MAKING ON-LINE TRANSIENT STABILITY DECISIONS
WITH MULTI-LEVEL DECISION TREES

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MAKING ON-LINE TRANSPORT STABILITY DECISIONS
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Abstract

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During recent years, power systems are becoming more and more stressed, and thus more vulnerable to various contingencies. For specific instability cases, engineers can conduct off-line studies to devise preventive and remedial actions. This paper however, proposes a multi-level decision tree scheme, which not only evaluates the dynamic security of power systems as what other companion papers have done using decision trees, but also suggests ways of making real-time remedial control actions based on the proposed scheme. To achieve these goals, we collect pre-fault and post-fault data with the help of PMU and build the multi-level decision trees to make on-line decisions.

Keywords: PMU, Decision Trees, On-line, PMU, Transient Stability Prediction, Remedial Actions
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1. **INTRODUCTION**  
1.1. **Motivation and Objective**

Due to the unpredictable features of the real power systems and the limited amount of time required to make real-time decisions after contingencies, much of the work done in the area of transient stability is focused on making preventive and remedial control actions based on presumed loading and generation scenarios, which gives us limited information to make on-line decisions in real-time.

The advent of decision tree methods and other automatic machine learning techniques gives us inspiration to utilize them in power systems. The first use of decision tree method in power systems dates back to 1980s [1]. And subsequent applications of DTs to transient stability assessment of power systems can be found in [2]-[4]. The advantage of utilizing decision tree method is that it does not require tedious calculations and can make real-time decisions within seconds or even milliseconds.

After realizing the potential of using DT method in power systems, more and more people are trying to apply it to the area of transient stability. In [5], a real-time transient stability prediction method was proposed using the post-fault phase angles collected by PMU, and it was tested on the New England 39-bus test system under various operation points. And in [6], a decision tree assisted method is proposed to evaluate load shedding, which is considered as an effective corrective action when power systems face instability after certain contingencies. Besides the application in dynamic security assessment, the DT method was also used in the area of islanding in [7]-[8]. When deciding preventive control actions [9]-[10], DTs were built primarily to provide us with security boundaries of generations. So combining with an OPF
method, it suggested ways of preventive control actions; and similarly, combining with the security boundaries, corrective actions such as load shedding can also be determined.

This paper presents an approach for making on-line decisions using multi-level decision trees and PMU measurements. Unlike how DTs were built in many other papers, the DTs trained in this paper still have good accuracy for different distribution of generations within certain range of load levels. And besides the ability of dynamic security assessment, the proposed scheme also helps us determine on-line remedial control actions, and thus shows great potential for real-time use.

1.2. Power System Security Assessment and Trajectory Sensitivity

1.2.1. Power System Security

Power system security is concerned with the evolution of the system when a disturbance results in a change in system condition [11]. Although "small changes" do cause changes in system performance and can be studied as small signal security [12], system security is generally concerned with large changes, which are called contingencies. These may be faults due to tripping of lines, lightning flashover, failure of a major piece of equipment, etc. These cause relays to operate and cause a change in system configuration, such as loss of a component or in rare cases, cascaded tripping of lines.

When changes occur, the system undergoes changes in variables, and hopefully they will reach a new equilibrium. The mathematical analysis of the responses and computation of the new equilibrium constitutes security analysis. If the analysis computes only the post-disturbance steady state operating point (post-disturbance
equilibrium), it is caved static security assessment (SSA). If, after SSA is done, the operating state is found to be unacceptable from an engineering point of view (engineering constraints), corrective action can be taken. If these constraints are factored into the mathematical analysis, and generation is rescheduled by minimizing an objective function, then it is called security-constrained optimal dispatch [13]. Much research has been done in SSA and preventive control [14].

If the mathematical analysis involves computing the dynamic response and evaluating it for possible instability, then it is called dynamic security assessment (DSA). DSA has also been formally defined as an evaluation of the ability of a certain power system to withstand a defined set of contingencies and to survive the transition dynamically to an acceptable steady state condition.

1.2.2. Dynamic Security Problem

Security in power systems became an issue after the Northeast blackout in 1965 [15]. Since then a lot of research has been done investigating both static and dynamic aspects of security. While a lot of success has been achieved on the static front [16], such is not the case with dynamic security. Also, the structure of power systems has changed dramatically in recent years due to deregulation. In the deregulated environment, the existing transmission system often operates at its limit due to inadequate capacity and multilateral transactions. In addition, power systems must be operated to satisfy the transient stability constraints for a set of contingencies. In these situations dynamic security assessment plays a crucial role. Prior to the 1970s, transient stability assessment (TSA) had been studied based on off-line conditions. In this process the time domain analysis is carried out for each credible contingency
under a variety of conditions. Based on the response, the system is classified as stable or unstable. This is a time-consuming process. Also, in real time operation, the system conditions usually do not match the one studied off line. Hence, off line studies are of limited use. TSA needs to be studied based on on-line conditions under normal operation. The aim of on-line DSA is to assess the stability of the system to a set of predefined contingencies. These contingencies are user specified or are chosen automatically through some procedure such as a filtering process. For each contingency if the system is stable, it can also provide a security margin based on the technique used. For instance, if critical clearing time is computed, then $t_{cr} - t_{cl}$ is the margin. On the other hand, if the transient energy function (TEF) is used, then $V_{cr} - V_{cl}$ is the margin. The security margin can be used to provide the operators with guidelines to system security while at the same time maintaining economic operation. This is known as security-constrained optimization. An extensive report on DSA in North America is contained in [17].

Dynamic security assessment comprises the following main tasks: contingency selection/screening, security evaluation, contingency ranking, and limit computation. Attention is focused on both voltage transient and system stability. This then is followed by some form of preventive control. In the contingency selection step, a predefined contingency list is provided for analysis. This list is based on past operating experiences that may affect the system security. In the contingency screening step, a process of filtering contingencies is done that separates the contingencies into critical and noncritical ones. The critical ones are saved for further study, and the noncritical ones are discarded to reduce the computational burden.
There are several approaches reported in the literature for selecting the critical contingencies: intelligent-systems technique [18], single machine equivalent (SIME) technique [19], artificial neural networks (ANN) [20-21], and TEF technique [22-23]. Of these the TEF and SIME techniques are considered the most promising ones by several researchers. However the advances in artificial intelligence (AI)/ANN have made it a promising tool as well [24]. In security evaluation of the critical contingencies step, a detailed time domain simulation is done to study the effects of a contingency on power system security. In the contingency ranking step, contingencies are ranked in order of severity, so that the operators can alter and move the operating point to a more secure one if the contingency is considered severe. It will be desirable to have a computer algorithm for this purpose. In the limit computation step, stability limits such as power transfer between areas are computed and used as in economic dispatch.

The literature on preventive control is largely tied to the TEF method, namely, to enhance the stability margin as defined by the difference between critical energy \( V_{cr} \) and energy at clearing \( V_{cr} \). The TEF method [25-29] based on the Lyapunov's method, hybrid method, and second kick methods [30-32] combine numerical simulation and direct method. These methods suffer from the fundamental drawbacks of model limitation and the need to compute the controlling unstable equilibrium point (u.e.p.), which is quite complicated. The energy functions beyond the classical model are not analytical in form. Hence, in spite of extensive research, the TEF method is rarely used by the industry.
Trajectory sensitivity analysis is based on linearizing a system around a nominal trajectory and rather than around an equilibrium point. Therefore it is possible to determine directly the change in system trajectory with respect to changes in initial conditions or any parameters. Trajectory sensitivity was originally used in the area of control and parameter estimation; however more and more people are applying it in power systems recently.

In this dissertation we propose a new technique based on trajectory sensitivities (TS) [33-38] to DSA and remedial control. As it will be shown, this new technique has several advantages over all the other techniques:

1) No restriction on complexity of the model.
2) Extension to systems with discrete events in possible.
3) Information other than mere stability can be obtained.
4) Limits to any parameter in the system affecting stability can be studied.
5) Identification of weak links in the transmission network is possible.
6) Preventive strategies can be incorporated easily

Trajectory sensitivity is used to help us make remedial control actions. After certain contingency happens in power systems, the system encounters transient stability problem as well as voltage problems. The so-called TSI index is created based on trajectory sensitivity theory to help us determine where to put SVCs and help the system survive from such contingency.

