; $d\varepsilon_{\text{cre}}(t, \xi)$ is the creep strain at time t>$\xi$; and $d\varepsilon_{\text{sh}}(\xi)$ is the differential shrinkage strain.

The creep strain $d\varepsilon_{\text{cre}}(t, \xi)$ is expressed by means of a tangent modulus and creep coefficient, as presented in Equation (2.35).

$$d\varepsilon_{\text{cre}}(t, \xi) = \frac{d\sigma(\xi)}{E_c} \phi(t, \xi)$$  \hspace{1cm} (2.35)

where $d\varepsilon_{\text{cre}}(t, \xi)$ is the creep strain at time t>$\xi$; $d\sigma(\xi)$ is an infinitesimal stress increment; $E_c$ is the tangent modulus; and $\phi(t, \xi)$ is the creep coefficient that can be obtained by experiments.

Based on a similar concept, other studies (Kim and Lee 1998, Rahman et al. 2000, Yuan and Wan 2002) in which the effects of creep are considered can be found in the literature.
2.6.3 Strength

The capacity of CSM to undertake shrinkage stress can be characterized in terms of tensile strength, a critical stress intensity factor, $K_{IC}$, and a fracture resistance curve (R-curve), etc. These parameters are all time-dependent in CSM due to hydration.

Weiss et al. (1998) measured the compressive strength and the critical stress intensity factor of both normal strength concrete (NSC) and high strength concrete (HSC) at 1, 3, 7, 14, and 28 days. 0%, 1%, and 2% shrinkage-reducing admixtures (SRA) were used for the different specimens. Figure 2.13 shows the development of the compressive strength, elastic modulus, and critical stress intensity factor with respect to age for the NSC. Figure 2.13 shows that within seven days the compressive and fracture properties increased quickly. After seven days, the curves become flat. The critical stress intensity factor even decreased between 14 days and 28 days. The Weiss et al. study provided no explanation of this phenomenon.

Very few studies have focused on the development of the tensile strength of CSM. Bofinger and Sullivan (1971) investigated the tensile strength development of cement-stabilized soil. Their results are shown in Figure 2.2.
2.6.4 Shrinkage Cracking

The risk of shrinkage cracking in any given structure is related to many factors, including the magnitude and distribution of free shrinkage, rate of shrinkage, degree of stress relaxation, degree of structural restraint, and how the material strength and stiffness develop (Weiss and Shah 2002).

1. Criteria based on tensile strength or empirical equation

Empirical models that are based on field investigations or models that are based on tensile strength have been proposed to predict crack spacing that occurs due to shrinkage (McCullough 1983, George 1968b, George 1969, Penev and Kawamura 1993).

George (1968b) derived formulae to predict crack spacing and crack widths in cement-treated layers in pavements, as shown in Equations (2.36) and (2.37).
equations indicate that crack spacing is a function of the tensile strength of the material and crack widths are primarily a function of the total shrinkage.

\[ L_{\text{max}} = \frac{2\sigma_u}{\mu \gamma} \]  
(2.36)

\[ \delta_T = \varepsilon_c L - \frac{\mu \gamma L^2}{4E_t} \]  
(2.37)

where \( L_{\text{max}} \) is the slab length at which the tensile stress becomes critical (ft); \( \sigma_u \) is the ultimate tensile strength (lb/ft\(^2\)); \( \mu \) is the coefficient of sliding friction; \( \gamma \) is the unit weight of the material (lb/ft\(^3\)); \( \delta_T \) is the total crack width (ft); \( \varepsilon_c \) is total shrinkage (in./in.); and \( E_t \) is the modulus of elasticity of the soil-cement in tension (lb/ft\(^2\)).

However, Equations (2.36) and (2.37) are based on the assumption of elasticity. The predictions were shown to be in extremely poor agreement with the measured data for crack widths (George 1969). After considering the viscoelastic property of the soil-cement based on Burger’s model and assuming that the subgrade resistance is linearly time-dependent and that the space variation of stress is linear, Equations (2.36) and (2.37) can be rewritten as Equations (2.38) and (2.39).

\[ L_{\text{max}} = \frac{2\sigma_u}{\mu \gamma} \]  
(2.38)

with: \( \sigma_u = \left( \frac{E_k + E_m}{t_1} \right) E_m \left[ \frac{\eta_m}{E_m} + \frac{1}{\beta - \alpha} \exp(-\alpha t) + \frac{1}{\beta - \alpha} \exp(-\beta t) \right] \)

where \( \alpha = \frac{1}{2} \left( \frac{E_k + E_m}{\eta_k} + \frac{E_m}{\eta_m} \right) + \frac{1}{2} \sqrt{\left( \frac{E_k + E_m}{\eta_k} + \frac{E_m}{\eta_m} \right)^2 - 4 \frac{E_k E_m}{\eta_k \eta_m}} \)

\( \beta = \frac{\eta_k}{E_k} \)

\( \frac{E_k}{t_1} \) = the shrinkage rate;
E_k, E_m, η_k, η_m are the Burger’s model parameters.

\[
\delta_T = \varepsilon_c L - \frac{\mu \gamma L^2}{4E_t} = \delta_1 - \delta_2 \tag{2.39}
\]

where \(\delta_1\) accounts for the contraction caused by shrinkage, which is not affected by the viscous property.

\[
\delta_2 = \frac{\mu \gamma L^2}{4} \left\{ \frac{1}{E_m} + \frac{1}{E_k} + \frac{t}{2\eta_m} \right\} t + \frac{1}{E_k} \frac{\tau}{t_1} \left[ \exp \left( -\frac{t}{\tau} \right) - 1 \right] \quad \text{for } t < t_1
\]

\[
\delta_2 = \frac{\mu \gamma L^2}{4} \left\{ \frac{1}{E_m} + \frac{1}{E_k} + \frac{1}{\eta_m} \left( t - \frac{t_1}{2} \right) + \frac{1}{E_k} \frac{\tau}{t_1} \left[ \exp \left( -\frac{t}{\tau} \right) - \exp \left( -\frac{t - t_1}{\tau} \right) \right] \right\} \quad \text{for } t > t_1
\]

However, neither the stress distribution nor the effect of warping is considered in these equations.

Using a viscoelastic model, the occurrence of shrinkage cracking can be determined by the interactions of the shrinkage rate, tensile strength, deformational properties of the material, and time. The ultimate strain (limiting strain) at the cracks depends on the strain history of the material before failure. The ultimate strain of the material is greater at a low shrinkage rate than at a high shrinkage rate, as shown in Figure 2.14, which indicates that the shrinkage rate has an effect on shrinkage cracking. George (1969) concluded that by using viscoelastic modified models, the agreement between the actual and the predicted crack width is excellent.
2. Criteria based on fracture mechanics

The tensile strength-based approach for shrinkage cracking analysis is unreliable and does not account for energy-absorbing mechanisms, such as the addition of fibers (Weiss et al. 1998). Therefore, fracture (or damage) mechanics have been used by some investigators to analyze shrinkage cracking.

Penev and Kawamura (1993) derived a model based on the following assumptions:
(a) the coefficient of subgrade resistance $\mu$ increases with the displacement of the slab (a function of free shrinkage); (b) cracks occur when the free shrinkage strain exceeds the bearing tensile strain of the material (ultimate strain) and the free shrinkage strain achieves its maximum value at the time of crack formation; (c) the crack width no longer increases when it reaches final failure, which is defined as the state in which the elastic strain energy in the cracked region reaches the fracture energy ($G_f$); (d) between the two time points of (b) and (c), cracks are in a slow growth stage in which the stress transfers substantially
through the ligament (post-cracking resistance). The results predicted by this model are reported to be in good agreement with those obtained from *in situ* measurements.

Weiss et al. (1998) carried out a study on the shrinkage cracking of restrained concrete slabs. They reported that not only the shrinkage strain and, therefore, the resultant shrinkage stress were time-dependent, but also the concrete properties, such as compressive strength, elastic modulus, critical stress intensity factor ($K_{IC}$), and critical crack tip opening displacement ($CTOD_C$) also were time-dependent.

The failure criterion used in the Weiss et al. study is based on the concept of a fracture resistance curve (R-curve), as shown in Figure 2.15. The strain energy release rate, also called the crack driving energy curves (G-curves) are shown in Figure 2.15 by dashed lines. An initial flaw with the length of $a_0$ is assumed in the slab. With the increase of the stress (load), the G-curve moves from (b) to (c) as the crack propagates ($\Delta a$) in a stable manner until the G-curve reaches (d) at which $G=R$ and $\frac{\partial G}{\partial (\Delta a)} = \frac{\partial R}{\partial (\Delta a)}$. At state (d), the crack propagation is unstable and failure occurs.

![Figure 2.15: Fracture Failure Criteria Showing G-Curves (Dashed Lines) and R-Curve(Solid Line) (after Weiss et al. 1998)](image-url)
Considering the time dependency of the material properties and the effect of the material creep under shrinkage stress, this failure criterion that is based on fracture mechanics can be used to predict the age at which the slab will crack. Weiss et al. (1998) noted that the numerical process was time-consuming due to the time-dependent nature of the material properties. A favorable correlation was reported when comparing the model results with the experimental data.

Weiss and Shah (2002) extended the previous modeling procedure to incorporate a concrete ring shrinkage test by considering the shrinkage stress as not only time-dependent but also space-dependent (shrinkage stress gradient due to moisture loss). Figure 2.16 shows their simulation results, which clearly indicate the distribution of humidity, shrinkage strain, residual stress (shrinkage stress), and the resultant axial load in the ring specimen at two different ages of 14 and 90 days. Based on the time- and space-dependent shrinkage stress, the predicted age up to cracking is reported to have improved.

![Figure 2.16: Humidity, Shrinkage Strain, Shrinkage Stress, and Resultant Axial Load in Ring Specimen at Two Different Ages (after Weiss and Shah 2002)](image-url)
However, Weiss and Shah’s model (2002) did not consider the effect of a possible slow crack growth stage before failure. The previous models were extended further by Neithalath et al. (2005) to account for time-dependent crack growth based on the crack band concept proposed by Bazant and Oh (1983) and Hillerborg et al. (1976), which is illustrated by Figure 2.17.

![Figure 2.17: Schematic Illustration of Crack Band Model (after Neithalath et al. 2005)](image)

With regard to the crack band theory, Neithalath et al. (2005) explain that:

. . . . [T]he specimen could be considered to have two distinct regions: an undamaged or bulk region and the damaged or cracked region. The ultimate shrinkage strain in the bulk and cracked region is the same as the free shrinkage of the specimen. However the shrinkage is applied to the bulk section uniformly across the cross-section and the residual stress is computed while the shrinkage in the cracked section is divided into a series of elements along its depth and the stress in each of these layers is computed.
For each layer, the shrinkage strain is calculated from the pore RH within the layer. These stresses are modeled in both the cracked and bulk sections using a step-wise method. By applying force equilibrium and displacement compatibility and assuming that the stress distribution in the specimen is linearly proportional to the shrinkage strain, the stress at any layer in the cracked region can be solved. Neithalath et al. (2005) explain:

_As drying progresses, it can be assumed that the elements will start cracking as the stresses in a particular layer exceed the tensile strength of the material. The stress in the cracked layer can be thought to be redistributed equally among all the un-cracked layers._

At a certain age, the specimen will crack completely. Neithalath et al. (2005) also noted that the equal distribution of the stress in the cracked layer to all the uncracked layers is just an approximation and that further studies were needed to account for post-peak stress and stress redistribution. It should be noted also that the predicted time to complete failure was not validated by experiments in their study.

3. Other criteria

Other shrinkage cracking criteria have been suggested, such as the energy minimization principle proposed by Chen and Baker (2004). They introduced an analytical model to predict the minimum and maximum crack spacing in concrete pavements. In their analytical model, the pavement is characterized by a longitudinal beam, and all the damage within a certain distance is assumed to be localized into a cohesive crack. Therefore, a pavement could be represented by cohesive cracks and an elastic bar, as shown in Figure 2.18. The friction of the subgrade is represented by distributed springs. From the
equilibrium and compatibility of both the elastic bar and the cohesive cracks, the strain and stress in the elastic bar and the tensile stresses of the cohesive cracks can be calculated. The strain energy of the elastic bar, the energy stored in the distributed springs, and the energy stored in the cohesive cracks are then evaluated by varying the displacement of the two ends of the elastic bar. The energy profiles show minimal values that correspond to the length of the elastic bar (crack spacing).

![Elastic bar, Spring, Crack](image)

Figure 2.18: Modeling of Crack Spacing (after Chen and Baker 2004)

2.7 CONCLUSIONS OF LITERATURE REVIEW

Based on the literature review, the major conclusions are listed as below.

1. Cementitious stabilization can substantially improve the engineering properties of unbound materials, such as the strength, stiffness, durability, and stability, etc. But it has its own engineering problems. The primary distresses associated with the use of CSM in pavement are load induced fatigue cracking and shrinkage cracking.

2. Many factors have effect on the CSM properties, including host material, binder type, binder content, moisture content, curing condition, age, etc.
3. The shrinkage of CSM can be separated into four categories: autogeneous shrinkage, drying shrinkage, thermal shrinkage, and carbonation shrinkage. Autogenous shrinkage is relatively small. Temperature shrinkage can be conveniently superposed with drying shrinkage. Carbonation shrinkage is a long term reaction. Drying shrinkage dominates the shrinkage strain. It is directly related to moisture loss and caused by matric suction.

4. Moisture loss from any porous material depends mainly on the surface area, the lengths of the moisture migration pathways, and the drying environment. Non-linear moisture diffusion theory should be used to describe the drying process of CSM. Although there are many studies about the moisture diffusion in concrete, few study focus on CSM.

5. The shrinkage coefficient could be used to express the change in shrinkage strain as a function of moisture or relative humidity change. There are models developed for shrinkage coefficient for concrete. But for CSM, the model is not well established.

6. The shrinkage stress is caused by restraint of shrinkage strain. If the restraint condition is know, the shrinkage stress can be solved based on equilibrium and compatibility equations. However, during this procedure, the stiffness change of the material due to moisture loss and cementitious hydration should be considered.
7. Shrinkage of stabilized materials is coupled with relaxation. Relaxation of stabilized material reduces the shrinkage stresses and mitigates the occurrence of shrinkage cracking.

8. Different shrinkage cracking criteria are used in literatures, such as the criterion based on empirical, tensile strength, fracture mechanics, and energy minimization principle. The shrinkage crack prediction model are not sufficiently verified by tests for CSM;
CHAPTER 3: MATERIALS AND EXPERIMENTS

3.1 MATERIALS

Four types of materials were used in this study to study the moisture and cement hydration effects on the shrinkage and mechanical properties of soil and cement soil: silty clayey sand, clay, cement silty clayey sand, and cement clay. The silty clayey sand and clay used in this study were obtained from Pullman, Washington and Madison, Wisconsin, respectively. The primary material properties and the grain size distribution of the materials are shown in Table 3.1 and Figure 3.1, respectively. The cement contents were determined to be 5% for cement silty clayey sand and 12% for cement clay, following the method developed by the Portland Cement Association (PCA) (1992). Sieve analysis was performed to determine the distribution of the coarse particles, and the hydrometer analysis method was used to determine the distribution of the fine particles (less than 0.1 mm), following the procedure found in ASTM D422.

Table 3.1: Properties of Soils and Cement Soils Used in this Study

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>AASHTO Classification</th>
<th>Liquid Limit</th>
<th>Plastic Index</th>
<th>Optimum Moisture Content (%)</th>
<th>Maximum Dry Density (g/cm$^3$)</th>
<th>Specific Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty Clayey Sand</td>
<td>A-2-4</td>
<td>33</td>
<td>6</td>
<td>11.64</td>
<td>1.930</td>
<td>2.645</td>
</tr>
<tr>
<td>Clay</td>
<td>A-6</td>
<td>39</td>
<td>16</td>
<td>19.12</td>
<td>1.720</td>
<td>2.675</td>
</tr>
<tr>
<td>Cement Silty Clayey Sand</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>12.85</td>
<td>1.870</td>
<td>2.657</td>
</tr>
<tr>
<td>Cement Clay</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>17.98</td>
<td>1.650</td>
<td>2.764</td>
</tr>
</tbody>
</table>
3.2 EXPERIMENTS

Table 3.2 provides a summary of the experiments performed in this study. These experiments were designed to achieve two objectives: (1) to investigate the effects of drying and/or cement hydration on the moisture distribution, shrinkage potential, and the mechanical properties of soils and cement soils; and (2) to provide time-dependent material parameters for finite element (FE) modeling.

