

QUANTITY DISCOUNTS, CAPACITY DECISIONS, AND CHANNEL
STRUCTURE CHOICES IN SUPPLY CHAINS

By

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To the Faculty of Washington State University:

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Abstract

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This dissertation takes a two-pronged approach into exploring best practices to optimize supply chains through procurement and distribution. I develop three models to help managers make better business decisions regarding their procurement policies when facing quantity discounts and their usage of different distribution channels when there are varying types and levels of retail competition.

Chapters Two and Three develop solution techniques to the shared resource allocation problem in the presence of quantity discounts and with the opportunity to expand or reduce the capacity level. This type of multi-item inventory system has two common ordering structures. The first considers each item independently, and thus the firm must keep its capacity level such that it can handle the worst-case situation where all items are ordered simultaneously. The second ordering structure links the timing of orders for all items to avoid a situation where all items are ordered simultaneously. I develop models to determine efficient procurement policies when facing these two ordering structures in Chapters Two and Three, respectively.

In Chapter Four I develop a model to help manufacturers determine their equilibrium distribution policy. My model prescribes whether they should sell their product directly through a manufacturer-owned retailer or online, or indirectly through an intermediary such as an independent retailer. Introducing asymmetries in product substitutability and brand equity, as well as different forms of competition, causes the equilibrium channel structure to vary widely. Depending on the levels of asymmetry and competition, there are multiple equilibrium channel structures, including a case where it is beneficial to utilize dual channels.

Outside of the modeling chapters, in Chapter Five I report results from two managerial surveys related to common practices of quantity discounts from both buyers' and sellers' perspectives. The benefits of these surveys are two-fold: first, they provide an up-to-date perspective into industry's usage of quantity discounts, and second, they identify research gaps that can help unify the interests of researchers and practitioners. A major area for future exploration is the development of quantity discount training scenarios. The survey results indicate a lack of fundamental understanding of the basic guidelines associated with quantity discounts.

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Dedication

This dissertation is dedicated to my mom and dad, who provided
unwavering support throughout this journey.

CHAPTER ONE

INTRODUCTION

There are many definitions for supply chain management, but one of my favorites is: “A set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements” (Simchi-Levi et al. 2008). There are countless approaches to attempt to optimize a supply chain: some approaches have been studied extensively, others are still in their infant stages of analysis and understanding, and some have yet to be explored. Since the 1980s, supply chain management has been a hot topic in operations management research, and given the multi-faceted nature of the topic, I envision it will continue to be a vibrant research field.

This dissertation approaches the general theme of supply chain management using a two-pronged approach. First, I analyze the procurement policies of firms in the presence of quantity discounts. Quantity discounts remain omnipresent in both business-to-business (B2B) and business-to-consumer (B2C) transactions. Munson and Rosenblatt (1998) report that a majority of B2B transactions involve some form of quantity discounts. Additionally, as consumers, we cannot go into town without noticing quantity discounts—whether we “upsized” our fast food lunch, make sure to get our punch on our punch card at our favorite local coffee shop, or overload on items at Costco to get a cheaper unit cost. Despite this overwhelming popularity of quantity discounts, there is still evidence that firms are not optimizing their procurement and sales policies to minimize costs either locally or within their supply chain. My dissertation focuses on understanding the needs of practitioners with respect to quantity discounts and seeks to provide applicable and beneficial quantity discount models for real-world situations.

My second approach to better understand how to optimize supply chains is through the determination of appropriate distribution policies. I look downstream from the manufacturer's perspective, where every competing manufacturer must decide which channel(s) to use for the distribution of its products. The manufacturer can choose to vertically integrate and thus sell the product directly. Alternatively, the manufacturer can sell the product through an intermediary, such as an independent retailer, who then resells the item to the end consumer. Lastly, the manufacturer can take advantage of *both* distribution channels and utilize a dual channel setup. Through the introduction of multiple forms of competition and asymmetry between the competing supply chains on brand equity and product substitutability, I help manufacturers determine the ideal distribution plan.

Chapter Two entails my first dissertation essay in which I develop a mathematical model to solve the traditional shared resource allocation problem in the presence of quantity discounts with the ability to incorporate the capacity level as a decision variable. My model is the first of its kind to be able to approximate any functional form of capacity cost (associated with a capacity expansion or potentially a reduction) through the use of a piecewise-linear approximation. This allows me to simultaneously solve for the appropriate order quantity for each item and the shared resource capacity level in the presence of all-units and incremental quantity discounts. The model is able to efficiently and accurately solve large-scale problems with up to 15,000 items in under an hour. This has potential to be an invaluable tool for manufacturers or retailers who routinely receive quantity discounts on their orders but also face capacity restrictions. These can be special restrictions (e.g., warehouse square footage) or budgetary restrictions (e.g., available cash).