1.3. Outline
This dissertation comprises 6 chapters and is outlined as follows. The difficulties of making on-line transient stability decisions, advantages of decision tree algorithm and introduction of trajectory sensitivities are described in Chapter 1.

In the next Chapter, the basic rules about decision trees are reviewed, fundamental theory about PMU and its application in power system is introduced.

In Chapter 3, we describe in detail about the structure of the proposed scheme, how the DTs are built off-line and used on-line and how the data from PMU is used in the scheme.

The theory of trajectory sensitivity is reviewed in Chapter 4, and trajectory sensitivity index is defined and used to help us make remedial control actions in power systems.

In Chapter 5, we show the scheme’s performance based on results tested on IEEE 39 bus system, WECC 179 bus system as well as real western system, and different remedial control actions are considered.

A summary of the findings in this dissertation will be provided in Chapter 6.
2. Decision Trees, Phasor Measurement Units and Its Application in Power System

2.1. The structure of DT

Among various machine learning algorithms, decision tree is an effective supervised machine learning tool to solve the classification problems in a high dimensional data space and is gaining increasing popularity in power systems. The underlying function for a DT is to form a predictive model from existing set of cases and apply it to predicting unseen cases within certain range of accuracy. It converts the complicated classification problem into a few “if-then” processes in terms of the thresholds of the input attributes or their linear combinations. More details about training a DT is included in [39] and is not discussed in this thesis.

The software WEKA is used to train and test DTs in this paper. Basically, it is a tool for classification with various machine learning algorithms besides decision tree [40]. But after comparing with other techniques, decision tree seems to be a relatively good method with higher accuracy. In order to train decision trees with better performance, a sufficiently large amount of cases should be created first. Each case consists of a number of pre-selected attributes, such as the active power outputs of generators, active power of loads, and phase angles of generators. By default, the last attribute is considered as the class/target variable, the attribute which should be predicted as a function of all other attributes. When all the training cases are sent to WEKA, it produces a DT model which is a complex mapping from all-but-one dataset attributes to the class attribute by identifying critical attributes that affect this objective most effectively and directly and setting thresholds for these attributes.
Starting from the top node of a DT, an optimal way is found each time to split each node into the two most “purified” (stable or unstable) subsets, creating two types of nodes, the “non-terminal node” with successors and the “terminal node” without any successor. For the non-terminal nodes, the node splitting procedure is applied repeatedly to build the corresponding sub-trees until the stop splitting criterion is met and thus a terminal node emerges. Each terminal node is labeled as stable or unstable, depending on the majority of the class attribute of the cases in the node. Ideally, when the class attribute of all cases in terminal nodes are purely stable or unstable, a DT is perfect with a 100% correct rate for the training data. Fig. 1 is a simple DT, where K and S are the thresholds for attributes A and B respectively.

Figure 1. Simple DT example

However, the correct rate for the training data is not the only criterion for evaluating how well the DT is built. WEKA can carry out cross-validation test and even evaluate a DT’s performance on an independent set of testing data. In the cross-validation test, a number of folds n is first specified, and the dataset is randomly
reordered and then split into n folds of equal size. During each iteration, one fold is used for testing and the other n-1 folds are used for training and building the DT. After each fold is used as the testing data once, the total number of testing cases that are misclassified by the n DTs is summed and the correct rate for the cross-validation test is provided to give an estimate of the DT’s ability to predict unseen cases. Needless to say, an independent set of testing data can also evaluate the DT’s ability to predict unseen cases, but for different testing data, the correct rate may change dramatically and to large extent depend on how the testing data is selected.

2.2. Phasor measurement units (PMU)

Synchronized phasor measurement units (PMUs) were first introduced in early 1980s, and since then have become a mature technology with many applications which are currently under development around the world. The occurrence of major blackouts in many major power systems around the world has given a new impetus for large-scale implementation of wide-area measurement systems (WAMS) using PMUs and phasor data concentrators (PDCs) in a hierarchical structure. Data provided by the PMUs are very accurate and enable system analysts to determine the exact sequence of events which have led to the blackouts, and help analyze the sequence of events which helps pinpoint the exact causes and malfunctions that may have contributed to the catastrophic failure of the power system.

2.2.1. Classical Definition of a Phasor

A pure sinusoidal waveform can be represented by a unique complex number known as a phasor. Consider a sinusoidal signal:
\[ x(t) = X_m \cos(\omega t + \varphi) \quad (2.1) \]

The phasor representation of this sinusoid is given by:

\[ \frac{X_m}{\sqrt{2}} e^{j\phi} = \frac{X_m}{\sqrt{2}} (\cos \phi + j \sin \phi) \quad (2.2) \]

Note that the signal frequency \( \omega \) is not explicitly stated in the phasor representation. The magnitude of the phasor is the RMS value of the sinusoid \( \frac{X_m}{\sqrt{2}} \), and its phase angle is \( \phi \), the phase angle of the signal in (2.1). The sinusoidal signal and its phasor representation given by (2.1) and (2.2) are illustrated in Fig.2.

![Figure 2. Phasor representation of a sinusoidal signal. (a) Sinusoidal signal. (b) Phasor representation](image)

Note that positive phase angles are measured in a counter-clockwise direction from the real axis. Since the frequency of the sinusoid is implicit in the phasor definition, it is clear that all phasors which are included in a single phasor diagram must have the same frequency. Phasor representation of the sinusoid implies that the signal remains stationary at all times, leading to a constant phasor representation. These concepts must be modified when practical phasor measurements are to be carried out when the input signals are not constant, and their frequency may be a variable.
2.2.2. **Phasor Measurement Concepts**

Although a constant phasor implies a stationary sinusoidal waveform, in practice it is necessary to deal with phasor measurements which consider the input signal over a finite data window. In many PMUs the data window in use is one period of the fundamental frequency of the input signal. If the power system frequency is not equal to its nominal value (it seldom is), the PMU uses a frequency-tracking step and thus estimates the period of the fundamental frequency component before the phasor is estimated. It is clear that the input signal may have harmonic or non-harmonic components. The task of the PMU is to separate the fundamental frequency component and find its phasor representation.

The most common technique for determining the phasor representation of an input signal is to use data samples taken from the waveform, and apply the discrete Fourier transform (DFT) to compute the phasor. Since sampled data are used to represent the input signal, it is essential that antialiasing filters be applied to the signal before data samples are taken. The antialiasing filters are analog devices which limit the bandwidth of the pass band to less than half the data sampling frequency (Nyquist criterion).

If \( x_k \{k = 0, \ldots, N - 1\} \) are the \( N \) samples of the input signal taken over one period, then the phasor representation is given by [41]:

\[
X = \frac{\sqrt{2}}{N} \sum_{k=0}^{N-1} x_k e^{-j k \frac{2\pi}{N}} \quad (2.3)
\]

The multiplier in front of the summation sign may need some explanation. Note that for real input signals, the components of the signal at a frequency \( \omega \) appear in the
DFT at $\pm \omega$ and are complex conjugates of each other. They can be combined, giving a factor of 2 in front of the summation sign in (2.3). The peak value of the fundamental frequency thus obtained is then converted to RMS value by dividing by $\sqrt{2}$. The DFT calculation eliminates the harmonics of the input signal. However, the non-harmonic signals and any other random noise present in the input signal leads to an error in estimation of the phasor. The error of estimation due to these effects has been discussed in the literature.