Table 3.2: Test Plan for Experimental Study

<table>
<thead>
<tr>
<th>Material</th>
<th>Experiment</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty Clayey Sand Clay</td>
<td>• Humidity Isotherm Test&lt;br&gt;• Diffusion Coefficient Test&lt;br&gt;• Moisture Gradient Verification Test</td>
<td>Drying process modeling</td>
</tr>
<tr>
<td></td>
<td>• Free Shrinkage Test</td>
<td>Shrinkage potential modeling</td>
</tr>
<tr>
<td>Cement Silty Clayey Sand</td>
<td>• Soil Water Characterization Curve Test&lt;br&gt;• Indirect Tensile Strength Test&lt;br&gt;• Unconfined Compressive Strength Test</td>
<td>Effective stress modeling&lt;br&gt;Tensile strength modeling</td>
</tr>
<tr>
<td>Cement Clay</td>
<td>• Tensile Stress-Strain Relationship Test&lt;br&gt;• Restrained Shrinkage Test</td>
<td>Shrinkage cracking modeling</td>
</tr>
</tbody>
</table>

Figure 3.2 presents the procedures for the shrinkage cracking modeling, the parameters used in each step, and the related laboratory experiments.
Figure 3.2: Procedure of Shrinkage Cracking Modeling and Parameters Used in Each Step
All the specimens used for the experiments listed in Table 3.2 were fabricated at the optimum moisture content (OMC) to the maximum dry density (MDD), as shown in Table 3.1, to obtain the same initial soil structure. This objective was achieved by compacting the soil of a specified weight at the OMC to reach the target volume.

### 3.2.1 Humidity Isotherm Test

The humidity isotherm provides the relationship between the equilibrium degree of saturation $S$ or relative moisture content $\theta$ (moisture content normalized to initial moisture content) and the pore RH of the specimen at a given temperature (Kodikara and Chakrabarti 2005). The water content at any location in a specimen is usually in equilibrium with the vapor pressure and is determined by the pore RH (Akita et al. 1997). For this study, glass desiccators that contain salt solutions placed at the bottom were used to maintain a constant ambient RH. The concentration and chemical composition of the salt solution determined the ambient RH within the glass desiccators (Akita et al. 1997). Table 3.3 shows the sulfuric acid concentrations used in this study and the ambient RH measured in the sealed desiccators.

<table>
<thead>
<tr>
<th>Sulfuric Acid Concentration</th>
<th>Measured Ambient Relative Humidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.90%</td>
<td>0.10%</td>
</tr>
<tr>
<td>57.90%</td>
<td>30.30%</td>
</tr>
<tr>
<td>47.80%</td>
<td>49.20%</td>
</tr>
<tr>
<td>38.40%</td>
<td>65.30%</td>
</tr>
<tr>
<td>26.10%</td>
<td>83.60%</td>
</tr>
<tr>
<td>0.00%</td>
<td>97.50%</td>
</tr>
</tbody>
</table>
Very small specimens, each with a diameter of around 35 mm and thickness of around 5 mm, were used to accelerate the equilibrium process for the humidity isotherm tests, but the specimens were too small to control the compaction quality. Therefore, they were carefully cut from larger specimens, which are easier to compact, to ensure that their material structure and quality were the same as those of the other specimens. This cutting procedure is described in the literature for undisturbed specimens to study the suction measurements of undisturbed and disturbed soil specimens (Petry and Jiang 2003).

Figure 3.3 (a) shows the humidity isotherm test specimens used in this study. The prepared humidity isotherm specimens were subjected to drying in the glass desiccators at various ambient RH values, as shown in Figure 3.3 (b), and were weighed periodically until their weights became practically constant. At the equilibrium state, the pore RH was assumed to be the same as the ambient RH. Next, the water content of the specimens at equilibrium was measured, and the degree of saturation of the specimens was calculated using Equation (3.1). The humidity isotherm can be established by plotting the degree of saturation against the pore RH.

\[
S = \frac{\omega}{1/\rho_d-1/G_s} \times 100\% \tag{3.1}
\]

where S is the degree of saturation (%); \(\omega\) is the moisture content; \(\rho_d\) is the dry density (g/cm\(^3\)); and \(G_s\) is the specific gravity.
3.2.2 Diffusion Coefficient Test

Extensive studies of diffusion coefficient measurements have been conducted for concrete (Bazant and Najjar 1972a, Kodikara and Chakrabarti 2005, Sakata 1983, Asad et al. 1997). However, soil and cementitiously-stabilized soil are different from concrete because they are not dried under saturated conditions. Kodikara and Chakrabarti (2005) modified the method used for concrete to make it applicable to soils.

This study follows the Kodikara and Chakrabarti procedure. Three types of short specimens with lengths of 40 mm, 80 mm, and 120 mm, each with a 100 mm by 100 mm cross-section, were fabricated for each type of material used in this study. Subsequently, four sides, except the ends (measuring 100 mm by 100 mm), of the specimens were covered with wax and plastic film. The specimens were then kept at 35% ambient RH and
20°C as the environmental condition. Figure 3.4 shows the shapes and dimensions of the short specimens. Under the drying condition, moisture evaporated from the two ends only. Therefore, the moisture diffusion can be considered as a one-dimensional phenomenon. The moisture loss of the three specimens was measured by weighing the specimens regularly during the drying process.

![Diagram of diffusion coefficient test specimens and moisture simulation schematic.](attachment:diagram.png)

**Figure 3.4**: (a) Dimensions and (b) Shapes of Diffusion Coefficient Test Specimens, and (c) Schematic of Cross-section for Moisture Simulation
Considering a longer specimen under the same drying conditions as used for the shorter three specimens, the relative moisture content $\theta$ at the cross-sections with distances of 10 mm, 30 mm, and 50 mm from the exposed ends could be calculated based on the total moisture loss of the three shorter specimens at time $t$, as shown by Figure 3.4 (c), using Equation (3.2) (Sakata 1983).

$$\theta_i(x_i, t) = \left(1 - \frac{w_i(t) - w_{i-1}(t)}{w_0(L_i - L_{i-1})A}\right) \times 100\%$$  \hspace{1cm} (3.2)

where $\theta_i(x_i, t)$ is the relative moisture content (the moisture content normalized to the initial moisture content) of the long specimen at the surface with a distance of $x_i$ from the end surface at time $t$; $x_i$ is 10 mm, 30 mm, or 50 mm when $i$ is 1, 2, or 3, respectively; $w_i(t)$ is the absolute moisture loss (from the initial moisture) of the short specimens with lengths of 40 mm, 80 mm, or 120 mm when $i$ is 1, 2, or 3, respectively, at time $t$; and $i = 1, 2, \text{ or } 3$; $w_i(t) = 0$ when $i = 0$; $w_0$ is the diffusible moisture in a unit volume of the specimen, which is the difference between the unit weight of the specimen at the start of the drying and its unit weight for oven drying at 105°C after the experiment; $L_i$ is the length of the short specimens, $L_0 = 0$ mm, $L_1 = 40$ mm, $L_2 = 80$ mm and $L_3 = 120$ mm; and $A$ is the area of the drying surface.

Moisture loss from a porous material depends mainly on the surface area, the lengths of the moisture migration pathways, and the drying environment (Chakrabarti and Kodikara 2006). Nonlinear moisture diffusion theory commonly is used to describe the drying process of porous materials. Moisture flux, $J$, can be expressed two different ways. It can be defined in terms of the pore RH gradient or in terms of the free water content gradient, as shown by Equation (3.3) (Abbasnia et al. 2009).
\[ J = -D \text{grad} (\text{RH}) \quad \text{or} \quad J = -D \text{grad} (w_e) \]  

(3.3)

where \( J \) is the moisture flux in cm/hour, which is defined as the mass of water that passes through a unit area perpendicular to \( J \) per unit time; \( D \) is the diffusion coefficient in cm\(^2\)/hour, corresponding to either pore RH or evaporable water content; \text{grad} is the gradient function; \( \text{RH} \) is the pore relative humidity, which is the ratio of the vapor pressure to the vapor pressure at saturation; and \( w_e \) is the evaporable water content. The pore RH is difficult to measure directly and typically is determined based on its relationship with the degree of saturation determined from the humidity isotherm test.

According to Bazant and Najjar (1972a), the rate of diffusion of water can be characterized by the moisture flux, \( J \). If the temperature is assumed to be constant and the rate of hydration is negligible (e.g., as is the case in the late stage of cementitiously-stabilized soil), Equation (3.3) can be expressed as Equation (3.4).

\[ \frac{\partial \rho_{RH}}{\partial t} = \text{div}(D \text{grad} \rho_{RH}) \]  

(3.4)

where \( \rho_{RH} \) is the pore relative humidity; \( t \) is the drying time in hour; \text{div} is the derivative function; and \( D \) is the diffusion coefficient that is the function of RH.

From Equations (3.1) and (3.2), and the definition of relative moisture content \( \theta \), the degree of saturation at cross-sections 10 mm, 30 mm, and 50 mm away from the exposed ends of the specimen at any time \( t \) can be obtained. Once the humidity isotherm relationship is established, the degree of saturation can be converted to the corresponding pore RH. The differential diffusion equation, as expressed by Equation (3.4), is a function of time \( t \) and spatial position \((x, y, z)\). In one-dimensional moisture diffusion, Equation (3.4) can be written in terms of pore RH as Equation (3.5).
\[
\frac{\partial \text{RH}}{\partial t} = \frac{\partial}{\partial x} \left( D(\text{RH}) \frac{\partial \text{RH}}{\partial x} \right) 
\]

where RH is the pore relative humidity; D(RH) is the diffusion coefficient with respect to the pore RH; t is drying time; and x is the distance from the drying surface.

Once the parameter D(RH) is known, Equation (3.5) can be used to solve the pore RH at any time t and position x. However, because the pore RH values at time t and at 10 mm, 30 mm, and 50 mm are obtained from the measured weight and humidity isotherm, the parameter D(RH) can be backcalculated. If \( \lambda \) is defined as a function of the distance from the drying surface x and drying time t, \( \lambda = x/\sqrt{t} \), Equation (3.5), becomes Equation (3.6) based on the Boltzmann transformation (Asad et al. 1997, Crank 1995).

\[
D(\text{RH}_e) = \frac{1}{2} \frac{1}{d\lambda} \int_{\text{RH}_e}^{1} \lambda d\text{RH}_e 
\]

where \( \text{RH}_e \) is the effective pore RH, which is defined by Equation (3.7), at the initial state; \( \text{RH}_e = 1; \lambda = \infty \); and \( d\text{RH}_e/d\lambda = 0 \).

\[
\text{RH}_e = \frac{\text{RH} - \text{RH}_{\text{envi}}}{\text{RH}_{\text{init}} - \text{RH}_{\text{envi}}} 
\]

where \( \text{RH}_{\text{envi}} \) is the ambient RH, and \( \text{RH}_{\text{init}} \) is the initial pore RH.

The use of \( \text{RH}_e \) is for the convenience of the integration of Equation (3.6). After \( D(\text{RH}_e) \) is obtained, it can easily be converted to \( D(\text{RH}) \). The RH versus \( \lambda \) relationship can be established from the experimental results. The \( D(\text{RH}) \) at any pore RH can be calculated based on Equation (3.6). The resultant \( D(\text{RH}) \) can be plotted versus RH to obtain their relationship.
3.2.3 Moisture Gradient Verification Test

To verify the accuracy of the moisture distribution models, cuboid specimens, each with a length of 285 mm and cross-section of 100 mm by 100 mm, were sealed with wax, except at one end, to simulate the one-dimensional drying condition. The specimens were kept in a controlled or natural room at an ambient RH and at 20°C for several days, and then were slice-cut, as shown in Figure 3.5. The moisture content of each slice was measured and converted into the degree of saturation using Equation (3.1), and then to corresponding pore RH values using the humidity isotherm. The resultant pore RH gradients were compared to the modeled results.

![Slice-cuts of Specimen for Model Verification](image)

Figure 3.5: Slice-cuts of Specimen for Model Verification

3.2.4 Free Shrinkage Test

Free shrinkage tests were conducted to measure the shrinkage potential of different types of materials with respect to the pore RH. The shrinkage potential is defined in this study as the maximum possible drying shrinkage of a material at a given pore RH when there is no restraint. The specimens used for free shrinkage testing had the dimensions of 285 mm length, 100 mm width, and 25.4 mm thickness. The 25.4 mm thickness was
chosen to accelerate the equilibrium of the specimen pore RH with the ambient RH and reduce both external and internal restraints. The specimens were subjected to drying all surfaces, as shown in Figure 3.6. After equilibrium, there is no moisture gradient and therefore no internal restraint within the specimen. To reduce the bottom friction effect, a plastic sheet on top of a thin layer of small glass beads was placed on the smooth aluminum alloy plate before seating the specimen. In free shrinkage tests, the bulk displacement of the specimens can easily be measured from the two ends, as shown in Figure 3.6. Dial gauges with resolution of 0.0025 mm were used to measure the shrinkage displacement. After the shrinkage potential measurements were taken, the specimens were measured for moisture content and suction.

Figure 3.6: Setup for Shrinkage Potential Test

3.2.5 Soil Water Characteristic Curve (SWCC) Test

A dewpoint potentiometer (WP4) was used to measure the suction in the soil samples. The dewpoint potentiometer, also known as a chilled-mirror hygrometer, measures the water potential by equilibrating the liquid phase water of the specimen with the vapor phase water in a closed space above the specimen. Compared to the pressure
plate method and filter paper method, the dewpoint potentiometer is simple to use and fast. The suction measurements can be completed within five to fifteen minutes (Nam et al. 2009, Vikas et al. 2006). The relationship between the suction of the sample and the vapor pressure of the air in the sealed block chamber can be expressed by the Kelvin-Laplace Equation (3.8) (Decagon Devices, Inc. 2000):

$$
\psi = -\frac{RT}{w_v \left(1/\rho_w\right)} \ln \frac{p}{p_0}
$$  \hspace{1cm} (3.8)

where $\psi$ is the matric suction (MPa); $R$ is the universal gas constant (8.31 MPa·cm$^3$/(mol·K) or J/(mol·K)); $T$ is the temperature of the sample in Kelvin (K); $w_v$ is the molecular mass of water (18 g/mol); $\rho_w$ is the unit weight of the water (1 g/cm$^3$); $p$ is the vapor pressure of the air (kPa); $p_0$ is the saturation vapor pressure at the sample temperature (kPa); and $\frac{p}{p_0}$ is the relative humidity.

Similar to the specimens that were used for the humidity isotherm tests, the small specimens that fit within the measurement plastic cup were too small to control the compaction quality. Therefore, they were carefully cut from larger specimens, which are easier to compact, to ensure that their material structure and quality were the same as those of the other specimens.

Each specimen was put in a plastic cup for 24 hours to attain moisture equilibrium within the specimen. The open cup containing the sample was placed into the WP4 equipment to measure the suction value, as shown in Figure 3.7. After the suction measurement was taken, the mass of the specimen with the cup was measured. The specimen was left to air-dry using a fan to accelerate the drying process. When the mass loss of the specimen reached 0.1 g to 0.2 g, the specimen was sealed in the cup for 24
hours to achieve moisture equilibrium within the specimen for the next suction measurement. This procedure was repeated until the suction measurement exceeded the suction measurement capacity of the WP4 device, which is 80 MPa. After the drying process was completed, each specimen was put into an oven to determine the dry mass of the soil solid and the moisture content at which the suction values were measured. The degree of saturation is given by Equation (3.1). The measured suction can be correlated with the degree of saturation to form the SWCC.