2.2.3. Synchrophasor Definition and Measurements

Synchrophasor is a term used to describe a phasor which has been estimated at an instant known as the time tag of the synchrophasor. In order to obtain simultaneous measurement of phasors across a wide area of the power system, it is necessary to synchronize these time tags, so that all phasor measurements belonging to the same time tag are truly simultaneous. Consider the marker $t = 0$ in Fig. 2 is the time tag of the measurement. The PMU must then provide the phasor given by (2.2) using the sampled data of the input signal. Note that there are antialiasing filters present in the input to the PMU, which produce a phase delay depending upon the filter characteristic. Furthermore, this delay will be a function of the signal frequency. The task of the PMU is to compensate for this delay because the sampled data are taken after the antialiasing delay is introduced by the filter. This is illustrated in Fig. 3

The synchronization is achieved by using a sampling clock which is phase-locked to the one-pulse-per-second signal provided by a GPS receiver. The receiver
may be built in the PMU, or may be installed in the substation and the synchronizing pulse distributed to the PMU and to any other device which requires it.

![Diagram of PMU signal processing](image)

**Figure 3.** Compensating for signal delay introduced by the antialiasing filter

The time tags are at intervals that are multiples of a period of the nominal power system frequency.

It should also be noted that the normal output of the PMU is the positive sequence voltage and current phasors. In many instances the PMUs are also able to provide phasors for individual phase voltages and currents [42].

Fig. 4 shows a typical synchronized phasor measurement system configuration. The analog input signals are obtained from the secondaries of the voltage and current transformers. The analog input signals are filtered by anti-aliasing filter to avoid aliasing errors. Then the signals will be sampled by the A/D converter. The sampling clock is phase-locked to the GPS time signal. The GPS receivers can provide uniform time stamps for PMUs at different locations. The phasor microprocessor calculates the values of the phasor. The calculated phasors and other information are transmitted to appropriate remote locations through the modems.
2.3. Application Of Phasor Measurements Units

2.3.1. Power System Monitoring

Early PMU Applications: When the first commercial PMUs became available, post-event monitoring was their only application due to the low availability and high cost of communications channels required for real-time monitoring, control, and protection applications. The first recorded wide-area measurements came from staged tests performed as part of the EPRI Parameter Identification Data Acquisition System (PIDAS) project in 1992. The tests consisted of opening and closing of the Klondike and Bonaire 500 kV lines at Plant Scherer in Georgia. PMUs were placed at Plant Scherer and five other locations in Tennessee, Georgia, and Florida [43]. Following the successful results of the initial PMU tests, small numbers of PMUs were placed in different locations throughout systems in the United States and other countries to
validate and correct system models by simulating records collected by PMUs. Essentially the PMUs at this stage were digital system disturbance recorders (DSDRs). As a DSDR, PMUs have collected interesting information that has expanded the monitoring application of the devices. One of these events is the data collected in Texas in July 1993 for the Comanche Peak load rejection test [44]. The frequency oscillations collected by PMUs revealed an electromechanical wave propagating through the system and easily observed in the frequency measurements. This and similar measurements in other parts of the country eventually led to the development of a frequency monitoring network, FNET. PMU and FNET data have shown that any significant loss of generation in a system will cause an electromechanical oscillation that will propagate through the system at slow speed. FNET data have shown that it is not only possible to detect these oscillations at residential voltage levels but also to use the frequency change to predict the amount of generation loss. It is also possible with some limitation to use the traveling wave to determine the location of the event [45]. In the 2003 Northeastern U.S. blackout and the 1996 U.S. West Coast blackouts, the PMU monitoring capabilities were essential for the quick and accurate postmortem analysis of the events. One of the recommendations from the United States–Canada Task Force on the 14 August 2003 blackout is to “require use of time-synchronized data recorders” to all utilities [46]. This and other recommendations led to the creation of the Eastern Interconnection Phasor Project (EIPP), now known as North American Synchrophasor Project (NASPI). With PMU installation cost ranging from 10 k to 70 K (depending on the utility, location, and availability of communication channels) placing PMUs in the optimum locations is one of the first
steps of a wide-area monitoring system [47]. PMU placement aimed at system monitoring is usually developed for full system observability. Concepts such as depth of observability have been developed to allow for a maximum utilization of data through a staged deployment of a monitoring system for full observability [48]. Other placement algorithms have been developed for specific applications such as Islanding and enhanced state estimation.

State Estimation: Since the 1965 Northeast blackout, state estimation has become a critical application function at energy control centers. Existing state estimation algorithms [41] use measurements of line flows and injections, both real and reactive power, to estimate all bus voltage angles and magnitudes. The complex bus voltages are the state of the system. The complex power flows in lines and the complex power injections at buses can be determined from the bus voltages and an accurate model of the network. Prior to synchronized phasor measurements, the state could not be measured directly but only inferred from the unsynchronized power flow measurements. This fact and the process of getting large numbers of measurements into the control center (a scan) forced early state estimators to make compromises that persist today and have an influence on how phasor measurements are integrated into existing state estimation algorithms. The most significant of these assumptions is that the system did not change during the scan—that the system was static. While scans have become quicker, the introduction of phasor measurements forces reconsideration of the static assumption. The direct integration of a few synchronized phasor measurements into an existing nonlinear estimator is straightforward but results in an estimator that has most of the limitations of the original conventional estimator. The
new estimator is still recursive and still has the assumptions required for the static state estimator. The issue of how to place the precisely timed PMU data into the scanned data can also introduce a skew. If an estimate could be formed with only PMU data, then the issues of data scan and time skew could be eliminated. The PMU data would be time tagged and the static assumption removed. Since the voltage and current measurements are linear functions of the system state, the estimation problem is simple a linear weighted least squares problem which requires no iterations. Given PMU data can be reported at rates as high as 60 times a second, a truly dynamic estimate would be available. An estimate of a dynamic system at an instant in time corresponding to the measuring instant would be obtained. Issues that must be addressed include the need for redundancy to eliminate bad data and determination of how many PMUs are required. Recognizing that a PMU in a substation would have access to line currents in addition to the bus voltage reduces the number of PMUs needed. With a large number of PMUs the redundancy issue is addressed. On the other hand, the smallest number of PMUs needed to indirectly measure all the bus voltages and the optimum PMU location to achieve this has been a subject of a number of papers [48]–[50]. With different approaches, treatment of zero injection buses, and differing assumptions about types of PMU used, the consensus is that the minimum number of PMUs is approximately one-third the total number of buses. A compromise between the full nonlinear and linear formulations is described in [51]

### 2.3.2. Power System Control

Prior to the introduction of real-time phasor measurements power system control was essentially local. Some subsystems such as machines were controlled with only
local signals. Other control action was taken based on local measurements and a mathematical model of the external world. External equivalents are such models. The introduction of phasor measurements offers the possibility of control based on measurement value of remote quantities. Latency of the phasor measurements is an issue, but the fact that many of the processes are in the 0.2–2.0 Hz range is encouraging. The phasor data will be time tagged so that control could be based on the actual state of the system a short time in the past. The frequencies involved are representative of transient stability, electromechanical oscillations, and certain overload phenomena. The frequency of measurements of the order of 15–60 Hz is certainly sufficient to handle the control task.

Wide-area measurements allowed the system to react to threatening situations without employing continuous feedback control. Monitoring of angles to detect possible instabilities and taking discrete switching controls in an attempt to militate against these events is a form of control made possible with synchrophasors [52], [53]. The training of the switching logic for such controllers has used simulation to train support vector machines [54] and decision trees obtained from data mining [53]. Incorporation of synchrophasor measurements into new system integrity protection schemes (SIPS) has also been described [55].

Early applications with continuous feedback were aimed at problems where a few phasor measurements could be applied to problems in which the control objective were global in nature: for example an HVDC controller may be called upon to damp electromechanical oscillations between two widely separated areas of a power system. Control of HVDC systems, excitation control, power system stabilizers, and FACTS
devices control [56]–[64] were approached in a similar fashion. Because of the nonlinearity of these problems, the power system dynamics were linearized and linear feedback based on various robust control techniques employed. Uncertain parameters were used to model unknown time delays [64], unmodeled dynamics were used to deal with the unmeasured portion of the system [65], and LMI techniques used to design a robust $H_\infty$ or $H_2/H_\infty$ controller [56], [66].

These techniques can be used to coordinate a number of local controllers of different types. For example, a collection of power system stabilizers, dc lines, and FACTS devices could be coordinated to provide damping for a collection of inter-area modes with a variation of the technique used for PSSs alone [56]. A representative scheme for such controllers is shown in Fig. 5. The local controllers retain their local input signals but have additional inputs from the wide-area controller. Selection of the PMU locations to be used and verification of the robustness of the overall scheme are important issues.

![Diagram](image.png)

Figure 5. A typical control based on remote synchrophasor measurements
3. Making On-Line Transient Stability Decisions With Multi-Level Decision Tree

3.1. Flowchart of Proposed Scheme

In our scheme, the load prediction is used to divide the 24-h horizon into many periods of time, within which, the total load and load patterns do not change dramatically. The flowchart for the processes of building DTs within each period of time is shown in Fig. 6. So the processes are repeated and DTs are updated in each period of time. There are several main parts in the flowchart:

A. Off-line DT

1) Load Prediction and OPs

We can divide the 24-hour horizon into many periods of time based on the load prediction. The length of each period of time is primarily on hourly basis, but may depend on how much the loads and load patterns change. During each period of time, we can get an estimate of the loads and generations, and thus pick some operation points (OPs) representing the changes of loads and generations. Generally speaking, the more OPs we have, the more accurate the DTs will be, but more time will be required to build the DTs.

2) Contingency Screening, Ranking and Remedial Actions

During each period of time, contingency screening and ranking are performed for exhaustive full time-domain transient stability simulations (N-1, N-2 and even N-k contingencies) and then the critical list of contingencies that can lead to out of step swings for those OPs in each period of time is identified. For each of the contingencies in the critical list, one or more remedial actions are found under the maximum loading condition.
3) Real-time Simulation

For each contingency under each OP, we should carry out the simulation for the cases with and without the corresponding remedial actions, creating all the cases needed to build the multi-level DTs.

4) Collecting Data and Building Multi-level DTs:

After obtaining all the cases, the attributes are selected for the software WEKA to build the multi-level DTs. The details of which attributes are selected and how the multi-level DTs are organized are introduced in next section.
B. On-line DT

1) Real-time OPs and Simulations:

During each period of time, the OPs for building off-line DTs are not real OPs in power systems, although they are quite close to the real-time OPs. So in the on-line part, new OPs can be continuously picked and simulation performed to create real-time cases.

2) New DTs

With the new real-time cases, attributes are selected and new files are sent to the off-line DTs, generating new DTs for on-line use through the software WEKA.

3.2. Multi-level DTs

3.2.1. Transient Stability Criteria

Before the DTs are trained, a database consisting of different cases needs to be created first. Each case is represented by a vector of system parameters and a final class attribute. The criterion for assessing the security of each operation point under certain contingency is Power swing-based stability margin in [67]. This method consists of three steps in determining the stability index:

1) Identify critical cluster of generators that become or will likely become unstable at the more stressed system condition.

2) Form parametric one-machine-infinite-bus (OMIB) equivalent. The parameters of this equivalent are constantly updated using simulation results of the full system.

3) Determine stability of the system and compute stability margin.
3.2.2. **Structure of multi-Level DT**

The structure of using the proposed multi-level DT to predict the stability of a specific case before and after the remedial action is shown in Fig. 7. As introduced previously, for each contingency in the critical contingency list, a remedial control action is found in each time period, so two DTs for cases before and after remedial control action can be built respectively. For on-line decision-making, when a specific contingency from the critical list occurs, the real-time case is created and sent to the first DT. If the real-time data is predicted as unstable, then the second DT is used to predict whether the system will be stable after proposed remedial control action.

3.2.3. **Correct Rate**

In Fig. 8, after sent to the first DT, the testing cases are divided into four categories, correctly/incorrectly classified stable cases, and correctly/incorrectly classified unstable cases. Then the correctly and incorrectly classified unstable cases are sent into the second DT, and divided into four similar subsets each. For the multi-level DT, the correctly classified cases among the testing cases consist of three parts:

1) The cases that are correctly classified as stable cases after using the first DT

2) Among the correctly classified unstable cases, the cases that are correctly classified by the second DT

3) Among the incorrectly classified unstable cases, the cases that are correctly classified as stable cases by the second DT
Figure 7. Structure of Multi-level DT

Figure 8. Correct Rate of Multi-level DT
4. Trajectory Sensitivity Analysis and Remedial Control Action

4.1. Introduction

Sensitivity theory in dynamic systems has a long and rich history well documented in the books by Frank [68] and Eslami [69]. It can be traced back to the work of Bode [70] in designing feedback amplifiers where feedback is used to cancel the effect of unwanted disturbances and parameter variations. In control theory after the advent of state space methods, considerable research has been done in sensitivity theory [71, 72]. While bulk of the work is in the area of linear time invariant (LTI) systems, the fundamental theory is applicable to nonlinear systems as well. Applications of sensitivity theory or more specifically trajectory sensitivity theory (TST) to nonlinear dynamic systems are few. The books of Tomovic [73], Tomovic and Vukobratovic [74], and Cruz [75] contain control-oriented applications.

The problem of sensitivity in dynamic systems is important for several reasons. The influence of parameter variations on system performance both at design and operational stage of a control system is important. As explained in the survey paper by Kokotovic and Rutman [76], trajectory sensitivity theory is applicable in three categories called $\alpha$, $\beta$ and $\gamma$ variations. The $\alpha$ category pertains only to parameter variations, whereas the $\beta$ category pertains to changes in initial conditions. In $\gamma$ category, the system order changes; hence, the theory of singular perturbation [77] plays an important role. There have been applications of sensitivity theory to power systems but mostly for linear systems such as eigenvalue sensitivity [78, 79]. From a stability point of view it has been applied for computing sensitivity of the energy
margin while using the transient energy function method \[80, 81\]. Applications of trajectory sensitivity are somewhat recent \[82-84\].

In this thesis, we restrict ourselves to the theory pertaining to parameter variations and how it is applied to different situations in the power system context. Specifically, we consider the nonlinear dynamics and use the concept of trajectory sensitivity. The application in linear system arises from the linearization of a nonlinear system around an equilibrium point. In trajectory sensitivity analysis, we linearize around a nominal trajectory and try to interpret the variations around that trajectory. It is a challenge to develop a metric for those variations and relate it to stability of the nonlinear system. In \[73, 74\] the authors hint at the intimate connection between trajectory sensitivity analysis and Lyapunov stability. In this chapter we outline the theory of trajectory sensitivity and reformulate it for power system applications.

### 4.2. Trajectory Sensitivity Theory for Differential Equation (DE) Model (Parameter Variations)

A continuous time nonlinear dynamical system can be represented as

\[
\dot{x} = f(x, \lambda), \quad x(t_0) = x_0 \tag{4.1}
\]

Where \(x\) is an \(n\)-vector of the dynamic state variables, and \(\lambda\) is an \(r\)-vector of system parameters of interest.

The sensitivity model for (4.1) with respect to the parameters is derived as

\[
\dot{x_\lambda} = f_x x_\lambda + f_\lambda, \quad x_\lambda(t_0) = 0 \tag{4.2}
\]

Where \(x_\lambda = \frac{\partial x}{\partial \lambda}\), \(f_x = \frac{\partial f}{\partial x}\), and \(f_\lambda = \frac{\partial f}{\partial \lambda}\). Note that \(f_x\) and \(f_\lambda\) are time varying matrices of order \(n \times n\) and \(n \times r\), respectively, and are evaluated along the
trajectories. Note that $x_\lambda$ is also a matrix of order $n \times r$ and is called the sensitivity matrix. The system trajectories and the trajectory sensitivities are numerically obtained by integrating (4.1) and (4.2) simultaneously with any numerical technique.

**Example**

A single machine infinite bus (SMIB) system (Fig. 9) is used to illustrate this concept. The nominal trajectory is defined for a three phase fault occurring at the machine bus at $t = 0$ and cleared at $t = t_{cl}$. The parameters can be the machine inertia, damping coefficient, or the mechanical input power.