![Figure 3.7: Suction Test for Soil Water Characterization Curve](image)

3.2.6 Indirect Tensile (IDT) Test

The IDT test is commonly used to measure the tensile properties of geomaterials. The geometry of the specimens used for IDT testing in this study is 101.6 mm (4 in.) in diameter and 25.4 mm (1 in.) in thickness. The load was applied across the diameter of each specimen using two loading strips that were 19 mm (0.75 in.) wide, as shown in Figure 3.8. A load guide device (LGD) with four columns was used as the loading apparatus. All tests were performed in an IDT test mode using a GCTS servo-hydraulic
closed-loop testing machine. The tests were carried out with a constant loading rate of 838.5 N/minute (15 lb/min).

A closed-form elastic solution for an IDT test specimen shows that the stress state at the center point of the IDT specimen can be expressed by Equation (3.9) (Hondros 1959, Wen 2001), based on the assumption of plane stress. The IDT strength equals the $\sigma_x$ under the peak load.

\[
\sigma_x = -\frac{2P}{\pi wt} \quad (3.9.a)
\]

\[
\sigma_y = -3.1\sigma_x = \frac{6.2P}{\pi wt} \quad (3.9.b)
\]

\[
\sigma_z = 0 \quad (3.9.c)
\]
where $\sigma_x$ is the tensile stress at the center of the specimen across the loading diameter; $P$ is the applied load on the specimen; $w$ is the diameter of the specimen; $t$ is the thickness of the specimen; $\sigma_y$ and $\sigma_z$ are the principal stresses in the $y$ and $z$ directions, respectively.

A 50-mm gauge length was used to mount the linear voltage differential transducers (LVDTs) that were used to measure specimen deformation, as shown in Figure 3.8 (left). This gauge length was selected as an optimal length that minimizes the effect of large aggregate particles located in between the two gauge points and the stress concentration in the vicinity of the loading strips (Wen and Kim 2002). Two horizontal and two vertical LVDTs were used for each specimen (one horizontal and one vertical LVDT on each side).

The displacements measured by the LVDTs can be used to calculate the tensile strain at the center of the specimen along the loading diameter, as shown by Equations (3.10) and (3.11) (Wen and Kim 2002):

$$
\nu = -\frac{a_1 U(t) + V(t)}{a_2 U(t) + a_3 V(t)} \tag{3.10}
$$

$$
\varepsilon_{x=0} = U(t) \frac{\gamma_1 + \gamma_2 \nu}{\gamma_3 + \gamma_4 \nu} \tag{3.11}
$$

where $\nu$ is Poisson’s ratio; $\varepsilon_{x=0}$ is the tensile strain at the center of the specimen along the loading diameter; $U(t)$ is the horizontal displacement in m; $V(t)$ is the vertical displacement in m; parameters $a_1$, $a_2$, and $a_3$ depend on the specimen geometry and gauge length and are 3.673, 1.154, and 3.192, respectively, in this study; and parameters $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$ depend on the specimen geometry and gauge length and are 8.48, 25.6, 0.373, and 1.18, respectively, in this study (Wen and Kim 2002).
The IDT test specimens were fabricated at the OMC to the maximum dry density (MDD), as shown in Table 3.1. After fabrication, the specimens, as shown in Figure 3.9, underwent different procedures prior to testing based on different objectives, as listed below:

1. **Analysis of pore RH effect on the tensile strength and stress-strain relationship (referred to as a pure soil specimen in this study):** IDT specimens were fabricated using pure silty clayey sand or pure clay without cement. The specimens were air-dried to different target moisture contents and sealed with a plastic sheet for three days to obtain moisture equilibrium within the specimens prior to IDT testing. After the IDT tests, the broken specimens were used to measure the actual moisture contents, which could be converted to pore RH using the humidity isotherm.

2. **Analysis of the hydration effect on the tensile strength and stress-strain relationship (referred to as a cured specimen or control specimen in this study):** IDT specimens were fabricated using cement silty clayey sand or cement clay. The specimens were cured at 100% ambient RH and at 20°C for 3, 7, 14, 28, 56, and 108 days prior to IDT testing. After the IDT tests, the broken specimens were used to measure the actual moisture contents, which could be converted to pore RH using the humidity isotherm.

3. **Analysis of the combined effect of pore RH and hydration on the tensile strength and stress-strain relationship (referred to as an overlapped drying specimen or top, middle, or bottom specimen in this study):** IDT specimens were fabricated using cement silty clayey sand or cement clay. Then, three IDT specimens were overlapped and sealed with wax, except on the top surface, and were referred to respectively as the top,
middle, and bottom specimens, as shown in Figure 3.9 (c). The overlapped IDT specimens were kept at 35% ambient RH and at 20°C for 3, 7, 14, 28, 56, and 108 days prior to IDT testing. Because moisture can escape only from the top surface, the three overlapped IDT specimens would have different moisture contents, even at the same age. The mechanical properties of the IDT specimens reflect the combined effects of moisture content and cement hydration. After the IDT tests, the broken specimens were used to measure the actual moisture contents, which could be converted to pore RH using the humidity isotherm.

Figure 3.9: IDT Specimens for (a) Soil Undergoing Drying, (b) Cured Cement Soil without Drying, and (c) Three Overlapped Cement Soil Samples Undergoing Drying
3.2.7 Unconfined Compressive Strength (UCS) Test

As shown in Figure 3.10, the geometry of the specimens used for UCS testing in this study is 101.6 mm (4 in.) in diameter and 116.8 mm (4.6 in.) in height, following ASTM D1633 Method A. However, the loading rate was different from the standard. To be consistent with the IDT testing in this study, a loading rate of 838.5 N/minute (15 lb/min) was used.

Figure 3.10: UCS Specimen and Setup of UCS Test

The stress state can be calculated using Equation (3.12), and the UCS is the $\sigma_y$ stress that corresponds to the peak load.

$$\sigma_y = \frac{P}{A}$$  \hspace{1cm} (3.12.a)

$$\sigma_x = \sigma_z = 0$$  \hspace{1cm} (3.12.b)

where $A$ is the area of the specimen cross-section; $P$ is the applied load on the specimen; $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the principal stresses in the x, y, and z directions, respectively.
The UCS specimens were fabricated using silty clayey sand or clay. The specimens were air-dried to different target moisture contents and sealed with a plastic sheet for three days to obtain moisture equilibrium within the specimens prior to UCS testing. After the UCS tests, the broken specimens were used to measure the actual moisture contents.

3.2.8 Tensile Stress-Strain Relationship Test

The IDT test also was used to obtain the tensile stress-strain relationship (or elastic modulus in tension) for the shrinkage stress modeling. The tensile stress-strain relationship was used to calculate the shrinkage stress that is due to the restraint of the shrinkage potential for each time interval in the shrinkage stress modeling.

The tensile stress-strain relationship is moisture- and hydration- (time-) dependent. The tensile stress-strain relationships of all the specimens used for the IDT strength tests were recorded to study the effects of moisture and hydration on the tensile stress-strain relationship.

3.2.9 Restrained Shrinkage Test

A fully restrained shrinkage test was designed to measure the shrinkage load and to observe the shrinkage cracking in order to verify the shrinkage cracking prediction models and procedure. In this test, specimens with the dimensions of 285 mm in length, 100 mm in width, and 25.4 mm in thickness, which are the same dimensions as those used for the free shrinkage tests, were wax-sealed on all sides, except the two largest surfaces, for moisture evaporation, as shown in Figure 3.11. The specimens were then glued to the top and
bottom plates with epoxy. The bottom plate was fixed. The top plate was connected to the loading piston, which was connected to a load cell. After the drying process was initiated, the loading piston was fixed, and therefore, both the top and bottom ends of the specimen were fixed. The load cell could measure the development of the tensile load due to shrinkage. This scenario simulates a fully restrained condition in the vertical direction. During the test, the room RH was recorded as the environmental input for FE modeling.

Figure 3.11: Restrained Shrinkage Specimen and Setup of Restrained Shrinkage Test
CHAPTER 4: RESULTS ANALYSIS AND SHRINKAGE CRACK MODELING

This chapter contains five sections. Section 4.1 describes the methods and models used to predict and simulate the drying processes of the soil and cement soil. The moisture content and pore RH distribution during drying were predicted based on relevant fundamental material properties, such as the diffusion coefficient, humidity isotherm, and SWCC, etc. Section 4.2 discusses the relationship between the drying shrinkage potential and the pore RH; that is, a model to predict the drying shrinkage potential is developed. Section 4.3 discusses the models developed to predict the change in the tensile and compressive strength due to pore RH changes based on unsaturated soil mechanics. The accuracy of these models has been experimentally verified. The moisture and hydration effects on the tensile strength therefore could be separated. Section 4.4 proposes models for predicting the evolution of the tensile stress-strain relationship during drying and/or hydration; these models are critical to predicting the shrinkage stress if a specific restraint is applied to the shrinkage potential. After the aforementioned four steps, Section 4.5 describes case studies to verify the entire procedure, starting from the drying process to the shrinkage cracking, using the FE method. Figure 4.1 presents a flow chart of this modeling procedure. For the case studies, the constantly changing moisture distribution, drying shrinkage potential, tensile strength, and tensile stress-strain relationship of the clay and cement clay that underwent drying and/or hydration processes were simulated and predicted using the procedures and models developed in this study. The FE modeling results have been verified by laboratory experiments.
4.1 MODELING OF DRYING PROCESS

Drying shrinkage is the major part of shrinkage for soil and cement soil (Little 1992, Chakrabarti et al. 2002). This study focuses only on drying shrinkage. Because drying (moisture loss) is the major cause of shrinkage, the accurate characterization of the moisture gradient during drying is of utmost importance and is indispensable to the study of the shrinkage, shrinkage stress, and shrinkage cracking that may be incurred.

4.1.1 Humidity Isotherm Test Results

As discussed in Section 3.2.1, the humidity isotherm provides the relationship between the equilibrium degree of saturation S and the pore RH of the specimen at a given temperature. The equilibrium of the water content of small specimens with a stable ambient air RH in sealed glass desiccators took several months to achieve. After
equilibrium was reached, the moisture contents of the specimens were measured and converted to the degree of saturation using Equation (3.1) (introduced in Section 3.2.1, also shown below).

\[ S = \frac{\omega}{1/\rho_d - 1/G_s} \times 100\% \]  

(3.1)

where \( S \) is the degree of saturation (%); \( \omega \) is the moisture content; \( \rho_d \) is the dry density (g/cm\(^3\)); and \( G_s \) is the specific gravity.

Figure 4.2 shows the humidity isotherm test results for the four materials used in this study. Equation (4.1) is proposed in this study to describe the humidity isotherms.

\[ \text{RH} = 1 - \exp(a S^b) \]  

(4.1)

where \( \text{RH} \) is the pore relative humidity; \( S \) is the degree of saturation; and \( a \) and \( b \) are the regression parameters.

Figure 4.2 indicates that for the same ambient RH, the clay could maintain a higher degree of saturation than the silty clayey sand. Adding cement to the silty clayey sand changed the humidity isotherm slightly. The effect of adding cement on the humidity isotherm of the clay was less obvious. Table 4.1 presents the regression parameters of Equation (4.1) for the four materials used in this study.

Table 4.1: Values of Parameters a and b for the Four Types of Materials

<table>
<thead>
<tr>
<th>Type of Material</th>
<th>Silty Clayey Sand</th>
<th>Cement Silty Clayey Sand</th>
<th>Clay</th>
<th>Cement Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-8.5137</td>
<td>-8.0521</td>
<td>-5.5568</td>
<td>-5.2529</td>
</tr>
<tr>
<td>b</td>
<td>1.1041</td>
<td>1.1686</td>
<td>1.2505</td>
<td>1.1474</td>
</tr>
</tbody>
</table>
4.1.2 Diffusion Coefficient Test Results

As introduced in Section 3.2.2, the weights of the short specimens measured at different drying times were processed to obtain the diffusion coefficients. When solving Equation (3.6) (introduced in Section 3.2.2, also shown below), the relationships between RHe and λ are obtained first, which is expressed by Equation (4.2) and plotted in Figure 4.3. The regression parameters, p, are 0.9877, 0.5767, 0.7696, and 0.5654 for silty clayey sand, cement silty clayey sand, clay, and cement clay, respectively.

\[ D(RH_e) = \frac{1}{2} \frac{1}{dRH_e \, d\lambda} \int_{RH_e}^{1} \lambda dRH_e \]

(3.6)

where \( RH_e \) is the effective pore RH, which is defined by Equation (3.7), at the initial state; \( RH_e = 1, \lambda = \infty \); and \( dRH_e/d\lambda = 0 \).

\[ RH_e(\lambda) = 1 - \frac{p^2}{p^2 + \lambda^2} \]

(4.2)
where RH is the pore relative humidity, $\lambda = x/\sqrt{t}$, with x as the distance from the drying surface and t as the drying time; and p is the regression parameter.

Figure 4.3: Relationship between RH and $\lambda$ for: (a) Silty Clayey Sand, (b) Cement Silty Clayey Sand, (c) Clay, and (d) Cement Clay
By solving Equation (3.6), the diffusion coefficient in terms of pore RH is expressed by Equation (4.3) and is strongly nonlinear with respect to the pore RH, as shown in Figure 4.4. The regression parameters in Equation (4.3) for the four materials used in this study are provided in Table 4.2.

\[
D(RH) = D_0 + c \left( \frac{RH}{1-RH} \right)^d
\]  

(4.3)

where \(D(RH)\) is the diffusion coefficient; and \(D_0\), \(c\), and \(d\) are the regression parameters.

![Figure 4.4: Diffusion Coefficients with Respect to Pore Relative Humidity](image)

Table 4.2: Regression Parameters in Diffusion Coefficient Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Silty Clayey Sand</th>
<th>Clay</th>
<th>Cement Silty Clayey Sand</th>
<th>Cement Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_0)</td>
<td>0.856453</td>
<td>0.519988</td>
<td>0.291935</td>
<td>0.280666</td>
</tr>
<tr>
<td>(c)</td>
<td>0.435122</td>
<td>0.264182</td>
<td>0.148318</td>
<td>0.142591</td>
</tr>
<tr>
<td>(d)</td>
<td>1.026836</td>
<td>1.026836</td>
<td>1.026837</td>
<td>1.026838</td>
</tr>
</tbody>
</table>
Figure 4.4 also shows that silty clayey sand has a larger diffusion coefficient than clay, probably due to the difference in their pore structure distributions. Adding cement can reduce the diffusion coefficient for both silty clayey sand and clay. This phenomenon could be due to the change in pore structure during hydration. It is reported that during hydration, the volume of the compound produced by the reaction between the binder material and the water is smaller than the original total volume, but larger than the original solid volume (Tazawa et al. 1995, Neville 2012). This phenomenon could result in the reduction of the total pore volume and may reduce the diffusion coefficient, thereby affecting the moisture diffusion property. However, the reduction of the diffusion coefficient that is due to the addition of cement is less significant for clay than for silty clayey sand. After adding cement, the diffusion coefficient curves of cement silty clayey sand and cement clay almost merge. Further study is needed to explain if this result is just a coincidence.

4.1.3 Finite Element Modeling of Drying Process

An FE program, ADINA, was used for moisture diffusion modeling in this study. The environmental flux boundary condition that defines the moisture evaporation (moisture loss) from the material surface is determined by the environmental flux condition at the specimen surface, as shown by Equation (4.4) (Chakrabarti and Kodikara 2006, Kodikara and Chakrabarti 2005), which is analogous to the heat flux.

\[ E_s = -D(RH) \left( \frac{\partial w}{\partial n} \right)_s = f(RH_s - RH_{en}) \]  

(4.4)
where $E_s$ is the evaporation from the material surface; $D(RH)$ is the diffusion coefficient; $w$ is the moisture content; $n$ is the unit vector normal to the drying surface; $f$ is a surface factor that accounts for wind velocity, surface temperature, and surface roughness; and $RH_{\text{a}}$ and $RH_s$ are the ambient RH and the pore RH at the material surface, respectively.

The specimens used for the verification tests were simulated in ADINA to solve the constitutive differential equation of moisture diffusion (introduced in Section 3.2.2 as Equations (3.4) and (3.5) and also provided below) to predict the pore RH at any distance from the drying surface at any drying time under given ambient RH conditions.

\[
\frac{\partial RH}{\partial t} = \text{div}(D \text{ grad}RH) \quad (3.4)
\]

where $RH$ is the pore relative humidity; $t$ is the drying time in hours; div is the derivative function; and $D$ is the diffusion coefficient that is the function of $RH$.

\[
\frac{\partial RH}{\partial t} = \frac{\partial}{\partial x} \left( D(RH) \frac{\partial RH}{\partial x} \right) \quad (3.5)
\]

where $RH$ is the pore relative humidity; $D(RH)$ is the diffusion coefficient with respect to the pore RH; $t$ is the drying time; and $x$ is the distance from the drying surface.