Figure 9. Single machine infinite bus system

If the classical model is used for the synchronous machine, the system dynamics can be described as

$$M \ddot{\delta} + D \dot{\delta} = P_M, \quad 0 < t \leq t_{cl}, \delta(0) = \delta_0, \dot{\delta}(0) = 0 \quad (4.3)$$

$$M \ddot{\delta} + D \dot{\delta} = P_M - P_{em} \sin \delta, \quad t > t_{cl}$$

Where

$\delta$ : rotor angle

$\theta_s = 0$

$E$ : machine internal voltage magnitude
\( M \): machine inertia

\( D \): damping coefficient

\( P_M \): mechanical input power

\[
P_{em} = \frac{E V_s}{x_{d}'+x_l}
\]

If the parameter is chosen as \( P_M \), the sensitivity model corresponding to (4.3) is

\[
M \ddot{u} + D \dot{u} = 1 \quad 0 < t \leq t_{cl} , \quad u(0) = 0 , \; \dot{u}(0) = 0 \quad (4.4)
\]

\[
M \ddot{u} + D \dot{u} = 1 - (P_{em} \cos \delta)u \quad t > t_{cl}
\]

Where the parameter \( \lambda \) in this case is chosen as \( P_M \), and \( u = \frac{\partial \delta}{\partial P_M} \) is the sensitivity of rotor angle with respect to mechanical input power. One can also choose any other parameter such as line reactance, damping coefficient, etc. Note that in (4.4) \( \delta \) is a function of time, and hence, the sensitivity model is a linear time varying system.

4.3. Trajectory Sensitivity Theory for Differential Algebraic Equation (DE) Model

A more accurate description of the power system model is represented by a set of differential algebraic equation (DAEs) of the form:

\[
\dot{x} = f(x, y, \lambda) \quad (4.5)
\]

\[
0 = f(x) = \begin{cases} g^-(x, y, \lambda) & s(x, y, \lambda) < 0 \\ g^+(x, y, \lambda) & s(x, y, \lambda) > 0 \end{cases} \quad (4.6)
\]

And a switching occurs when \( s(x, y, \lambda) = 0 \).

In the above model, \( x \) are the dynamic state variable of the machines such as angles, velocities, etc. \( y \) are the algebraic variables such as load bus voltage
magnitudes and angles; and \( \lambda \) are the system parameters such as line reactance, generator mechanical power, and fault clearing time. Note that the state variables \( x \) are continuous while the algebraic variable \( y \) can undergo step changes at the switching instants.

The initial conditions for (4.5) - (4.6) are given by:

\[
x(t_0) = x_0 \tag{4.7}
\]
\[
y(t_0) = y_0 \tag{4.8}
\]

Where \( y_0 \) satisfies the equation

\[
g(x_0, y_0, \lambda) = 0 \tag{4.9}
\]

For compactness of notation, the following definitions are used

\[
\mathbf{x} = \begin{bmatrix} x \\ \lambda \end{bmatrix}
\]
\[
\mathbf{f} = \begin{bmatrix} f \\ 0 \end{bmatrix}
\]

With these definitions, (4.5) – (4.6) can be written in a compact form as

\[
\dot{x} = f(x, y) \tag{4.10}
\]

\[
0 = \begin{cases} 
g^-(x, y) & s(x, y) < 0 \\ 
g^+(x, y) & s(x, y) > 0 \end{cases} \tag{4.11}
\]

The initial conditions for (4.10) – (4.11) are

\[
x(t_0) = x_0 \tag{4.12}
\]
\[
y(t_0) = y_0
\]

4.3.1. Trajectory Sensitivity Calculation For Non-switching Periods
This section gives the analytical formula for calculating sensitivities $x_{x_0}(t)$ and $y_{x_0}(t)$ on the non-switching time intervals as discussed in [85]. On these intervals, the DA systems can be written in the form

$$\dot{x} = f(x, y)$$

(4.13)

$$0 = g(x, y)$$

(4.14)

Differentiating (4.13) and (4.14) with respect to the initial conditions $x_0$ yields

$$\dot{x}_{x_0} = f_x(t)x_{x_0} + f_y(t)y_{x_0}$$

(4.15)

$$0 = g_x(t)x_{x_0} + g_y(t)y_{x_0}$$

(4.16)

Where $f_x, f_y, g_x,$ and $g_y$ are time-varying matrices and are calculated along the system trajectories.

Initial conditions for $x_{x_0}$ are obtained by differentiating (4.12) with respect to $x_0$ as:

$$x_{x_0}(t_0) = I$$

(4.17)

Where $I$ is the identity matrix.

Using (4.17) and assuming that $g_y(t_0)$ is nonsingular along the trajectory, initial condition for $y_{x_0}$ can be calculated from (4.16) as

$$y_{x_0}(t_0) = -[g_y(t_0)]^{-1}g_x(t_0)$$

(4.18)

Therefore, the trajectory sensitivities can be obtained by solving (4.15) and (4.16) simultaneously with (4.13) and (4.14) using any numerical method with (4.12), (4.17), and (4.18) as the initial conditions. At switching instants, it is necessary to calculate the jump conditions that describe the behavior of the trajectory sensitivities at the discontinuities.
4.3.2. Sensitivity Calculations At Switching Instants

The discussion in this section closely follows [84]. A switching instant is signified by

\[ s(x(\tau), y(\tau)) = 0 \]  \hspace{1cm} (4.19)

Where \((x(\tau), y(\tau))\) is called the junction point and \(\tau\) is called the junction time.

At time \(\tau^-\) just before the switching instant, the state variables \(x\) and the algebraic variables \(y\) satisfy the equations:

\[ x^- = x(\tau^-) \]  \hspace{1cm} (4.20)
\[ y^- = y(\tau^-) \]  \hspace{1cm} (4.21)

Where

\[ 0 = g^-(x^-, y^-) \]  \hspace{1cm} (4.22)

And

\[ \lim_{\tau^- \to \tau} s(x^-, y^-) = s(x(\tau), y(\tau)) = 0 \]  \hspace{1cm} (4.23)

From (4.19), the following equation is obtained at \(\tau = \tau^-\)

\[ s_x d\bar{x}^- + s_y dy^- = 0 \]  \hspace{1cm} (4.24)

And from (4.22)

\[ g^- x d\bar{x}^- + g^- y dy^- = 0 \]  \hspace{1cm} (4.25)

Where all the partial derivatives are evaluated at \(\tau^-\). Again, from the assumption that \(g^- y\) is nonsingular, \(dy^-\) can be solved from (4.25) and is then substituted in (4.24) to get

\[ (s_x - s_y (g^- y)^{-1} g^- x)|_{\tau^-} = 0 \]  \hspace{1cm} (4.26)
Differentiating (4.20) with respect to \( x_0 \) gives

\[
\frac{dx^-}{dx_0} = \left( \frac{\partial x}{\partial x_0} + \frac{\partial x}{\partial \tau} \frac{d\tau}{dx_0} \right) \bigg|_{\tau^-} \tag{4.27}
\]

Also, from (4.13)

\[
\frac{\partial x}{\partial \tau} \bigg|_{\tau^-} = \dot{x}(\tau^-) = f\left( x(\tau^-), y(\tau^-) \right) \tag{4.28}
\]

Substituting (4.27) and (4.28) into (4.26) yields

\[
(s_x - s_y(g_y)^{-1} g_x^-) (x_{x_0}^- (\tau^-) + f(x(\tau^-), y(\tau^-))) \frac{d\tau}{dx_0} (\tau^-) = 0 \tag{4.29}
\]

Rearranging and taking the limit of (4.29) as \( \tau^- \to \tau \) gives

\[
\frac{d\tau}{dx_0} (\tau^-) = \tau_{x_0}^- = -\frac{(s_x - s_y(g_y)^{-1} g_x^-) x_{x_0}^- (\tau^-)}{(s_x - s_y(g_y)^{-1} g_x^-) f(x(\tau^-), y(\tau^-))} \tag{4.30}
\]

It is obvious that assumption of non-singularity of \( g_y^- \) is necessary in the above calculation process. Otherwise, the junction time \( \tau \) will be infinitely sensitive to the initial condition \( x_0 \).