Figure 4.5 shows an example of the ADINA simulation results for silty clayey sand after 480 hours of drying.
4.1.4 Moisture Gradient Verification Test Results

Figures 4.6 through 4.9 show the comparisons of the pore RH predicted by the FE method and the pore RH measured by the verification tests at different drying times for silty clayey sand, cement silty clayey sand, clay, and cement clay, respectively. The FE method used in this study shows reasonable accuracy in predicting the pore RH under one-dimensional drying conditions for both soil and cement soil. It should be noted that, for the silty clayey sand and cement silty clayey sand, ambient RH values of 60% and 20%, respectively, were controlled. For clay and cement clay, the various room ambient RH values were recorded every day and simulated in the FE model incrementally. The FE method is especially convenient for predicting moisture loss under variable ambient RH conditions, which are common in engineering practice.
Figure 4.6: Comparison of the FE Modeling Results and the Experimental Results for Silty Clayey Sand (Ambient Relative Humidity =60% at 20°C)

Figure 4.7: Comparison of the FE Modeling Results and the Experimental Results for Cement-Silty Clayey Sand (Ambient Relative Humidity =20% at 20°C)
Figure 4.8: Comparison of the FE Modeling Results and the Experimental Results for Clay (Ambient Relative Humidity = Variable at 20°C)

Figure 4.9: Comparison of the FE Modeling Results and the Experimental Results for Cement-Clay (Ambient Relative Humidity = Variable at 20°C)
4.2 MODELING OF DRYING SHRINKAGE POTENTIAL

The objective of this section is to bridge the knowledge gap between soil pore RH and soil shrinkage. During drying and/or hydration, the distribution of the pore RH can be used to obtain the shrinkage potential distribution. The drying shrinkage can be considered as the result of matric suction within the pore structure (Chakrabarti and Kodikara 2006). The matric suction can be related to the soil pore RH by the Kelvin-Laplace Equation (4.5).

\[ \psi = -\frac{RT}{w_v \left( \frac{1}{\rho_w} \right)} \ln \text{RH} \]  (4.5)

where \( \psi \) is the matric suction (MPa); \( R \) is the universal gas constant (8.31 MPa·cm\(^3\)/mol·K) or J/(mol·K)); \( T \) is the temperature of the sample in Kelvin (K); \( w_v \) is the molecular mass of water (18 g/mol); \( \rho_w \) is the unit weight of water (1 g/cm\(^3\)); and RH is the pore relative humidity.

After the pore RH profiles are obtained, they can be related to matric suction using the Kelvin-Laplace equation, Equation (4.5). Assuming the material exhibits isotropic stress generation and nonlinear elastic behavior during drying, the shrinkage stress increment \( \Delta \sigma_{sh} \) could be related to the shrinkage strain increment \( \Delta \varepsilon_{sh} \) by using Equation (4.6) (Chakrabarti and Kodikara 2006).

\[ \Delta \varepsilon_{sh} = \left( \frac{1-2\nu}{E} \right) \Delta \sigma_{sh} + \frac{\Delta \psi}{H} \]  (4.6)

where \( \Delta \varepsilon_{sh} \) is the strain increment; \( E \) is the tangent elastic modulus, \( \nu \) is Poisson’s ratio; \( \Delta \sigma_{sh} \) is the stress increment; \( \Delta \psi \) is the increment in moisture suction; and \( H \) is the suction modulus (a tangent modulus with respect to suction).

For the free shrinkage condition, the shrinkage stress should be zero (no restraint). The total shrinkage strain can be expressed as Equation (4.7).
\[
\Delta \varepsilon_{sh} = \frac{\Delta \psi}{H} \quad (4.7)
\]

Combining Equations (4.5) and (4.7), the free shrinkage can be expressed as Equation (4.8).
\[
\Delta \varepsilon_{sh} = \frac{\Delta \left( -\frac{RT \ln RH}{W_v} \right)}{H} \quad (4.8)
\]

This model relates the shrinkage strain to the pore RH using the suction modulus H. The suction modulus H can be obtained by correlating the measured free shrinkage with the pore RH.

### 4.2.1 Free Shrinkage Test Results

As discussed in Section 3.2.4, in order to measure the shrinkage potential at different pore RH values, free shrinkage tests were conducted at several ambient air RH levels until the equilibrium of the pore RH and the ambient air RH was reached. After taking shrinkage potential measurements, the specimens were measured for their moisture content and suction. For the four materials used in this study, Table 4.3 lists the measured shrinkage potential found from the free shrinkage tests at different pore RH levels (which were equilibrated with the ambient RH levels). Table 4.3 indicates that at the same pore RH, clay and cement clay have a much larger shrinkage potential than silty clayey sand and cement silty clayey sand. This finding could be due to the differences in the soil particles and pore structures that control the suction and shrinkage modulus (i.e., reflect the resistance of the deformation). Table 4.3 also indicates that at the same pore RH, the cement soil has significantly less shrinkage potential than the corresponding soil (i.e., without cement stabilization). This result could be because the cement hydration changed
the pore structure and increased the stiffness to withstand the deformation. However, Chakrabarti and Kodikara (2003) reported that for crushed rock stabilized by blended cement, the shrinkage decreased with an increase in binder content up to 4%, and then appeared to increase again.

Table 4.3: Free Shrinkage Test Results

<table>
<thead>
<tr>
<th>Silty Clayey Sand</th>
<th>Cement Silty Clayey Sand</th>
<th>Clay</th>
<th>Cement Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore RH</td>
<td>Shrinkage Potential</td>
<td>Pore RH</td>
<td>Shrinkage Potential</td>
</tr>
<tr>
<td></td>
<td>(Micro Strain)</td>
<td></td>
<td>(Micro Strain)</td>
</tr>
<tr>
<td>37.62%</td>
<td>2709</td>
<td>39.60%</td>
<td>1576</td>
</tr>
<tr>
<td>61.78%</td>
<td>1921</td>
<td>61.78%</td>
<td>1220</td>
</tr>
<tr>
<td>86.25%</td>
<td>1526</td>
<td>86.25%</td>
<td>563</td>
</tr>
<tr>
<td>99.00%</td>
<td>0</td>
<td>99.00%</td>
<td>0</td>
</tr>
</tbody>
</table>

4.2.2 Modeling of Drying Shrinkage Potential

Equation (4.9) is proposed in this study to express suction $\Psi$ as a function of shrinking potential $\varepsilon_{sh}$:

$$\Psi = f(\varepsilon_{sh}) = e^{\left(\frac{\varepsilon_{sh}}{m}\right)^n} - 1$$  \hspace{1cm} (4.9)

where $\Psi$ is the matric suction in MPa; $\varepsilon_{sh}$ is the shrinkage potential; and $m$ and $n$ are the regression parameters.

From Equations (4.8) and (4.9), the suction modulus can be expressed as Equation (4.10).

$$H = \frac{\Delta \Psi}{\Delta \varepsilon_{sh}} = \frac{n \left(\frac{\varepsilon_{sh}}{m}\right)^n e^{\left(\frac{\varepsilon_{sh}}{m}\right)^n}}{\varepsilon_{sh}}$$  \hspace{1cm} (4.10)
where $\psi$ is the matric suction in MPa; $H$ is the suction modulus in MPa; and the free shrinkage increment is $\Delta e_{sh} = \frac{\Delta \psi}{H}$.

Table 4.4 lists the regression parameters $m$ and $n$ in Equation (4.9), obtained from the test results shown in Table 4.3. Figure 4.10 presents comparisons of the measured and modeled relationships between the matric suction and shrinkage potential.

Figure 4.10: Relationship between Matric Suction and Shrinkage Potential: (a) Silty Clayey Sand, (b) Cement Silty Clayey Sand, (c) Clay, and (d) Cement Clay
Table 4.4: Regression Parameters of Free Shrinkage Model

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty Clayey Sand</td>
<td>0.000156</td>
<td>0.5575</td>
</tr>
<tr>
<td>Cement Silty Clayey Sand</td>
<td>0.000060</td>
<td>0.4824</td>
</tr>
<tr>
<td>Clay</td>
<td>0.015275</td>
<td>1.6783</td>
</tr>
<tr>
<td>Cement Clay</td>
<td>0.000451</td>
<td>0.4904</td>
</tr>
</tbody>
</table>

Using Equations (4.5) and (4.9), the shrinkage potential $\varepsilon_{sh}$ can be expressed as a nonlinear function of pore RH, as expressed in Equation (4.11). Figure 4.11 presents the plots of the shrinkage potential versus pore RH.

$$
\varepsilon_{sh} = m \left[ \ln \left( 1 - \frac{RT}{w_v (\frac{1}{\rho_w}) \ln \text{RH}} \right) \right]^{\frac{1}{n}}
$$

(4.11)

where $\varepsilon_{sh}$ is the shrinkage potential; m and n are the regression parameters; R is the universal gas constant (8.31 MPa·cm³/(mol·K) or J/(mol·K)); T is the temperature of the sample in Kelvin (K); $w_v$ is the molecular mass of water (18 g/mol); $\rho_w$ is the unit weight of water (1 g/cm³); and RH is the pore relative humidity.
Figure 4.11: Relationship between Shrinkage Potential and Pore RH: (a) Silty Clayey Sand, (b) Cement Silty Clayey Sand, (c) Clay, and (d) Cement Clay

The coefficient of thermal expansion can be defined as the amount of expansion (or contraction) per unit length of a material that results from a one-degree change in
temperature. Similarly, in this study, the so-called coefficient of moisture shrinkage is proposed and can be defined as the amount of contraction (or expansion) per unit length of a material that results from a one-degree change (1%) in pore RH. Referring to Equation (4.11), the coefficient of moisture shrinkage $\alpha$ can be expressed as Equation (4.12) in terms of pore RH or Equation (4.13) in terms of matric suction.

$$
\alpha = \frac{\varepsilon_{sh}}{\Delta RH} = \frac{m \left[ \ln \left( 1 - \frac{RT}{W \nu(1/\rho_w)} \frac{\ln RH}{1-RH} \right) \right]}{1-e^{-\psi \frac{W \nu(1/\rho_w)}{RT}}} \quad (4.12)
$$

$$
\alpha = \frac{\varepsilon_{sh}}{\Delta RH} = \frac{m \left[ \ln \left( 1 + \psi \frac{W \nu(1/\rho_w)}{RT} \right) \right]}{1-e^{-\psi \frac{W \nu(1/\rho_w)}{RT}}} \quad (4.13)
$$

The coefficient of moisture shrinkage $\alpha$ is a nonlinear function of pore RH or matric suction. Given different pore RH or matric suction values, $\alpha$ can be calculated from Equations (4.12) or (4.13). Table 4.5 lists the corresponding matric suction, suction modulus H, and coefficient of moisture shrinkage $\alpha$ at different pore RH values for the four materials used in this study. The nonlinear relationships between coefficient of moisture shrinkage $\alpha$ and pore RH or matric suction also are shown in Figures (4.12) through (4.15).

Figures (4.12) through (4.15) indicate that at high pore RH and low suction values, the behavior of the soil and cement soil differs. For soils (without cement stabilization), the coefficient of moisture shrinkage $\alpha$ kept increasing substantially with an increase in pore RH or decrease in suction. However, for the cement soils, the curves became flat at high pore RH or low suction values. Again, this behavior is assumed to be related to cement hydration, which changes the pore structure and stiffness of the soil. The hydration effect is more obvious in the early stages of stabilization.
Table 4.5: Coefficient of Moisture Shrinkage at Different Pore RH Values

<table>
<thead>
<tr>
<th>Pore RH</th>
<th>Matric Suction (MPa)</th>
<th>Suction Modulus H (MPa)</th>
<th>α</th>
<th>Pore RH</th>
<th>Matric Suction (MPa)</th>
<th>Suction Modulus H (MPa)</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.62%</td>
<td>133.21</td>
<td>137508.43</td>
<td>0.00432</td>
<td>39.60%</td>
<td>126.22</td>
<td>187664.83</td>
<td>0.00261</td>
</tr>
<tr>
<td>60.00%</td>
<td>65.62</td>
<td>67970.31</td>
<td>0.00535</td>
<td>60.00%</td>
<td>65.62</td>
<td>122076.40</td>
<td>0.00307</td>
</tr>
<tr>
<td>86.25%</td>
<td>20.16</td>
<td>46174.59</td>
<td>0.00839</td>
<td>86.25%</td>
<td>20.16</td>
<td>48066.18</td>
<td>0.00440</td>
</tr>
<tr>
<td>99.00%</td>
<td>1.37</td>
<td>n/a</td>
<td>0.01196</td>
<td>99.00%</td>
<td>1.37</td>
<td>n/a</td>
<td>0.00441</td>
</tr>
</tbody>
</table>

Clay

<table>
<thead>
<tr>
<th>Pore RH</th>
<th>Matric Suction (MPa)</th>
<th>Suction Modulus H (MPa)</th>
<th>α</th>
<th>Pore RH</th>
<th>Matric Suction (MPa)</th>
<th>Suction Modulus H (MPa)</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.93%</td>
<td>178.76</td>
<td>38496.85</td>
<td>0.05578</td>
<td>39.60%</td>
<td>126.22</td>
<td>187664.83</td>
<td>0.00261</td>
</tr>
<tr>
<td>59.87%</td>
<td>69.90</td>
<td>13499.19</td>
<td>0.09029</td>
<td>60.00%</td>
<td>65.62</td>
<td>122076.40</td>
<td>0.00307</td>
</tr>
<tr>
<td>79.96%</td>
<td>30.47</td>
<td>6637.66</td>
<td>0.14868</td>
<td>86.25%</td>
<td>20.16</td>
<td>48066.18</td>
<td>0.00440</td>
</tr>
<tr>
<td>99.00%</td>
<td>1.37</td>
<td>n/a</td>
<td>1.39879</td>
<td>99.00%</td>
<td>1.37</td>
<td>n/a</td>
<td>0.03339</td>
</tr>
</tbody>
</table>

Cement Clay

Figure 4.12: (a) Coefficient of Moisture Shrinkage versus Pore RH, and (b) Coefficient of Moisture Shrinkage versus Matric Suction for Silty Clayey Sand
Figure 4.13: (a) Coefficient of Moisture Shrinkage versus Pore RH, and (b) Coefficient of Moisture Shrinkage versus Matric Suction for Cement Silty Clayey Sand

Figure 4.14: (a) Coefficient of Moisture Shrinkage versus Pore RH, and (b) Coefficient of Moisture Shrinkage versus Matric Suction for Clay
Figure 4.15: (a) Coefficient of Moisture Shrinkage versus Pore RH, and (b) Coefficient of Moisture Shrinkage versus Matric Suction for Cement Clay

4.3 MODELING OF TENSILE STRENGTH

The mechanical properties of soil, such as tensile strength and stiffness, are affected significantly by moisture content. For cement-stabilized soil, in addition to the effects of moisture, its mechanical properties are affected by hydration as well. To study and predict the shrinkage cracking of soil and cement soil, the evolution of their mechanical properties along with the development of drying and/or hydration needs to be predicted with accuracy.

4.3.1 Test Results for Tensile Strength Growth Due to Moisture Loss and Hydration

For this study, IDT tests were used to measure the tensile strength of different types of specimens, as discussed in Section 3.2.6. Figure 4.16 shows the IDT strength values of
the cured specimens (control specimens) and overlapped drying specimens (in terms of top, middle, and bottom specimens) for cement silty clayey sand and cement clay, respectively.

The cured (control) cement soil specimens were cured at 100% RH and at 20°C at different ages of the specimens to study the effect of hydration on IDT strength, with the assumption of no moisture loss. However, for the overlapped drying specimens of cement soils, because they were wax-sealed except on the top surface from which the moisture could evaporate during the drying process, even for the same ages of the specimens, the top, middle, and bottom specimens had different moisture contents. The difference in moisture contents may or may not have affected the hydration rate, depending on if the moisture was still sufficient for cement hydration. Therefore, the IDT strength of the overlapped drying specimens is a result of the combined effects of moisture loss and hydration.