The jump condition can be determined as follows. First, rewrite (4.27) as

\[
\frac{dx^-}{dx_0} (\tau^-) = x_{x_0}^- (\tau^-) + \frac{\partial x}{\partial \tau} \big|_{\tau^-} x_{x_0}^- \tag{4.31}
\]

Then substitute (4.28) into (4.31) and rearrange terms to obtain

\[
x_{x_0}^- (\tau^-) = \frac{dx}{dx_0} (\tau^-) - f\left( x(\tau^-), y(\tau^-) \right) \tau_{x_0}^- \tag{4.32}
\]

Similar results can also be obtained at \( \tau^+ \) just after the junction time as

\[
\frac{dx}{dx_0} (\tau^+) = x_{x_0}^+ (\tau^+) + \frac{\partial x}{\partial \tau} \big|_{\tau^+} x_{x_0}^+ \tag{4.33}
\]

And
\[
\frac{\partial x}{\partial \tau} \bigg|_{\tau^+} = \dot{x}(\tau^+) = f\left(\underline{x}(\tau^+), y(\tau^+)\right) 
\] (4.34)

Hence, from (4.33)

\[
x_{\underline{x}_0}(\tau^+) = \frac{dx}{dx_{\underline{x}_0}}(\tau^+) - f\left(\underline{x}(\tau^+), y(\tau^+)\right) \tau_{x_0} 
\] (4.35)

Where \(\underline{x}(\tau^+)\) and \(y(\tau^+)\) satisfy the equation

\[
0 = g^+\left(\underline{x}(\tau^+), y(\tau^+)\right) 
\] (4.36)

Now, using the above relationship, the sensitivities at just after the junction time can be obtained as follows. Because \(\underline{x}\) is continuous, its value at just before and just after the junction time are equal, \(\underline{x}(\tau^-) = \underline{x}(\tau^+)\).

Therefore,

\[
\frac{dx}{dx_{\underline{x}_0}}(\tau^-) = \frac{dx}{dx_{\underline{x}_0}}(\tau^+) 
\] (4.37)

The jump condition can be obtained from (4.32), (4.35), and (4.37) as

\[
x_{\underline{x}_0}(\tau^+) = x_{\underline{x}_0}(\tau^-) - \left(f^+ - f^-\right) \tau_{x_0} 
\] (4.38)

Where the following notation have been used in (4.38)

\[
f^+ = f\left(\underline{x}(\tau^+), y(\tau^+)\right) 
\] (4.39)

\[
f^- = f\left(\underline{x}(\tau^-), y(\tau^-)\right) 
\]

Differentiating and rearranging (4.36) gives

\[
y_{\underline{x}_0}(\tau^+) = -\left(\left(g^+_y\right)^{-1} g^+_x\right) \bigg|_{\tau^+} x_{\underline{x}_0}(\tau^+) 
\] (4.40)

Equations (4.38) and (4.40) together with (4.30) and (4.39) give the trajectory sensitivity jump condition at the switching instant \(\tau\).
Note that when the switching instant $\tau$ does not depend on the values of the state variable, i.e. $\tau_{x_0} = 0$, there will be no jump in trajectory sensitivities. Equation (4.38) becomes

$$x_{x_0}(\tau^+) = x_{x_0}(\tau^-) \quad (4.41)$$

When the trajectory sensitivities are known, the perturbed trajectories can be estimated by first-order approximation without redoing simulation as

$$\Delta x(t) \approx x_{x_0}(t) \Delta x_0 \quad (4.42)$$

$$\Delta y(t) \approx y_{x_0}(t) \Delta x_0 \quad (4.42)$$

### 4.4. Trajectory Sensitivity And Remedial Control Action

#### 4.4.1. Review the Application of Trajectory Sensitivity in Power Systems

Sensitivity theory has been applied in power system stability primarily to investigate the effect of parameter variations. In [86] sensitivity of the normalized energy margin with respect to the parameters is used to find the optimal value of the system parameters. In [87] the sensitivity of the energy margin with respect to total power supply and scheduled interchange power is used to calculate the maximum power supply capability of the system and the maximum interchange capability between areas of the system. The analytical expressions for sensitivities are obtained by using trajectory sensitivities. In [88] sensitivity of the energy margin with respect to generation power is used to estimate the generation limits, tie-line power flow limits, total power supply capability, and to apply it for preventive control. The sensitivity of the energy margin with respect to changes in generation, load, and network is proposed in [81] to calculate various stability limits of power systems.
Trajectory sensitivities are used in [83] to compute the influence of control signals at certain switching instant on the trajectories of the system. In [89] the sensitivity of the transient time margin (TTM), which is the difference between the critical clearing time and the actual clearing time, is used to estimate the system stability limits. Expression for TTM is obtained by first reducing the multi-machine system to a single machine equivalent system. In [90] trajectory sensitivities of tie line power with respect to input power are used to rank the severity level of contingencies and to identify the generator contributing to the stability of the system. Power system parameter estimation based on trajectory sensitivities and an optimization technique is proposed in [91]. In [92], on-line simulation with detailed dynamics is done, and the rank-ordered rotor angle separations are used to identify the critical machines before deriving a single machine equivalent. When the single machine equivalent is established, two stability margins are calculated for two different values of a chosen parameter and then extrapolated to obtain the critical value of that parameter. In [93] the normalized trajectory sensitivities of the post-fault system are used as indicators to show the stress level of the system, to rank the severity of contingencies, and to predict which machine is likely to go unstable. Since only the post-fault system is considered, the sensitivities are obtained with respect to initial conditions at the fault clearing time. Different values of \( t_{cl} \), give indications of the proximity of the system to the region of attraction. Hence the method cannot be applied to the system as a whole, i.e., for both \( t \leq t_{cl} \) and \( t \leq t_{cl} \) periods.

The trajectory sensitivities were used to analyze successfully the Nordel power grid disturbance on January 1, 1997 [85]. The sequence of events that occurred in the
Nordel system on that day was simulated, and the trajectory sensitivities were calculated to analyze the system stability. Sensitivities of relative rotor angle with respect to the line impedances were used to identify the line in the system which was most sensitive with respect to system stability. After identifying the critical line in the system, sensitivities of rotor angles with respect to this line impedance were used to indicate the effect of this line with respect to different generators in the system. Sensitivities to switching time of devices were used to indicate the significant contribution of the devices in the process of stabilizing the system after disturbances.

4.4.2. Remedial Control Action Based On Trajectory Sensitivity

For most of the cases where the system encounters angular stability problem after contingency, one simple way is to disconnect the unstable generators. However there are situations that the system will have both angular stability issues and low voltage problems, which we will see in the example of a real western system. In this case, putting SVCs in certain locations can be the possible remedial control action. But one question needs to be determined, where to put the SVCs? In this section, we will introduce Trajectory sensitivity theory, and then develop the so-called Trajectory sensitivity index to help us determine the best locations.

As discussed in the previous section, power system can be represented as:

\[
\dot{x} = f(x, y, \lambda) \quad (4.43)
\]

\[
0 = g(x, y, \lambda) \quad (4.44)
\]
Where \( x \) represents state variables, \( y \) represents algebraic variables, and \( \lambda \) represents system parameter. The sensitivity model for the system with respect to the system parameter is derived as:

\[
\dot{x}_\lambda = f_x x_\lambda + f_y y_\lambda + f_\lambda \tag{4.45}
\]

\[
0 = g_x x_\lambda + g_y y_\lambda + g_\lambda \tag{4.46}
\]

Where \( f_x = \frac{\partial f}{\partial x}, f_y = \frac{\partial f}{\partial y}, f_\lambda = \frac{\partial f}{\partial \lambda}, x_\lambda = \frac{\partial x}{\partial \lambda}, y_\lambda = \frac{\partial y}{\partial \lambda}, g_x = \frac{\partial g}{\partial x}, g_y = \frac{\partial g}{\partial y}, g_\lambda = \frac{\partial g}{\partial \lambda} \), then we have \( y_\lambda = -g_y^{-1}(g_x x_\lambda + g_\lambda) \) from (4.46). And substituting into equation (4.45), we get

\[
\dot{x}_\lambda = Ax_\lambda + B \tag{4.47}
\]

Where \( A = f_x - f_y g_y^{-1}g_x, B = f_\lambda - f_y g_y^{-1}g_\lambda \)

\( x_\lambda \) is called the sensitivity matrix. The system trajectory and trajectory sensitivities are numerically obtained by integrating (4.45), (4.46) and (4.47) simultaneously.