The test results shown in Figure 4.16 indicate that cement silty clayey sand and cement clay behave differently. For the cement silty clayey sand, at the same age, the specimens at the middle and bottom of the three overlapped drying specimens had higher IDT strength values than the cured (control) specimens, whereas the specimens at the top had lower IDT strength values than the cured (control) specimens. However, for cement clay, the IDT strength values of the top, middle, and bottom specimens were all higher than those of the cured (control) specimens at corresponding ages. This phenomenon is assumed to be due to the combined effects of moisture loss and cement hydration and is examined further in the following sections.
Figure 4.16: Tensile Strength Growth Due to Moisture Loss and Hydration: (a) Cement Silty Clayey Sand and (b) Cement Clay
4.3.2 Moisture Effect on Tensile Strength Growth

A practical way to approach the study of the combined effects of moisture and cement hydration is first to understand the moisture effect without cement hydration.

4.3.2.1 Introduction

It is well known that the strength of a geomaterial is affected by its moisture content. Therefore, the effect of moisture on strength must be taken into account. Furthermore, in pavement engineering and other geotechnical practices, usually soils are neither completely saturated nor completely dry, but rather in an unsaturated condition in which the moisture content keeps changing with the environment. Therefore, characterizing the effect of moisture on the mechanical properties, such as tensile strength and UCS, of geomaterials such as soils is of paramount importance to engineering practice.

The field of unsaturated soil mechanics has gained the attention of researchers over the past decade due to the fact that most geotechnical practices involve soils in an unsaturated condition. The concept of effective stress has had the most profound impact on the characterization of the mechanical properties of unsaturated soil. Research into effective stress in unsaturated soil dates back to the 1950s when Bishop (1959) proposed an effective stress parameter, $\chi$, in an effective stress relationship that is related to matric suction ($u_a - u_w$), as shown in Equation (4.14).

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w)$$  \hspace{1cm} (4.14)

where $\sigma'$ is effective stress; $\sigma$ is total stress; $u_a$ is the pore air pressure; $u_w$ is the pore water pressure; $\chi$ is the effective stress parameter, which equals 1.0 for the saturated state and 0
for the dry state; \((\sigma - u_a)\) is the net stress; \((u_a - u_w)\) is the matric suction; and \(\chi(u_a - u_w)\) is the part of the effective stress that is attributed to the saturation state or internal stress.

The effective stress parameter, \(\chi\), describes the contribution of matric suction to the macroscopic stress of a solid skeleton (Khalili and Zargarbashi 2010). Although this concept has been used for many years (Craig 1992, Khalili and Khabbaz 1998, Zienkiewicz et al. 1999), some researchers (Muraleetharan and Wei 1999, Jennings and Burland 1962, Fredlund and Morgenstern 1977, Khalili et al. 2004, Lu and Likos 2006) have shown that Equation (4.14) is valid only under certain conditions due to the fact that effective stress parameter \(\chi\) depends not only on saturation, but also on the soil structure and stress path (Khalili and Zargarbashi 2010, Heath et al. 2004). Oh et al. (2012) showed that one of the theoretical difficulties of Equation (4.14) is that it predicts effective stress to be zero for any soil in a dry state without external stress, which may not be true. Many soils, such as clays, under dry conditions can have effective stress levels of the order of several hundred kPa (Lu and Likos 2006, Lu et al. 2010).

Lu and Likos (2006, 2004) and Lu et al. (2010) published a series of papers to introduce the concept of suction stress, which is a function of matric suction, to characterize the effective stress in an unsaturated soil. The concept of suction stress can unify the effective stress under both saturated and unsaturated conditions via one closed-form equation, as shown by Equation (4.15) (Lu and Likos 2006, Oh et al. 2012).

\[
\sigma' = (\sigma - u_a) - \sigma^s
\]  

(4.15)
where $\sigma_s$ is the suction stress, and the other parameters have the same meanings as given heretofore.

The suction stress characteristic curve (SSCC) of the soil defines the relationship between the suction stress and the matric suction or the effective degree of saturation.

Lu et al. (2010) showed that suction stress can be described as

$$\sigma_s = -(u_a - u_w)S_e$$ \hspace{1cm} (4.16)

where $S_e$ is the effective degree of saturation, which is defined by

$$S_e = \frac{S - S_r}{1 - S_r}$$ \hspace{1cm} (4.17)

where $S$ = the degree of saturation; $S_r$ = the residual degree of saturation at which a significantly large suction change is required to remove additional water from the material, thereby defined as the transition between the funicular and pendular states in a SWCC (Kim and Sture 2008).

Based on the Van Genuchten (Van Genuchten et al. 1991) SWCC model, the effective degree of saturation $S_e$ is expressed by

$$S_e = \left\{\frac{1}{1+\left[\alpha(u_a - u_w)^n\right]}\right\}^{1-1/n}$$ \hspace{1cm} (4.18)

where $\alpha$ and $n$ are the empirical fitting parameters of the unsaturated soil properties for which $\alpha$ is the inverse of the air entry pressure for water-saturated soil and $n$ is the pore size distribution parameter, and the other parameters have the same meanings as given heretofore.

Lu et al. (2010) derived closed-form solutions for suction stress for the full range of saturation or matric suction levels, as follows.
\[
\sigma^s = -\frac{S_e}{\alpha} \left( S_e^{\frac{n}{1-n}} - 1 \right)^{\frac{1}{n}} \quad 0 \leq S_e \leq 1 \quad (4.19)
\]

\[
\sigma^s = -(u_a - u_w) \quad u_a - u_w \leq 0 \quad (4.20a)
\]

\[
\sigma^s = -\frac{(u_a - u_w)}{1 + \left[\alpha (u_a - u_w)\right]^{n(1-n)/n}} \quad u_a - u_w \geq 0 \quad (4.20b)
\]

Based on the concept of suction stress, the SSCC can be correlated directly with the SWCC. The closed-form solutions can be defined using only two parameters, \(\alpha\) and \(n\), in Equations (4.19) and (4.20) (Oh et al. 2012). The theory has been validated using sufficient experimental data obtained from the literature or via tests (Lu and Likos 2006, Oh et al. 2012, Lu et al. 2010, Khosravi et al. 2010). However, most of the validations were from shear strength tests at relatively high moisture contents and, thus, low suction levels (typically less than 700 kPa) (Oh et al. 2012, Lu et al. 2010). The validation of the suction stress theory at high suction levels remains inconclusive (Oh et al. 2012). No research has been undertaken that focuses on the application of the SSCC in pavement engineering practice that typically uses different material parameters, such as tensile strength and UCS. The former is a critical property to characterize the cracking in soils (either bound or unbound) due to drying shrinkage, and the latter is a commonly used material property for unsaturated soils. The IDT test is commonly used to measure the tensile strength of soils.

Therefore, this study focuses on the validation and application of the effective stress theory, which is based on suction stress and SCCCs, to IDT strength and UCS tests. One advantage of these tests is that they are easy to conduct and a triaxial cell is not needed because neither of these two tests requires confining pressure. The objective of this study is that, based on the IDT strength or UCS at two unsaturated moisture contents and the SWCC, the IDT strength or UCS at any other unsaturated moisture content of interest
can be predicted. Thus, these mechanical properties can be related to the fundamental material parameters of \( \alpha \) and \( n \) of SWCCs.

### 4.3.2.2 SWCC, IDT strength, and UCS test results of pure soil

#### 1. SWCC Test Results

Figures 4.17 and 4.18 present the SWCC test results for the silty clayey sand and clay, respectively. The Van Genuchten model, expressed as Equation (4.18), also is plotted. For the silty clayey sand, the curve-fitting results in \( \alpha = 0.01837 \) and \( n = 1.53339 \). For the clay, the curve-fitting results in \( \alpha = 0.00202 \) and \( n = 1.67731 \).

Prior to curve fitting, the degree of saturation, \( S \), was calculated based on Equation (3.1). To calculate the effective degree of saturation \( S_e \), the residual degree of saturation \( S_r \) was obtained from the measured SWCC, as shown in Figure 4.19. From Figure 4.19, the residual degree of saturation \( S_r \) was obtained easily as 0.12 and 0.22 for the silty clayey sand and clay, respectively, following the procedure used by Kim and Sture (2008). Then, the effective degree of saturation was calculated based on Equation (4.17).

![SWCC of Silty Clayey Sand](image)

**Figure 4.17: SWCC of Silty Clayey Sand**
Figure 4.18: SWCC of Clay

Figure 4.19: Relationship between Matric Suction and Degree of Saturation

2. IDT Strength and UCS Test Results

Figures 4.20 and 4.21 present the IDT strength and UCS versus their corresponding moisture contents, respectively. Figure 4.20 indicates that, for both the silty clayey sand and clay, the IDT strength increases with a decrease in moisture content, but follows different paths.
Similar to Figure 4.20 for IDT strength, Figure 4.21 shows that the UCS increases with a decrease in moisture content, following different paths for the silty clayey sand and clay used in this study.
4.3.2.3 Modeling the moisture effect on tensile strength

1. Verification of the Effective Stress Theory Based on Suction Stress and SSCCs

   a. Verification of suction stress and SSCCs: The test results are presented in terms of mean total stress $p$ versus deviatoric stress $q$, and mean effective stress $p'$ versus deviatoric stress $q$. The $p$, $q$, and $p'$ can be expressed by:

   \[ p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \]  \hspace{1cm} (4.21)

   \[ q = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} \]  \hspace{1cm} (4.22)

   \[ p' = (p - u_a) - s = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) - u_a - s \]  \hspace{1cm} (4.23)

   where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the principal stresses determined by the stress state; $p$ is the mean total stress; $q$ is deviatoric stress; $p'$ is mean effective stress; and the other parameters are defined as heretofore expressed.

   In the $p$-$q$ plot in Figure 4.22 for silty clayey sand, the relationships between deviatoric stress and mean total stress, as obtained from the IDT strength and UCS tests, are separate, whereas in the $p'$-$q$ plots, the relationships between deviatoric stress and mean effective stress collapse onto one failure line. The same results were found for the clay, as shown in Figure 4.23. These findings indicate the applicability of the suction stress and SSCC concept to IDT strength and UCS at low moisture contents or high matric suction levels. The moisture contents and matric suction levels range from 2.56% to 18.9% and from 600 kPa to 16,000 kPa, respectively, which are the typical ranges for many unsaturated soil conditions, such as for the base, sub-base, and subgrade materials in pavements.
b. Verification of the internal friction angle: The internal friction angle of unsaturated soils is assumed to be invariant to the degree of saturation or matric suction level and equal to the effective friction angle obtained from shear strength tests for the saturated state (Lu and Likos 2006, Oh et al. 2012). The slope of the linear failure line, M, can be used to determine the internal friction angle assuming a Mohr-Coulomb failure criterion. The internal friction angle can be expressed as follows (Oh et al. 2012):
\[ \phi' = \sin^{-1} \left[ \frac{3M}{6+M} \right] \]  

(4.24)

where \( \phi' \) is the internal friction angle, and M is the slope of the mean effective total stress (\( p' \)) versus deviatoric stress (q) plot.

Table 4.6 presents the calculated internal friction angles (\( \phi' \)) at different moisture contents. The coefficients of variation of the calculated internal friction angles are within 6.15%, indicating the effectiveness and reasonableness of using suction stress (\( \bar{\sigma}^s \)) to calculate effective stress (\( \bar{\sigma}' \)) are reasonable.

Table 4.6: Angle of Failure Envelope at Different Moisture Contents

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Moisture Content ( \omega ) (%)</th>
<th>UCS (kPa)</th>
<th>Interpolated IDT Strength (kPa)</th>
<th>Internal Friction Angle ( \phi' ) (Deg)</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty Clayey Sand</td>
<td>10.9</td>
<td>365.3</td>
<td>-22.9</td>
<td>36.1</td>
<td>6.82%</td>
</tr>
<tr>
<td></td>
<td>8.7</td>
<td>509.3</td>
<td>-29.6</td>
<td>37.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.8</td>
<td>683.6</td>
<td>-39.7</td>
<td>35.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.9</td>
<td>793.3</td>
<td>-46.4</td>
<td>35.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.9</td>
<td>960.6</td>
<td>-57.2</td>
<td>34.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>1268.5</td>
<td>-84.6</td>
<td>30.4</td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>18.2</td>
<td>528.4</td>
<td>-88.0</td>
<td>28.7</td>
<td>4.49%</td>
</tr>
<tr>
<td></td>
<td>15.6</td>
<td>917.9</td>
<td>-155.1</td>
<td>32.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13.5</td>
<td>1299.6</td>
<td>-208.8</td>
<td>32.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.1</td>
<td>1786.5</td>
<td>-271.5</td>
<td>31.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.9</td>
<td>2262.2</td>
<td>-327.2</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.8</td>
<td>2207.3</td>
<td>-356.6</td>
<td>32.2</td>
<td></td>
</tr>
</tbody>
</table>

2. Prediction of IDT Strength and UCS Values at Different Moisture Contents

Based on the aforementioned analysis, a simple procedure can be used to predict the IDT strength or UCS of soils for any unsaturated condition. The procedure for
predicting IDT strength is summarized below. The procedure for UCS prediction is similar, except for Step 7.

1. Determine the soil properties for maximum dry density and specific gravity by experimental testing.

2. Measure the suction at different moisture contents and create the SWCCs to obtain parameters $\alpha$ and $n$ in Equation (4.18) and the residual degree of saturation, $S_r$.

3. Perform IDT tests at two unsaturated moisture contents

4. Calculate the effective degree of saturation $S_e$, the suction stress $\bar{\sigma}_s$, and the effective stress $\bar{\sigma}$ using Equations (4.17), (4.19), and (4.15), respectively, for the IDT test results.

5. Plot the effective stress results in the $p' \cdot q$ coordinate and obtain slope $M$ and the intersection $N$ for the $p' \cdot q$ plot.

6. For any unsaturated moisture content of interest, calculate the effective degree of saturation $S_e$ and the corresponding suction stress $\bar{\sigma}_s$ using Equations (4.17) and (4.19), respectively.

7. The IDT strength at the moisture content of interest can be obtained by solving the group of functions below, based on the stress state of the IDT strength test specimen:
\[ b_x = \text{IDT} \]
\[ b_y = -3.1 \text{ IDT} \]
\[ q = M p' + N \]
\[
q = \frac{1}{\sqrt{2}} \sqrt{(b_1 - b_2)^2 + (b_1 - b_3)^2 + (b_2 - b_3)^2} = \frac{1}{\sqrt{2}} \sqrt{b_y^2 + (b_y - b_x)^2 + b_x^2}
\]
\[ p' = \frac{1}{3} (b_1 + b_2 + b_3) - u_a - \sigma^s = \frac{1}{3} (b_x + b_y) - u_a - \sigma^s \]

The resulting IDT strength can be expressed as

\[
\text{IDT} = \left[ \frac{M(\sigma^s + u_a) - N}{3.7 - 0.7M} \right]
\]

(4.25)

where \( M \) is the slope of the \( p' \)-q plot, and \( N \) is the intersection of the \( p' \)-q plot.

Using the stress state of the UCS test specimens, the UCS at any moisture content of interest can be expressed as

\[
\text{UCS} = \left[ \frac{3M(\sigma^s + u_a) - 3N}{M - 3} \right]
\]

(4.26)

Using the test results presented in this study, the IDT strength and UCS test results at two water contents can be used to predict the IDT strength and UCS at other water contents. Figure 4.24 presents a comparison of the predicted and measured IDT strength and UCS values. The figure shows that the predicted IDT strength or UCS values are close to the measured values, indicating the applicability of this prediction procedure.
3. Discussion

The dewpoint potentiometer method (WP4) can only measure suction accurately within the range from -1.0 MPa to -80 MPa. The model expressed by Equation (4.18) does not perfectly fit the measured data, which may be one of the reasons that the data points shown in Figures 4.22 and 4.23 do not fall perfectly along the failure line. The SWCC fitting may be improved by accurately measuring the suction beyond the aforementioned range. The SSCC is very sensitive to parameters $\alpha$ and $n$ from the SWCC fitting.

Figure 4.24 indicates slight discrepancies between the predicted and measured IDT strength and UCS values, probably due to two reasons. First, in the prediction procedure, only the IDT strength or UCS tests at two moisture contents were used to obtain the slope and intersection for the $p' - q$ plot, which may have resulted in a large experimental error. This type of error can be reduced by adding more tests at different moisture contents and
fitting the \( p' - q \) plot by regression. Second, the repeatability of the IDT strength and UCS tests may be compromised when a specimen is more brittle at a relatively low moisture content.

An accurate characterization of the strength of a soil as a function of moisture variation is critical in order to predict the performance of the soil. This section provides experimental verification of the effective stress theory using IDT strength and UCS tests, both of which are widely used in engineering practice. The verification is extended to low water contents ranging from 2.56\% to 18.9\% and high matric suction levels, which are typical ranges used in engineering practice. A new method is proposed and experimentally validated in this section to predict the soil’s UCS and tensile strength values at any moisture content of interest by taking simple SWCC measurements and conducting UCS and IDT strength tests at two unsaturated moisture contents. The mechanical properties of the soils are related to the fundamental material parameters of \( \alpha \) and \( n \) of the SWCCs.

4.3.2.4 Moisture effect on tensile strength growth

The models and procedure outlined in Section 4.3.2.3 can be used to calculate how much tensile strength growth is due to moisture loss only without considering the effect of hydration. To this end, the IDT strength of cement soils at several moisture content levels, excluding the effect of hydration, needs to be measured. Specimens of cement silty clayey sand and cement clay were fabricated at their OMCs to obtain MDD values following the specimen fabrication procedure to ensure the same initial soil structure. Then, the specimens were dried to different moisture content levels by keeping them in a
temperature-controlled chamber at 3°C to minimize cement hydration (NRMCA 1998). In this way, specimens with the same material structure and properties of cement soil, without cement hydration, but with different moisture content levels, were fabricated in order to study the effect of moisture on tensile strength.

Table 4.7 presents the moisture contents, IDT strength test results, calculated degrees of saturation, suction stress levels, and p’ and q values. Figure 4.25 presents the p’–q plots of cement silty clayey sand and cement clay, respectively, in which the M and N are 0.373 and -119.8 kPa for the cement silty clayey sand, and 0.228 and 384.6 kPa for the cement clay, as listed in Table 4.8.

Table 4.7: IDT Test Results and M and N Calculations of Cement Soils

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>w(%)</th>
<th>IDT</th>
<th>S</th>
<th>Se</th>
<th>Suction (kPa)</th>
<th>Suction Stress σs (kPa)</th>
<th>IDT σx (kPa)</th>
<th>IDT σy (kPa)</th>
<th>p (kPa)</th>
<th>p' (kPa)</th>
<th>q (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement Silty Clayey Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.52</td>
<td>47971</td>
<td>0.79</td>
<td>0.77</td>
<td>931.8</td>
<td>-713.2</td>
<td>-48.0</td>
<td>148.7</td>
<td>33.6</td>
<td>645.5</td>
<td>177.6</td>
<td></td>
</tr>
<tr>
<td>12.43</td>
<td>42868</td>
<td>0.78</td>
<td>0.76</td>
<td>953.3</td>
<td>-723.1</td>
<td>-42.9</td>
<td>132.9</td>
<td>30.0</td>
<td>651.8</td>
<td>158.7</td>
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<tr>
<td>10.54</td>
<td>39126</td>
<td>0.67</td>
<td>0.63</td>
<td>1427.4</td>
<td>-892.2</td>
<td>-39.1</td>
<td>121.3</td>
<td>27.4</td>
<td>818.2</td>
<td>144.9</td>
<td></td>
</tr>
<tr>
<td>9.49</td>
<td>52054</td>
<td>0.60</td>
<td>0.55</td>
<td>1757.8</td>
<td>-967.9</td>
<td>-52.1</td>
<td>161.4</td>
<td>36.4</td>
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<tr>
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<td>56477</td>
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<td>0.36</td>
<td>3163.6</td>
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<td>-98.7</td>
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<td>-116.0</td>
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<td>0.02</td>
<td>66733.1</td>
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<td>427.8</td>
<td>96.6</td>
<td>1647.1</td>
<td>511.0</td>
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<tr>
<td>Cement Clay</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>0.67</td>
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<td>-179.3</td>
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<td>125.5</td>
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<td>0.66</td>
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<td>-1094.9</td>
<td>-160.2</td>
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<td>112.2</td>
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<td>168069</td>
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<td>0.53</td>
<td>2634.4</td>
<td>-1406.5</td>
<td>-168.1</td>
<td>521.0</td>
<td>117.6</td>
<td>1422.8</td>
<td>622.3</td>
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<td>3672.5</td>
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<td>133.1</td>
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<td>806.8</td>
<td>182.2</td>
<td>2325.0</td>
<td>963.7</td>
<td></td>
</tr>
<tr>
<td>9.38</td>
<td>253806</td>
<td>0.38</td>
<td>0.28</td>
<td>8303.8</td>
<td>-2333.8</td>
<td>-253.8</td>
<td>786.8</td>
<td>177.7</td>
<td>2410.2</td>
<td>939.8</td>
<td></td>
</tr>
<tr>
<td>7.72</td>
<td>350767</td>
<td>0.32</td>
<td>0.20</td>
<td>14396.3</td>
<td>-2903.1</td>
<td>-350.8</td>
<td>1087.4</td>
<td>245.5</td>
<td>3047.3</td>
<td>1298.8</td>
<td></td>
</tr>
<tr>
<td>5.14</td>
<td>414081</td>
<td>0.21</td>
<td>0.08</td>
<td>66979.8</td>
<td>-5249.1</td>
<td>-414.1</td>
<td>1283.7</td>
<td>289.9</td>
<td>5437.7</td>
<td>1533.2</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.25: p’-q Plots for (a) Cement Silty Clayey Sand and (b) Cement Clay

Table 4.8: Material Parameters Used for Modeling the Moisture Effect on the IDT Strength of Cement Soils

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\alpha$</th>
<th>n</th>
<th>$S_r$</th>
<th>M</th>
<th>N (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement Silty Clayey Sand</td>
<td>0.000927</td>
<td>1.896555</td>
<td>0.11</td>
<td>0.373</td>
<td>-119.8</td>
</tr>
<tr>
<td>Cement Clay</td>
<td>0.000917</td>
<td>1.618228</td>
<td>0.14</td>
<td>0.228</td>
<td>384.6</td>
</tr>
</tbody>
</table>

Table 4.8 presents the regressed parameters $\alpha$ and n found in Equation (4.18) and the residual degree of saturation $S_r$ for the cement soils based on the SWCC test results that are presented in Figures 4.26 and 4.27.
Using Equations (4.17), (4.19), and (4.25), the IDT strength growth due to moisture loss, without considering the effect of cement hydration, can be calculated for the overlapped drying specimens of cement silty clayey sand and cement clay, as shown in Figures 4.28 and 4.29, respectively.
Figures 4.28 and 4.29 also show the moisture loss effect on the cured (control) specimens. As expected, no IDT strength growth occurred (the plot is flat) due to moisture loss for the cured (control) specimens, because there was no moisture loss. Because the top specimens always have lower moisture contents than the corresponding middle and bottom
specimens at the same age, the IDT strength growth due to moisture loss for the top specimens is always higher than that of the corresponding middle and bottom specimens at the same age.

**4.3.3 Hydration Effect on Tensile Strength Growth**

As discussed earlier, the IDT strength test results for each overlapped specimen stem from the combined effect of moisture loss and cement hydration. In Section 4.3.2.4, the effect of moisture loss on IDT strength growth could already be predicted. By subtracting the IDT strength growth due to moisture loss from the measured IDT strength, the remaining IDT strength growth is due to cement hydration, as shown in Figures 4.30 and 4.31 for cement silty clayey sand and cement clay, respectively.

![Figure 4.30: Hydration Effect on IDT Strength: Cement Silty Clayey Sand](image)

Figure 4.30: Hydration Effect on IDT Strength: Cement Silty Clayey Sand
4.3.3.1 Hydration effect with sufficient water

After excluding the moisture effect on IDT strength, Figures 4.30 and 4.31 show that the hydration effect on IDT strength growth for the middle and bottom specimens is very close to that of the cured (control) specimens. These results indicate that the moisture loss during drying for the middle and bottom specimens had almost no effect on hydration. In other words, for the middle and bottom specimens, the moisture that was left after moisture loss was still sufficient for hydration. It should be noted that the moisture diffusion within the three overlapped specimens differs from the moisture diffusion for one large specimen, because there is no continuity between the surfaces of the three overlapped specimens. However, even without covering the top surface (as is the case for cement-stabilized pavement layers without curing), the drying has little or no effect on the hydration of the cement soil below 2.54 cm (1 in.) of the drying surface, at least for the
cement contents used in this study (5% for cement silty clayey sand and 12% for cement clay).

For the cured (control) specimens, the cement hydration is assumed to be fully developed. Therefore, the hydration effect on the IDT strength of the middle and bottom specimens should not exceed that of the cured (control) specimens. Figure 4.30 shows that for the cement silty clayey sand, the plots for IDT strength due to hydration for the middle and bottom specimens almost merge with those of the cured (control) specimens. However, in Figure 4.31, for cement clay, the hydration effect on the IDT strength of the middle and bottom specimens is greater than that of the cured (control) specimens, which indicates that the moisture effect on IDT strength growth of cement clay was somehow underestimated.

4.3.3.2 Hydration effect without sufficient water

It is worth further analyzing the moisture and hydration effects on IDT strength for the top specimens among the three overlapped drying specimens, because shrinkage cracking typically initiates from the drying surface.

For the top specimens, the effects of moisture and hydration on IDT strength are compared in Figure 4.32 for cement silty clayey sand and cement clay. Figure 4.32 shows that for the cement clay, the moisture effect and hydration effect on IDT strength are more significant than for the cement silty clayey sand, probably because cement clay has finer particles and higher cement content than cement silty clayey sand. For the cement silty clayey sand, the moisture loss-induced IDT strength is about half of the hydration-induced IDT strength at the same age. Both moisture loss- and hydration-induced IDT strength
increased quickly within seven days, but the increments slowed considerably after seven days. For the cement clay, the IDT strength due to hydration increased quickly within seven days, but the increments slowed considerably after seven days, similar to the cement silty clayey sand. The moisture that was left was not sufficient for cement hydration after seven days of drying. Therefore, the effect of hydration on IDT strength ceased. However, unlike for the cement silty clayey sand, the IDT strength due to moisture loss continued to increase almost linearly up to 56 days for the cement clay. This increase in IDT strength is assumed to be related to the particle size distributions and pore structures of the cement silty clayey sand and cement clay. Further study is needed to explain this phenomenon. The results also indicate that at a spot near the drying surface, where the shrinkage cracking typically initiates, the effect of moisture loss on tensile strength is significant and should not be neglected.

Figure 4.32: Moisture and Hydration Effects on IDT Strength: Top Specimens
4.4 MODELING OF TENSILE STRESS-STRAIN RELATIONSHIP

Similar to the effects of moisture and hydration on tensile strength, moisture loss and hydration also affect other mechanical properties of soils and cement soils, such as the stress-strain relationship. With the loss of moisture and/or the development of hydration, the soils or cement soils become stiffer, which changes the stress-strain relationship.

4.4.1 Moisture Effect on Tensile Stress-Strain Relationship

Figures 4.33 and 4.34 present the tensile stress-strain curves derived from IDT strength tests at different moisture contents for the pure soil specimens of silty clayey sand and clay, respectively. The curves clearly indicate the effects of moisture loss. With the loss of moisture (and therefore the reduction of pore RH), the IDT strength increases and the tensile stress-strain curves become steeper.

A two-parameter model, shown in Equation (4.27), is proposed in this study to characterize the tensile stress-strain relationships of pure soils.

\[ \delta = \delta_{\text{ult}} \left( 1 - \frac{1}{1+\varepsilon} \right) \]  

(4.27)

where \( \delta \) and \( \varepsilon \) are the tensile stress and strain, respectively, and \( \delta_{\text{ult}} \) and \( c \) are the regression parameters.

Figures 4.33 and 4.34 present the plots for the regression models.
Figure 4.33: Tensile Stress-Strain Relationship at Different Moisture Contents: Silty Clayey Sand

Figure 4.34: Stress-Strain Relationship at Different Moisture Contents: Clay
Table 4.9 presents the regression parameters, $\delta_{\text{ult}}$ and $c$, at different moisture contents and pore RH values for both the silty clayey sand and clay. Figures 4.35 and 4.36 present the relationship between $\delta_{\text{ult}}$ and pore RH and the relationship between $c$ and pore RH for the silty clayey sand and clay, respectively. These plots indicate that the regression parameters $\delta_{\text{ult}}$ and $c$ have a strong correlation with the pore RH (moisture contents) for both of the soils used in this study.

Table 4.9: Regression Parameters of $\delta_{\text{ult}}$ and $c$ in Stress-Strain Curves of Pure Soil

<table>
<thead>
<tr>
<th>Soil</th>
<th>Specimen</th>
<th>w%</th>
<th>S</th>
<th>RH</th>
<th>$c$</th>
<th>$\delta_{\text{ult}}$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty Clayey Sand</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.7</td>
<td>0.55</td>
<td>0.99</td>
<td>1.8270E-04</td>
<td>34780</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6.6</td>
<td>0.47</td>
<td>0.98</td>
<td>1.8089E-04</td>
<td>47687</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.8</td>
<td>0.34</td>
<td>0.92</td>
<td>1.5116E-04</td>
<td>68392</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.7</td>
<td>0.26</td>
<td>0.86</td>
<td>1.2978E-04</td>
<td>90167</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.7</td>
<td>0.19</td>
<td>0.75</td>
<td>1.0222E-04</td>
<td>121918</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>0.15</td>
<td>0.64</td>
<td>5.0228E-05</td>
<td>157948</td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>1</td>
<td>17.3</td>
<td>0.83</td>
<td>0.99</td>
<td>3.4842E-04</td>
<td>96540</td>
</tr>
<tr>
<td>2</td>
<td>15.3</td>
<td>0.74</td>
<td>0.98</td>
<td>4.1325E-04</td>
<td>157123</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>13.2</td>
<td>0.64</td>
<td>0.96</td>
<td>4.1284E-04</td>
<td>228929</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9.4</td>
<td>0.46</td>
<td>0.87</td>
<td>2.3989E-04</td>
<td>318034</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7.7</td>
<td>0.37</td>
<td>0.80</td>
<td>2.7451E-04</td>
<td>370023</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5.1</td>
<td>0.25</td>
<td>0.62</td>
<td>1.7255E-04</td>
<td>456342</td>
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</tr>
<tr>
<td>7</td>
<td>3.9</td>
<td>0.19</td>
<td>0.49</td>
<td>8.1606E-05</td>
<td>591018</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.35: Parameters in Stress-Strain Relationship versus Pore RH for Silty Clayey Sand: (a) $\delta_{\text{ult}}$ vs. Pore RH, and (b) $c$ vs. Pore RH

(a)

(b)
Figure 4.36: Parameters in Stress-Strain Relationship versus Pore RH for Clay: (a) $\sigma_{\text{ult}}$ vs. Pore RH, and (b) $c$ vs. Pore RH

(a) $\sigma_{\text{ult}}$-Clay

\[ y = -999,990.49x^2 + 633,583.43x + 497,448.65 \]
\[ R^2 = 0.95 \]

(b) $c$-Clay

\[ y = 0.000401x^{2.109002} \]
\[ R^2 = 0.93 \]
Two methods are presented in the following sections to study the relationship between parameters $\sigma_{\text{ult}}$ and $c$ in Equation (4.27) and the pore RH.

(1) Statistical Method

From the plots and regressions presented in Figure 4.35, the parameters $\sigma_{\text{ult}}$ and $c$ for silty clayey sand can be expressed as a function of pore RH in Equations (4.28) and (4.29), respectively.

$$\sigma_{\text{ult}} = -338646 \times RH + 376785$$  \hspace{1cm} (4.28)

$$c = 0.000368 \times RH - 0.000183$$  \hspace{1cm} (4.29)

where $\sigma_{\text{ult}}$ and $c$ are the regression parameters, and RH is the pore relative humidity.