After obtaining the system trajectory and trajectory sensitivity matrix, we can also calculate \( y_\lambda \) from (4.46). The trajectory Sensitivity Index is defined as:

\[
S_N = \sqrt{\sum_{i=1}^{m} \left( \frac{V_i}{V_{base}} \right)^2 \left( \frac{\partial V_i}{\partial C} \right)^2} \tag{4.48}
\]

Where \( \frac{\partial V_i}{\partial C} \) means the change of voltage for the ith bus with respect to the change of shunts in certain location. The index is developed to indicate how the change of shunt in certain location will impact the voltages for the whole system when the system suffers from low voltage problem, and thus the larger the index is, the better the location will be for the SVC.
4.4.3. Finding Effective Location For SVC As Remedial Control Action

The theory is tested in the real western system, which consists of more than 10000 buses, 10000 transmission lines and 2000 generators. The total electricity generation is around 100GW. The contingency is on one 287-KV line in British Columbia. Totally 10 generators with more than 800MW will go unstable as shown in Fig. 10. Fifty-eight buses above 230-KV will have voltage problem including nine 500-KV buses as shown in Fig. 11.

Figure 10. Phase angle of generators
The trajectory sensitivity index is calculated for the SVC in different locations, and the results are compared in Table I.

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Sn</th>
<th>Stability after SVC</th>
<th>KV Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>81896</td>
<td>14.81</td>
<td>yes</td>
<td>500</td>
</tr>
<tr>
<td>81895</td>
<td>14.65</td>
<td>yes</td>
<td>500</td>
</tr>
<tr>
<td>50455</td>
<td>14.23</td>
<td>yes</td>
<td>287</td>
</tr>
<tr>
<td>50463</td>
<td>13.48</td>
<td>yes</td>
<td>500</td>
</tr>
<tr>
<td>50464</td>
<td>10.13</td>
<td>yes</td>
<td>500</td>
</tr>
<tr>
<td>50457</td>
<td>9.65</td>
<td>yes</td>
<td>230</td>
</tr>
<tr>
<td>81894</td>
<td>8.66</td>
<td>no</td>
<td>500</td>
</tr>
<tr>
<td>81858</td>
<td>6.37</td>
<td>no</td>
<td>500</td>
</tr>
<tr>
<td>81857</td>
<td>6.35</td>
<td>no</td>
<td>500</td>
</tr>
<tr>
<td>81864</td>
<td>5.88</td>
<td>no</td>
<td>230</td>
</tr>
</tbody>
</table>

It is observed from the table that the system is more stable when the TSI is larger. Two cases are compared in Fig. 12 and Fig. 13 when SVC is put in different locations and thus with different TSI. When the SVC is put in the location in Fig. 13 where TSI
index is too small, the system is still unstable, but in Fig. 12, when the TSI index is large enough, the system becomes stable.

Figure 12. Case 1 with Sn=13.48

Figure 13. Case 2 with Sn=6.37
5. Case Study

5.1. IEEE 39 bus system

We use the IEEE-39 bus system in Fig. 14 as the test system. This section only considers building the off-line DTs, and verifies the training and testing errors. Because each multi-level DT corresponds to each contingency in the critical contingency list, we choose two different contingencies that will make the system become unstable when no actions are taken, and find the corresponding remedial actions.

Figure 14. IEEE 39-bus system

5.1.1. Database of Cases

For the training data, the total active load that we choose ranges from 5910.5 MW to 6390.5 MW with the same load pattern and power factor, which represents the
possible loading scenario for a period of time by the load prediction. The increment of active load and generation is 20 MW, and the changes for the generation are among 4 of totally 10 generators, considering economic constraints. So this creates 3639 operation points. Time domain simulations using Power-tech Lab’s TSAT software [94] are conducted for all the operation points with respect to different contingencies and remedial actions.

All the N-1 contingencies occurring at 50% of each line are performed, and during each case, the circuit breaker trips the line after 6 cycles. The following two N-1 contingencies are chosen and analyzed for each operation point.

i. Ctg1: three-phase fault at 50% of the line between bus 9 and bus 39 with a clearing time of 6 cycles.

ii. Ctg2: three-phase fault at 50% of the line between bus 26 and bus 28 with a clearing time of 6 cycles.

Needless to say, there are numerous ways of making remedial actions that will make the system become stable. But for those that make the system stable under all 3639 operation points, the DTs do not exist, which means when a real-time operation point is within that range, the second DT is not needed, because the system will be stable when remedial actions are taken. This makes sense, but is not good for demonstrating the validation of making on-line decision based on multi-level DTs. So the remedial control actions that we choose are like this: for ctg1 and ctg2, the remedial control actions are both to disconnect the generator in bus 39, 4 seconds after the fault. When the remedial control actions are taken, there will be 3567 and 1763 stable cases out of totally 3639 cases for ctg1 and ctg2 respectively. The phase
angles of all the generators under ctg1/ctg2, with/without remedial action are shown in Fig. 15.

Figure 15. Phase angle of generators for different cases
5.1.2. DT’s Performance for Training Data

When remedial control actions are decided and time domain simulations are carried out, python scripts are used to create files that can be directly processed by WEKA. Each file contains 3639 cases and the attributes that are selected by each case consist of three types:

Type-1(pre-fault attributes): The pre-fault attributes include active power outputs of generators (denoted by G1 to G10), active power of loads (denoted by L1 to L19), totally 29 attributes.

Type-2(post-fault attributes):

i. Terminal voltages (denoted by V1 to V10) and phase angles (denoted by P1 to P10) of all the generators 3 cycles after the faulted line is tripped (for the first DT) or 3 cycles after the remedial control actions are taken (for the second DT).

ii. The phase angles (denoted by P1_2 to P10_2) of all the generators 9 cycles after the faulted line is tripped (for the first DT) or 9 cycles after the remedial control actions are taken (for the second DT).

iii. The phase angle velocities (denoted by P1_ac to P10_ac) of generators between 3 and 9 cycles. The phase angle velocity for generator X is calculated by formula:

\[ PX_{ac} = (PX_2 - PX)/0.1 \]  \hspace{1cm} (5.1)

There are totally 40 post-fault attributes.

Type-3(class attribute): There is 1 class attribute that is labeled as stable or unstable in each case.
WEKA is then utilized to build DTs for the training data. There are totally 4 files, corresponding to totally 4 DTs: First/ Second DT for ctg1/ctg2. The correct rate of all DTs for training data is shown in Table II, and the first DT for ctg2 is shown in Fig. 16.

![First DT for ctg2](image)

**Figure 16. First DT for ctg2**

**TABLE II. Correct Rate for Training Data**

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ctg1_DT1</td>
<td>99.97%</td>
</tr>
<tr>
<td>Ctg1_DT2</td>
<td>99.86%</td>
</tr>
<tr>
<td>Ctg2_DT1</td>
<td>99.97%</td>
</tr>
<tr>
<td>Ctg2_DT2</td>
<td>99.97%</td>
</tr>
</tbody>
</table>

**5.1.3. DT’s Performance for Testing Data**
As mentioned previously, two sets of tests are carried out to evaluate DT’s performance of predicting unseen situations, which is very important for making real-time decisions.

i. Cross-validation test: The original training dataset is randomly reordered and then split into 10 folds of equal size. Each time, one fold is used for testing and the other 9 folds are used for training and building the DT. The total number of testing cases that are misclassified is summed. Since in the proposed scheme, load prediction is used to divide the 24-hour horizon into many periods of time, during which the total load and load patterns do not change much, the real-time operation points may probably be within the range of total load in the training data and even with the same load patterns. So the cross-validation test is shown in Table III, giving us relatively practical estimate of the DT’s ability to predict unseen situations.

ii. Independent testing data: There are countless cases that can be used as testing data, but most of them cannot give us satisfactory results because no DT is so powerful to predict cases that have nothing to do with the training data. What a DT can do and is doing well is to predict power systems’ stability under operation points that are between or around those operation points listed in the training data. Before obtaining the independent testing data, 11 operation points that represent different loading scenarios between 5910.5 MW and 6390.5 MW are identified. Then for each of the 11 operation points, a perturbation of 40 MW for the load and generation is distributed between 4 generators and 8 loads, creating $11 \times 352 = 3872$ independent cases as testing
data. The correct rate of the multi-level DT for ctg1 and ctg2 is shown in Table IV, according to the definition of correct rate.