If Equations (4.28) and (4.29) are substituted into Equation (4.27), the tensile stress-strain relationship of the silty clayey sand can be expressed as a function of pore RH in Equation (4.30).

$$\sigma = (-338646 \times RH + 376785) \left[1 - \frac{1}{1 + \frac{1}{c (0.000368 \times RH - 0.000183)}} \right]$$  \hspace{1cm} (4.30)

From the plots and regressions shown in Figure 4.36, the parameters $\sigma_{\text{ult}}$ and $c$ for silty clayey sand can be expressed as a function of pore RH in Equations (4.31) and (4.32), respectively.

$$\sigma_{\text{ult}} = -1207304 \times RH^2 + 985946 \times RH + 367317$$  \hspace{1cm} (4.31)

$$c = 0.000428 \times RH^{2.209482}$$  \hspace{1cm} (4.32)
If Equations (4.31) and (4.32) are substituted into Equation (4.27), the tensile stress-strain relationship of the silty clayey sand can be expressed as a function of pore RH in Equation (4.33):

$$
\sigma = (-1207304 \times RH^2 + 985946 \times RH + 367317) \left[ 1 - \frac{1}{1 + \left( \frac{c}{(0.000428 \times RH^{2.209482})} \right)^{\epsilon}} \right]
$$

(4.33)

The test results for the silty clayey sand and clay indicate that good correlations exist between the two parameters $\sigma_{ult}$ or $c$ in Equation (4.27) and the pore RH. However, for different materials, the correlation model could have different forms (linear, power, or polynomial, etc.) and might not be able to be expressed using the same form with only different regression parameters.

Figures (4.37) and (4.38) present a comparison of the parameters $\sigma_{ult}$ and $c$ obtained from the regression of the measured tensile stress-strain curves and the parameters calculated from the pore RH using Equations (4.28), (4.29), (4.31), and (4.32) for silty clayey sand and clay, respectively.
Figure 4.37: Comparison of Model Predicted and Measured Parameters in Stress-Strain Relationship for Silty Clayey Sand using Statistical Method: (a) $\sigma_{\text{ult}}$, and (b) $c$
Figure 4.38: Comparison of Model Predicted and Measured Parameters in Stress-Strain Relationship for Clay using Statistical Method: (a) $\sigma_{\text{ult}}$, and (b) $c$

(2) IDT method:

In Equation (4.27), when the tensile strain $\varepsilon$ is approaching $+\infty$, the tensile stress is approaching $\sigma_{\text{ult}}$. Parameter $\sigma_{\text{ult}}$ represents the asymptote of the tensile stress-strain curve.
Therefore, $\sigma_{ult}$ can be called *ultimate tensile strength*. If $\sigma_{ult}$ was is plotted against IDT strength, as shown in Figure 4.39 for the test results of both soils and cement soils in this study, it is clear that $\sigma_{ult}$ can be expressed as a linear function of IDT strength using Equation (4.34), no matter if it is silty clayey sand, clay, cement silty clayey sand, or cement clay.

$$\sigma_{ult} = f(IDT) = 1.2467 \text{ (IDT)}$$

(4.34)

The relationship between parameters $c$ and $\sigma_{ult}$ can be obtained from Figure 4.40 for the silty clayey sand and clay. Based on Figure 4.40, parameter $c$ can be expressed as a linear function of $\sigma_{ult}$ and therefore a linear function of IDT strength, as shown by Equations (4.35) and (4.36) for silty clayey sand and clay, respectively.

$$c = -1E - 09 \sigma_{ult} + 0.0002 = -1E - 09 [1.2467 \text{ (IDT)}] + 0.0002$$

(4.35)

$$c = -7E - 10 \sigma_{ult} + 0.0005 = -7E - 10 [1.2467 \text{ (IDT)}] + 0.0005$$

(4.36)

Figure 4.39: Relationship between Parameter $\sigma_{ult}$ and IDT Strength
Figure 4.4: Relationship between Parameters $\delta_{\text{ult}}$ and $c$: (a) Silty Clayey Sand, and (b) Clay

Figures 4.41 presents a comparison of parameters $\delta_{\text{ult}}$ and $c$, as obtained from the regression of the measured tensile stress-strain curves and the parameters calculated from IDT strength using Equations (4.34), (4.35), and (4.36) for both silty clayey sand and clay, which shows high prediction accuracy.
To this point, both parameters $\delta_{\text{ult}}$ and $c$ in Equation (4.27) have been expressed as a function of IDT strength. As indicated in Section 4.3.2.3, IDT strength also can be
predicted using suction stress theory and SWCC parameters $\alpha$ and $n$, as shown by Equations (4.17), (4.19), and (4.25) (introduced previously and also shown below).

\[
S_e = \frac{S - S_r}{1 - S_r} \tag{4.17}
\]

\[
\sigma_s = -\frac{S_e}{\alpha} \left( S_e^{\frac{n}{1-n}} - 1 \right)^{\frac{1}{n}} \quad 0 \leq S_e \leq 1 \tag{4.19}
\]

\[
\text{IDT} = \left[ \frac{M(\delta^s + u_a) - N}{3.7 - 0.7M} \right] \tag{4.25}
\]

The parameters used in Equations (4.17), (4.19), and (4.25) are listed in Table 4.10 in which the M and N are regressed using half of the experimental results. The other half of the experimental results is used to verify the IDT strength prediction accuracy. As shown in Figure 4.42, the prediction accuracy is good.

In summary, parameters $6_{\text{ult}}$ and $c$ in Equation (4.27) at a given moisture content can be predicted from the IDT strength. The IDT strength at a given moisture content can be correlated to SWCC parameters $\alpha$ and $n$ using suction stress theory. It should be noted that in order to obtain M and N in Equation (4.25), the IDT strength needs to be measured at at least two different moisture levels (the more moisture levels that are used, the more accurate the prediction).

**Table 4.10: Regression Parameters of M, N, Sr, $\alpha$, and $n$ for Pure Soil**

<table>
<thead>
<tr>
<th>parameter</th>
<th>sand</th>
<th>clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1.3785</td>
<td>0.8236</td>
</tr>
<tr>
<td>N</td>
<td>-170.43</td>
<td>-257.75</td>
</tr>
<tr>
<td>$S_r$</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.003124</td>
<td>0.0006961</td>
</tr>
<tr>
<td>$n$</td>
<td>1.872634</td>
<td>1.851416</td>
</tr>
</tbody>
</table>
Figure 4.42: Comparison of Model Predicted and Measured IDT Strength Values: (a) Silty Clayey Sand, and (b) Clay
4.4.2 Combined Effect of Moisture and Hydration on Tensile Stress-Strain Relationship

For cement soils that undergo the drying process, the tensile stress-strain relationship is controlled by the combined effects of pore RH (moisture content) and hydration. Furthermore, the change of pore RH (moisture loss due to drying) have an effect on the hydration rate, which makes the modeling of the tensile stress-strain relationship much more complicated than for pure soils.

The tensile stress-strain curves of cement soils are expressed using Equation (4.27) (introduced previously and also shown below), which was used also for the analysis of the tensile stress-strain curves of the pure soil. However, for the cement soil, the parameters $\delta_{\text{ult}}$ and $c$ should be related not only to the pore RH (moisture content), but also to age, which reflects the hydration rate.

$$\delta = \delta_{\text{ult}} \left(1 - \frac{1}{1+c}\right) \quad (4.27)$$

Figure 4.43 shows an example of the tensile stress-strain relationship in the IDT strength test results for the top specimen of cement clay at 28 days.
Tables 4.11 and 4.12 present the regression parameters $\sigma_{\text{ult}}$ and $c$ at different pore RH (moisture content) values and different ages for cement silty clayey sand and cement clay, respectively.

Table 4.11: Regression Parameters $\sigma_{\text{ult}}$ and $c$ of Cement Silty Clayey Sand
Table 4.12: Regression Parameters $\delta_{ult}$ and c of Cement Clay

<table>
<thead>
<tr>
<th>Location</th>
<th>Age (day)</th>
<th>w%</th>
<th>S</th>
<th>Pore RH</th>
<th>c</th>
<th>$\delta_{ult}$</th>
<th>Measured IDT Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>7</td>
<td>10.5</td>
<td>0.43</td>
<td>0.86</td>
<td>9.9733E-05</td>
<td>682988</td>
<td>616140</td>
</tr>
<tr>
<td>Top</td>
<td>7</td>
<td>10.1</td>
<td>0.42</td>
<td>0.85</td>
<td>8.6848E-05</td>
<td>649173</td>
<td>594706</td>
</tr>
<tr>
<td>Top</td>
<td>7</td>
<td>10.5</td>
<td>0.43</td>
<td>0.86</td>
<td>1.3963E-04</td>
<td>775223</td>
<td>613759</td>
</tr>
<tr>
<td>Middle</td>
<td>7</td>
<td>14.6</td>
<td>0.60</td>
<td>0.95</td>
<td>1.2317E-04</td>
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<tr>
<td>Middle</td>
<td>7</td>
<td>14.5</td>
<td>0.59</td>
<td>0.94</td>
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<td>515075</td>
<td>464402</td>
</tr>
<tr>
<td>Middle</td>
<td>7</td>
<td>14.7</td>
<td>0.60</td>
<td>0.95</td>
<td>1.2811E-04</td>
<td>618889</td>
<td>500125</td>
</tr>
<tr>
<td>Bottom</td>
<td>7</td>
<td>15.3</td>
<td>0.63</td>
<td>0.95</td>
<td>1.3000E-04</td>
<td>611047</td>
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</tr>
<tr>
<td>Bottom</td>
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<td>0.96</td>
<td>6.1362E-05</td>
<td>531644</td>
<td>495363</td>
</tr>
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<td>Bottom</td>
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<td>0.96</td>
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<td>490591</td>
<td>501145</td>
</tr>
<tr>
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<td>0.35</td>
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<tr>
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<td>0.79</td>
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<td>0.92</td>
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<tr>
<td>Middle</td>
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<td>0.92</td>
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<td>699817</td>
<td>568168</td>
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<td>Bottom</td>
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<td>14.1</td>
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<td>Bottom</td>
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<td>1.5479E-04</td>
<td>1024945</td>
<td>825378</td>
</tr>
</tbody>
</table>
As discussed in Section 4.4.1 and shown in Figure 4.39, the correlation between parameter \( \sigma_{\text{ult}} \) and IDT strength has no relation to soil type or whether the soil is stabilized or not. \( \sigma_{\text{ult}} \) and IDT strength follow the same linear function of Equation (4.34). Furthermore, for cement soil, the parameters \( c \) and \( \sigma_{\text{ult}} \) have the same correlation for both cement silty clayey sand and cement clay, as shown in Figure 4.44 and Equation (4.37). This behavior is different from that of soil in which the relationship between \( c \) and \( \sigma_{\text{ult}} \) follows different trends for silty clayey sand and clay, as shown in Figures 4.40 and Equations (4.35) and (4.36). Further study is needed to explain this phenomenon.

Figure 4.44: Tensile Stress-Strain Relationship: Cement Clay 28-day Top Specimen

\[
c = 2E - 10 \sigma_{\text{ult}} - 3E - 05 = 2E - 10 \left[1.2467 \text{ (IDT)}\right] - 3E - 05 \quad (4.37)
\]
Similar to the case for soils, for cement soils, both parameters $c$ and $\delta_{ult}$ can be correlated with IDT strength. However, for cement soils, the IDT strength includes the effect of hydration and therefore cannot be modeled and predicted in this study.

Using statistical analysis, parameters $\delta_{ult}$ and $c$ can be expressed as functions of pore RH and age for both cement silty clayey sand and cement clay, as follows.

(1) Cement Silty Clayey Sand:

The parameters $\delta_{ult}$ and $c$, as functions of pore RH and age, are shown in Equations (4.38) and (4.39), respectively. The parameters obtained from the regression of the measured tensile stress-strain curves and the parameters calculated from the pore RH and age using Equations (4.38) and (4.39) are compared in Figure (4.45) for cement silty clayey sand, which shows reasonable prediction accuracy.

\[
\delta_{ult} = -18558.39 \times \text{Age} - 435512.88 \times \text{RH} + 25743.12 \times \text{Age} \times \text{RH} + 665233.20
\]  
(4.38)

\[
c = -9.2106E - 06 \times \text{Age}^{1/2} - 5.6709E - 05 \times \text{RH}^{2} + 2.2066E - 10 \times \delta_{ult} + 4.1483E - 05
\]  
(4.39)
Figure 4.45: Comparison of Model Predicted and Measured Parameters in Stress-Strain Relationship for Cement Silty Clayey Sand using Statistical Method: (a) $\delta_{ult}$, and (b) $c$

(2) Cement Clay:

The parameters $\delta_{ult}$ and $c$ as functions of pore RH and age are shown in Equations (4.40) and (4.41), respectively. Figure (4.46) presents a comparison of the parameters obtained from the regression of the measured tensile stress-strain curves and the parameters calculated from the pore RH and age using Equations (4.40) and (4.41), which shows reasonable prediction accuracy.
\[
\delta_{\text{ult}} = -4060.88 \times \text{Age} + 1495775.48 \times \text{RH} + 13350.24 \times \text{Age} \times \text{RH} + 1943891.76
\] (4.40)

\[
c = 1.1233E - 04 \times \text{Age}^{1/2} - 7.1442E - 04 \times \text{RH} - 1.1734E - 9 \times \delta_{\text{ult}} + 1.1494E - 03
\] (4.41)

Figure 4.46: Comparison of Model Predicted and Measured Parameters in Stress-Strain Relationship for Cement Clay using Statistical Method: (a) \(\delta_{\text{ult}}\), and (b) \(c\)
4.5 MODELING OF SHRINKAGE CRACKING

Drying shrinkage stress in soil or cement soil depends on environmental conditions as well as the material shrinkage potential and the restraint applied on it. When shrinkage stress reaches the material’s tensile strength, shrinkage cracking occurs. Even if the material shrinkage potential is great, no shrinkage stress occurs if there is no restraint on the shrinkage potential, and therefore, no shrinkage cracking occurs. In other words, the restraint of the shrinkage potential is the cause of shrinkage stress. The restraint of the shrinkage potential is controlled by the service conditions of the material.

Based on the analyses presented in Sections 4.1 through 4.4, when soil or cement soil undergoes drying, the constantly changing pore RH gradient within the specimen (which can be predicted using the models discussed in Section 4.1, Modeling of Drying Process) will cause a constantly changing shrinkage potential gradient (which can be predicted using the models discussed in Section 4.2, Modeling of Drying Shrinkage Potential). If a restraint is applied to this shrinkage potential gradient, a constantly changing tensile stress gradient within the specimen will result. Meanwhile, the tensile stress-strain relationship (which can be predicted using the models discussed in Section 4.4, Modeling of Tensile Stress-Strain Relationship) keeps changing with the pore RH and/or the development of hydration. For a given quantity of the so-called ‘restraint of the shrinkage potential’, which is basically tensile strain, the tensile stress-strain relationship controls how much shrinkage stress is produced.

Finally, the tensile strength within the soil or cement soil is neither uniform nor constant. The tensile strength gradient (which can be predicted using the models discussed
in Section 4.3, Modeling of Tensile Strength) keeps changing with the pore RH and/or the development of hydration. By comparing the shrinkage stress to the tensile strength during the whole drying process, the shrinkage cracking can be predicted. Figure 4.1 illustrates this procedure schematically.

4.5.1 Restrained Shrinkage Test Results

In order to verify the procedure and models proposed in this study, fully restrained shrinkage tests were performed to measure the tensile load due to shrinkage and observe the shrinkage cracking. As introduced in Section 3.2.9, specimens that were 285 mm in length, 100 mm in width, and 25.4 mm in thickness, which reflect the same dimensions as for the free shrinkage tests, were wax-sealed on all sides, except the two largest surfaces for moisture evaporation, as shown in Figure 4.47 for the restrained shrinkage test setup. Figure 4.47 (left-hand side) also shows the shrinkage crack of a clay specimen after 18 hours of drying.

The specimens were then glued to the top and bottom plates with epoxy. The bottom plate was fixed. The top plate was connected to the loading piston, which was connected to a load cell. After the drying process was initiated, the loading piston was fixed. Therefore, both the top and bottom ends of the specimen were fixed. The load cell was used to measure the development of the tensile load due to shrinkage. This scenario simulates a fully restrained condition in the vertical direction. During the test, the room RH was recorded as environmental input for FE modeling.
Figure 4.48 shows the development of the tensile loads due to shrinkage for the clay and cement clay specimens. Table 4.13 lists the drying times and maximum tensile loads at the time of cracking, as obtained from the test results. Table 4.13 also provides the ambient RH during the test. The test results indicate that the clay specimen cracked after 0.5 hours (1800 seconds) of drying at the room RH of 14.1%, and the maximum tensile load at cracking was -89.6 N. The cement clay specimen cracked after 1.25 hours (4500 seconds) of drying at the room RH of 16.3%, and the maximum tensile load at cracking was -477.8 N.
Figure 4.48: Development of Tensile Load due to Shrinkage: (a) Clay and (b) Cement Clay

Table 4.13: Tensile Load and Drying Time at Shrinkage Crack Initiation

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Maximum Tensile Load (N)</th>
<th>Drying Time at Crack Initiation (s)</th>
<th>Ambient RH (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>-89.6</td>
<td>1800.3</td>
<td>14.1</td>
</tr>
<tr>
<td>Cement Clay</td>
<td>-477.8</td>
<td>4500.8</td>
<td>16.3</td>
</tr>
</tbody>
</table>
4.5.2 Finite Element Modeling Results and Laboratory Verification

The scenarios used in the restrained shrinkage tests were simulated in FE modeling, based on the measured environmental RH, measured material properties, and models proposed in this study. The measured material diffusion coefficients and the recorded environmental RH were input into the FE model, and the pore RH gradient evolution within the specimens was predicted. The pore RH gradient is related to the shrinkage potential by the ‘coefficient of moisture shrinkage’ model. By applying restraint conditions, the shrinkage stress could be calculated. During the modeling, the material property models, which are pore RH- and cement hydration-dependent, were embedded in ADINA using the User-Coded Material function.

The FE modeling results are shown in Figures 4.49 through 4.52. Figures 4.49 and 4.50 show the pore RH distribution and shrinkage stress distribution, respectively, of the clay specimen after 0.5 hours drying at 14.1% room RH. Based on the shrinkage stress distribution, the tensile load along the cross-section was integrated as -107.5N, which is 20.0% higher than the measured -89.6N. Figure 4.50 presents a comparison of the shrinkage stress distribution and the model-predicted clay tensile strength that corresponds to the pore RH at the specimen surface, as indicated by the red line in the figure. It should be noted that the FE model does not consider crack criteria, crack propagation, and stress relaxation. Therefore, cracking can be predicted only by manually comparing the shrinkage stress to the tensile strength. Figure 4.50 also shows that the shrinkage stress near the specimen surface is much larger than the tensile strength. This finding indicates that before this time, micro-cracks already could have initiated on the drying surface. Therefore, the
shrinkage stress on the specimen surface already could have relaxed. This phenomenon also explains the reason that the FE model-predicted tensile load is slightly higher than the measured load. Also, the fact that material stress relaxation is not considered in this study makes the FE model prone to overestimating the shrinkage stress and tensile load. However, due to the relatively short drying time, the effect of relaxation and therefore the overestimating is not obvious. Based on the aforementioned analysis, and considering the cracking time, tensile strength, and tensile load comprehensively, the FE modeling results are considered to be reasonable.

Figures 4.51 and 4.52 show the pore RH distribution and shrinkage stress distribution, respectively, of the cement clay specimen after 1.25 hours drying at 16.3% room RH. Based on the shrinkage stress distribution, the tensile load along the cross-section was integrated as -577.4N, which is 20.8% higher than the measured -477.8N. Figure 4.52 presents a comparison of the shrinkage stress distribution of the model-predicted cement clay tensile strength that corresponds to the pore RH at the specimen surface and the specimen age, which is shown by the red line in the figure. Similar to the analysis of the clay specimen, the FE modeling results for the cement clay are considered to be reasonable as well.

The developments of the tensile load predicted by FE modeling are compared with the measured tensile load developments in Figure 4.48 for clay and cement clay specimens. The comparison indicates a bit over-prediction of the FE modeling, but overall, the prediction is reasonable. Again, the over-prediction could be because that in the modeling,
the stress relaxation and stress release due to microcracks on the specimen surface are not considered.

Figure 4.49: Pore RH Distribution of Cement Clay: After 0.5 Hours Drying under Full Restraint of the Top and Bottom Ends for Clay
Figure 4.50: Vertical Shrinkage Stress Distribution: After 0.5 Hours Drying under Full Restraint of the Top and Bottom Ends for Clay
Figure 4.51: Pore RH Distribution of Cement Clay: After 1.25 Hours Drying under Full Restraint of the Top and Bottom Ends for Cement Clay
Figure 4.52: Vertical Shrinkage Stress Distribution: After 1.25 Hours Drying under Full Restraint of the Top and Bottom Ends for Cement Clay
The two case studies described in this section verify the shrinkage cracking modeling procedure and the material models proposed in this study. By measuring some of the fundamental material properties of soil or cement soil, such as the humidity isotherm, diffusion coefficient, SWCC, shrinkage potential, IDT strength, and tensile stress-strain relationships, etc., using the models and procedure proposed in this study, the pore RH distribution, evolution of tensile strength, development of the tensile stress-strain relationship, shrinkage stress distribution, and shrinkage cracking can be predicted using the FE method. Comparing the model results with the laboratory test results, the models and procedure used in the modeling show reasonable accuracy in term of predicting shrinkage cracking time, shrinkage stress, and shrinkage cracking.
CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

Based on the literature review, laboratory experimental results, data analysis and model development, and the FE modeling results of this study, the following conclusions and recommendations can be made.

5.1 CONCLUSIONS

(1) Based on the literature review, drying shrinkage is the major component of shrinkage cracking that is associated with the use of cementitiously-stabilized materials (CSM). Drying shrinkage is believed to be related directly to moisture loss. An accurate prediction of the distribution of the pore RH is critical in order to assess shrinkage properties and related cracking.

   Moisture loss from any porous material depends mainly on the surface area, the lengths of the moisture migration pathways, and the drying environment. Nonlinear moisture diffusion theory is commonly used to describe the drying process of porous materials.

   During drying, the drop in moisture content occurs first in the surface layers and much later in the core. The shrinkage gradient that is due to non-uniform moisture loss can result in non-uniform stress within the soil or cement soil. The stress gradient keeps changing during drying. Although some studies have been conducted on the drying shrinkage cracking of concrete, few research efforts have focused on unsaturated soil and cement soil. Even fewer studies consider the
shrinkage stress distribution and evolution of the stress profile during drying in shrinkage cracking modeling. However, if the moisture gradient due to drying is high, the shrinkage stress could localize on the exposed surface and decrease sharply with depth. Therefore, the evaluation of shrinkage cracking without considering the stress gradient and its evolution due to drying could be misleading and result in significant errors.

(2) The humidity isotherm test results indicate that, for the same ambient RH, clay can maintain a higher degree of saturation than silty clayey sand. Adding cement to silty clayey sand changes the humidity isotherm slightly, but the effect of adding cement to clay is less obvious.

(3) The diffusion coefficient is found to have a strong nonlinear relationship with the pore RH. Silty clayey sand has a larger diffusion coefficient than clay, probably due to the difference in their pore structure distributions. Adding cement appears to reduce the diffusion coefficient for both silty clayey sand and clay. This phenomenon could be due to the change in their pore structures during hydration. Cement hydration could result in the reduction of the total pore volume and may reduce the diffusion coefficient, thereby affecting the moisture diffusion property. However, the reduction of the diffusion coefficient due to adding cement is less significant for clay than for silty clayey sand. After adding cement, the diffusion coefficient curves of the cement silty clayey sand and cement clay used in this study almost merge.
(4) A practical approach for modeling and predicting the moisture distribution during drying at a constant temperature is proposed in this study. A pore RH-dependent nonlinear diffusion coefficient is used in the moisture distribution model. The governing constitutive differential equation for moisture diffusion was solved using the FE method. The modeling procedure and results were verified using experimental results for the different ambient RH conditions with reasonable accuracy. The FE method was especially convenient for predicting moisture loss under variable ambient RH conditions, which are common in engineering practice.

(5) Shrinkage potential, which is defined in this study as the maximum possible drying shrinkage of a material at a given pore RH when there is no restraint, can be expressed as a nonlinear function of pore RH. At the same pore RH, the clay and cement clay used in this study exhibited much more shrinkage potential than the silty clayey sand and cement silty clayey sand. This phenomenon may be due to the differences in the soil particles and pore structures that control the suction and shrinkage modulus. At the same pore RH, the cement soil exhibited significant less shrinkage potential than the corresponding soil (without cement stabilization). This result could be because the cement hydration changed the pore structure and increased the stiffness to withstand the deformation.

(6) Analogous to the coefficient of thermal expansion, in this study, the ‘coefficient of moisture shrinkage’ is proposed and defined as the amount of contraction (or expansion) per unit length of a material that results from a one-degree change (1%) in the pore RH. The coefficient of moisture shrinkage $\alpha$ was found to be a nonlinear
function of the pore RH or matric suction. At high pore RH and low suction values, the behavior of the soil and cement soil were found to be different. For soils (without cement stabilization), the coefficient of moisture shrinkage $\alpha$ kept increasing substantially with the increase in pore RH or decrease in suction. However, for the cement soils, the curves became flat at high pore RH or low suction values. Again, this behavior is assumed to be related to cement hydration, which changes the pore structure and stiffness of the soil. The hydration effect was more obvious at the early age of stabilization.

(7) This study provides experimental verification of the effective stress theory using IDT strength and UCS tests, both of which are widely used in engineering practice. The verification was extended to low water contents ranging from 2.56% to 18.9% and high matric suction levels, which are all typically used in engineering practice. A new method was proposed and experimentally validated in this study to predict the soil’s unconfined compressive and tensile strength values at any moisture content of interest by taking simple SWCC measurements and conducting UCS and IDT strength tests at two unsaturated moisture contents. The mechanical properties of the soils were related to the fundamental material parameters of $\alpha$ and $n$ of the SWCCs.

(8) Overlapped drying specimens were used to study the mechanical properties at different moisture contents and hydration rates of cement soil. Using the methods developed in this study, the moisture effect and hydration effect on the IDT strength of cement soil could be separated. As expected, no IDT strength growth
due to moisture loss occurred for the cured (control) specimens, because there was no moisture loss. Because the top specimens always had lower moisture contents than the corresponding middle and bottom specimens at the same age, the IDT strength growth due to moisture loss for the top specimens was always higher than that of the corresponding middle and bottom specimens at the same age.

After excluding the moisture effect on the IDT strength, the hydration effect on the IDT strength growth for the middle and bottom specimens was found to be very close to that of the cured (control) specimens. This finding indicates that the moisture loss during drying for the middle and bottom specimens had almost no effect on hydration. In other words, for the middle and bottom specimens, the moisture that remained after moisture loss was still sufficient for hydration. This result suggests that, in engineering practice, even without covering the top surface (like cement-stabilized pavement layers without curing), drying has little or no effect on the hydration of the cement soil below 2.54 cm (1 in.) of the drying surface, at least for the cement contents used in this study (5% for cement silty clayey sand and 12% for cement clay).

For the cement clay, the moisture effect and hydration effect on IDT strength were found to be more significant than for the cement silty clayey sand, probably because cement clay has finer particles and higher cement content. For the cement silty clayey sand, the IDT strength due to moisture was about half of the IDT strength due to hydration at the same age as the cement clay. Both of the strength values increased quickly within seven days, but the incremental increases
became much slower after seven days. For the cement clay, the IDT strength due to hydration increased quickly within seven days but slowed considerably after seven days, similar to the cement silty clayey sand. This outcome is due to the fact that the moisture that remained was not sufficient for cement hydration after seven days of drying. Therefore, the effect of hydration on IDT strength ceased. Unlike for the cement silty clayey sand, the IDT strength due to moisture kept increasing almost linearly up to 56 days for the cement clay. This outcome is assumed to be related to the different particle size distributions and pore structures of the cement silty clayey sand and cement clay. The results also indicate that at a spot near the drying surface, where shrinkage cracking typically initiates, the effect of moisture on tensile strength is significant and should not be neglected.

(9) Moisture loss and, therefore, reduction of the pore RH have a significant effect on the tensile stress-strain curves obtained from the IDT strength tests for the soil specimens. With the loss of moisture and therefore the reduction in pore RH, the IDT strength increased and the tensile stress-strain curves became steeper.

A two-parameter (\(\sigma_{ult}\) and c) model is proposed in this study to characterize the tensile stress-strain relationships for both soil and cement soil. The test results of the silty clayey sand and clay indicate good correlations between the two parameters \(\sigma_{ult}\) or c in the tensile stress-strain model and the pore RH. However, for different materials, the correlation model could take different forms (linear, power, or polynomial, etc.) and might not be able to be expressed using the same form with only different regression parameters.
Parameter $\bar{\epsilon}_{\text{ult}}$ represents the asymptote of the tensile stress-strain curve. Therefore, $\bar{\epsilon}_{\text{ult}}$ can be referred to as the ‘ultimate tensile strength’. Parameter $\bar{\epsilon}_{\text{ult}}$ was found to have the same linear correlation with IDT strength for all four types of soils used in this study, no matter whether it was silty clayey sand, clay, cement silty clayey sand, or cement clay. Parameter c showed good correlations with parameter $\bar{\epsilon}_{\text{ult}}$ for both soil and cement soil and, therefore, it also correlated well with IDT strength. 

(10) For the soil and cement soil, properties such as strength, stiffness, and shrinkage potential were found to be affected significantly by moisture content and/or cement hydration. As a result of drying and hydration, the tensile strength, stiffness, and shrinkage potential were neither uniform nor constant.

By measuring some fundamental material properties of the soil or cement soil, such as the humidity isotherm, diffusion coefficient, SWCC, shrinkage potential, IDT strength, and tensile stress-strain relationships, using the models and procedures proposed in this study, the pore RH distribution, evolution of the tensile strength, development of the tensile stress-strain relationship, shrinkage stress distribution, and shrinkage cracking could be predicted using the FE method. Comparing the model results with the laboratory test results, the models and procedure used in the modeling could be verified with reasonable accuracy in terms of predicting shrinkage cracking time, shrinkage stress, and shrinkage cracking.
5.2 RECOMMENDATIONS

(1) All the tests in this study were performed at the same temperature. Although the mechanical properties of the soils or cement soils may not be affected substantially by the temperature itself, the variation of the temperature could have a significant effect on the moisture diffusion and cement hydration and therefore have an effect on the shrinkage properties and shrinkage cracking of the soils and/or cement soils. Further study is needed to include the temperature effect in models for moisture distribution, shrinkage stress, and shrinkage cracking.

(2) This study does not consider material relaxation or creep. The modeling of the shrinkage cracking did not consider the possibility of a slow crack propagation stage before failure. Based on the literature, these effects could be substantial and need to be considered in future study.

(3) For engineering application purposes, the modeling procedure and tests of the material parameters used for shrinkage modeling need to be simplified. The findings are instrumental to investigating moisture behavior as a function of environmental conditions, the effect of moisture variation on the behavior of soils, and facilitating the implementation of stiffness-based performance prediction, e.g., to improve the Enhanced Integrated Climatic Model (EICM) in the Mechanistic-Empirical Pavement Design Guide (MEPDG).

(4) The mechanisms associated with shrinkage are complicated. It is commonly believed that moisture movement and diffusion are related to the pore structure. A fundamental multiscale study that relates the microstructure to the macro-properties
is needed to explain some phenomena in this study; e.g., why do the diffusion coefficient curves of the cement silty clayey sand and cement clay nearly merge after cement is added, and why do the IDT strength values that are due to moisture differ substantially for cement silty clayey sand and cement clay.

(5) Different types of soils, especially granular soils, need to be studied to expand the application of the findings. In this study, the models and procedure for shrinkage stress and shrinkage cracking were verified in the laboratory. Further in situ verification in the field is needed.
CHAPTER 6: REFERENCES


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