### Table III. Correct Rate for Testing Data

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ctg1_DT1</td>
<td>99.81%</td>
</tr>
<tr>
<td>Ctg1_DT2</td>
<td>99.48%</td>
</tr>
<tr>
<td>Ctg2_DT1</td>
<td>99.45%</td>
</tr>
<tr>
<td>Ctg2_DT2</td>
<td>99.61%</td>
</tr>
</tbody>
</table>

### Table IV. Correct Rate for Multi-level DTs

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ctg1</td>
<td>94.83%</td>
</tr>
<tr>
<td>Ctg2</td>
<td>97.88%</td>
</tr>
</tbody>
</table>

5.2. WECC 179 bus system

5.2.1. Database of Cases

The one-line diagram of the WECC 179 bus system is shown in Fig. 17. For the training data, the total active load that we choose ranges from 59585.41 MW to 61985.41 MW with the same load pattern and power factor. The increment of active load and generation is 80 MW, and the changes for the generation are among 4 of totally 29 generators. So this creates 7751 operation points.

The following N-1 contingency is chosen and analyzed for each operation point.

i. Three-phase fault at 50% of the line between bus 8 and bus 17 with a clearing time of 6 cycles.

After the contingency, all the 7751 cases are unstable, which means the 1st DT does not exist, because no matter how the generation patterns change within that range, the system is always unstable.
The remedial control action is to disconnect the generator on bus 4, 4 seconds after the fault. When the remedial control actions is taken, there will be 3122 stable cases out of totally 7751 cases.

Figure 17. System diagram of the WECC 179 bus system
5.2.2. DT’s Performance for Training Data

When remedial control actions are decided and time domain simulations are carried out, python scripts are used to create files that can be directly processed by WEKA. Each file contains 7751 cases and the way how attributes are selected is the same as that for IEEE 39 bus system. The correct rate of all DTs for training data is shown in Table V, and the second DT for ctg1 is shown in Fig. 18.

![Second DT for the Ctg](image)

Figure 18. Second DT for the Ctg
5.2.3. DT’s Performance for Testing Data

As for the IEEE 39 bus system, two sets of tests are carried out to evaluate DT’s performance of predicting unseen situations.

i. Cross-validation test: The original training dataset is randomly reordered and then split into 10 folds of equal size. Each time, one fold is used for testing and the other 9 folds are used for training and building the DT. The total number of testing cases that are misclassified is summed. The cross-validation test is shown in Table V.

### TABLE V. Correct Rate for Training Data

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ctg2</td>
<td>99.97%</td>
</tr>
</tbody>
</table>

ii. Independent testing data: The total active load that we choose ranges from 60065.41 MW to 61505.41 MW with the same load pattern and power factor. The increment of active load and generation is 40 MW, and the changes for the generation are among 4 of totally 29 generators. So this creates 6154 totally different operation points. The correct rate of the multi-level DT is shown in Table VI, according to the definition of correct rate.

### TABLE VI. Correct Rate for Testing Data

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ctg2</td>
<td>99.89%</td>
</tr>
</tbody>
</table>

5.3. Real Western system

5.3.1. Database of Cases
The proposed scheme is also tested in the real western system, which consists of more than 10000 buses, 10000 transmission lines and 2000 generators. The total electricity generation is around 100GW. One critical contingency and its remedial control action are identified, and the generation pattern is changed in the sub area to test how it will impact the stability for the system with and without remedial control actions.

The critical contingency happens at one end of certain transmission line in British Columbia, and in the base case, totally 12 generators with more than 800MW will go unstable as shown in Fig. 19. Fifty-nine buses above 230-KV will have voltage problem including nine 500-KV buses as shown in Fig. 20.

The remedial control action is to install SVC on one 500-KV bus in the sub area. The Phase angle of generators and voltage of buses are shown in Fig.21 and Fig.22, indicating the remedial control action is effective for the base case.
Figure 20. Voltages of buses

Figure 21. Phase Angle of Generators
Figure 22. Voltages of buses

The generation pattern is changed within five critical generators that will affect the stability of the system. For the training data, 320MW active power is changed among those generators with an increment of 20MW, and this creates 2572 operation points. Time domain simulations using software PSLF developed by GE [96] are conducted for all the operation points with/without remedial control action. Before remedial control action is taken, 1812 out of 2572 cases are unstable, and after the remedial control action, 1318 out of 2572 cases are unstable.

5.3.2. DT’s Performance for Training Data

When remedial control actions are decided and time domain simulations are carried out, python scripts are used to create files that can be directly processed by WEKA. Each file contains 2572 cases and the way how attributes are selected is the
same as that for IEEE 39 bus system. The correct rate of all DTs for training data is shown in Table VIII, and the first and the second DTs are shown in Fig. 23.

<table>
<thead>
<tr>
<th>DTs</th>
<th>Correct Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>DT1</td>
<td>99.92%</td>
</tr>
<tr>
<td>DT2</td>
<td>99.88%</td>
</tr>
</tbody>
</table>

5.3.3. DT’s Performance for Testing Data
As for the IEEE 39 bus system, two sets of tests are carried out to evaluate DT’s performance of predicting unseen situations.

i. Cross-validation test: The original training dataset is randomly reordered and then split into 10 folds of equal size. Each time, one fold is used for testing and the other 9 folds are used for training and building the DT. The total number of testing cases that are misclassified is summed. The cross-validation test is shown in Table IX.

ii. Independent testing data: 1585 operation points with different generation patterns are randomly selected as the independent testing cases. The correct rate of the multi-level DT is shown in Table X, according to the definition of correct rate.

<table>
<thead>
<tr>
<th>TABLE IX. Correct Rate for Testing Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTs</td>
</tr>
<tr>
<td>DT1</td>
</tr>
<tr>
<td>DT2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE X. Correct Rate for Multi-Level DTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTs</td>
</tr>
<tr>
<td>DT</td>
</tr>
</tbody>
</table>
6. Conclusions And Future Work

6.1. Conclusions

Machine learning algorithms, especially DT method is gaining more and more attention in power systems. This paper proposes a multi-level DT scheme, which is capable of not only predicting system stability in real-time situations, but also suggesting possible remedial control actions. Besides, the advantage of little time being required to make decisions by DTs when certain contingencies really happen in power systems is very appealing and has good potential for applying to real-time decisions. Off-line simulations show that the proposed approach can build DTs with high accuracy for both training and testing data. Simulations on both 39 bus systems and real western system considering different remedial control actions suggest that the scheme will achieve good results in practical use.

6.2. Future Work

Future work to be done includes modifying the algorithm of decision tree that is used in the software Weka and improve its performance for not only the training of the decision tree, but also predicting unseen situations. So far, our focus is to demonstrate that decision tree together with the multi-level scheme is capable of helping us make decisions like whether the system will be stable and whether the remedial control actions are effective when contingencies happen. It has shown great potential in terms of different good correct rates. In our test, we choose small increment for the generation, but in reality for large systems, sometimes we have to choose large increment in order to limit the amount of training data and thus time for off-line training. It may affect the performance for the decision tree, so there is
always the dilemma of whether to have less data or better performance. One possible way to alleviate the problem is that through carefully selecting the attributes and improving the decision tree algorithm, better performance can be achieved even with less training data.

Besides, more details needs to be considered when applying this scheme on-line. We have to develop a more systematic way of how to divide the day ahead load into several periods of time, and how to choose the operation points that not only best represents the system during each period of time but also works well with the training of decision tree. After all these are considered, the proposed scheme will not only show great performance to help us make decisions, but will be more practical to be applied on-line.
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