Chapter Three presents an extension of Chapter Two. In Chapter Two, the mathematical model solution applies a commonly used ordering structure for which each item is treated independently; thus, the capacity level must be able to handle a worst-case scenario where all items are ordered simultaneously. An alternative ordering structure links all the items' replenishment points through a common replenishment cycle. For example, the firm could reorder inventory weekly. To prevent the worst-case scenario from Chapter Two, the replenishment point of each item is phased within the cycle; therefore, some items are ordered Monday, some Tuesday, etc. In Chapter Three I develop a mathematical model to solve the same traditional shared resource allocation problem, but utilizing a new ordering structure. Again, I allow for capacity expansion (or reduction) and consider all-units quantity discounts. In the numerical analysis, I compare the two ordering structures. In general, the independent cycle approach outperforms the common replenishment cycle approach, but the common replenishment cycle approach is valuable as it mirrors the concepts of a periodic review system seen in inventory management, which is covered in most introductory operations management texts (e.g., Heizer and Render 2014). The primary advantage of such a system is the convenience factor of placing orders and receiving goods on a fixed and repeatable schedule. The common replenishment cycle approach allows for this type of setup, and the convenience factor likely outweighs the added cost for many managers who do not want to monitor their inventory replenishment needs constantly.

Chapter Four incorporates my third dissertation essay, which helps manufacturers determine in which channel(s) to distribute their products. More specifically, the model prescribes if they should sell their product directly through a manufacturer-owned retail store or through an online outlet, or if they should sell their product through an intermediary, such as an

independent retailer. Each manufacturer may also choose a mix of both, or a dual channel strategy where they sell a proportion of their products through a direct channel and the rest through an indirect channel. My essay is unique in that it is the first of its kind to introduce asymmetry among the characteristics of each supply chain and its products. I determine the equilibrium channel structure for each manufacturer in a duopoly where the manufacturers compete via different forms of competition and with varying levels of asymmetry in brand equity and product substitutability. All three factors play an important role in determining the equilibrium channel structure for each manufacturer. Intuition leads us to believe that, if possible, selling directly and eliminating the “middle man” is preferable, but it turns out that under intense competition at the retail level, manufacturers benefit from removing themselves from that competition by selling and profiting through their sales to an independent retailer. This study demonstrates the need to incorporate these asymmetries and different forms of competition into the channel structure decision facing many manufacturers.

Chapter Five summarizes the findings from two managerial surveys regarding the common practices associated with quantity discounts from both buyers’ and sellers’ perspectives. The aim of the surveys is to update the common practices and uses of quantity discounts in industry to help motivate and identify future research opportunities. Recently, academic research on quantity discounts and the needs of practitioners who face quantity discounts seem to be going in different directions. This essay and these surveys intend to help identify ways to bridge the gap between academia and industry. Additionally, I hope to gain a better grasp of practitioners’ knowledge of some basic quantity discount guidelines as well as the extent of their use of profit or cost analyses in their decision making in the presence of quantity discounts. The results are informative and provide numerous potential avenues for future research. One avenue

is the development of quantity discount training scenarios. A major finding in my study is that purchasing and sales managers do not fully grasp the basics of quantity discounts. For example, when facing all-units quantity discounts it is *often* beneficial to order at a price breakpoint; meanwhile, when facing incremental quantity discounts it is *never* beneficial to order at a price breakpoint. A surprising number of survey respondents do not follow these guidelines, perhaps indicating a lack of institutional knowledge about quantity discounts. Ideally, these future scenarios can help us better understand the rationale behind decisions made by practicing managers in the presence of quantity discounts.

CHAPTER TWO

SHARED RESOURCE CAPACITY EXPANSION DECISIONS FOR MULTIPLE PRODUCTS WITH QUANTITY DISCOUNTS

Abstract

I analyze the traditional shared resource capacity allocation problem by incorporating the existence of quantity discounts for multiple products and converting the capacity level into a decision variable. By utilizing a piecewise-linear approximation for capacity cost, the algorithms can generate solutions regardless of the functional form of capacity cost (i.e., concave or convex). The model can accommodate both all-units and incremental quantity discounts, or even a mixture of both. I utilize numerical examples and sensitivity analysis to understand the key factors that influence the capacity expansion decision and the performance of the algorithms. The algorithms can incorporate simultaneous lot-sizing decisions for thousands of products in reasonable solution time.

2.1 Introduction

With ever-shrinking margins in today's marketplace, proper inventory management has become more vital than ever before. Purchasing managers typically procure many products simultaneously, often subject to an inventory-based resource constraint (e.g., warehouse space). Basic inventory models, such as the economic order quantity (EOQ) model, assume that procurement decisions relate to a single product with no constraints on the order size. These basic models are well-known and easy to solve analytically. However, the addition of a common resource constraint, multiple products, and quantity discounts significantly complicates the determination of optimal order quantities.

Inventory-based common resource constraints typically come in one of two forms: warehouse constraints or financial constraints. A warehouse can be constrained in terms of volume (or square footage), weight, or number of units. The most common warehouse constraint is square footage, as the drive toward "lean" in many industries today severely limits available warehouse space. Financial constraints might arise, for example, from a maximum inventory value that insurance covers or from a credit line available to purchase inventory. A natural, but often forgotten, question is, "Would it be valuable for our firm to increase the capacity of our common resource?" Too often managers may simply assume that capacity is fixed, but in reality, more warehouse space may be available for purchase, or unused warehouse space may be convertible for lease.

Any expansion decision comes with a cost, so what is the benefit? The vast majority of businesses have opportunities to receive quantity discounts for at least some of their purchased products (Munson and Rosenblatt 1998). By increasing the resource capacity, firms open up more opportunities to take advantage of quantity discounts. The model in this chapter

accommodates both of the common quantity discount forms: *all-units* and *incremental* (Hadley and Whitin 1963). Specifically, I address the following research question: When a firm faces all-units or incremental quantity discount schedules for multiple products in a common resource-constrained inventory system, how much capacity should exist, and how many units of each item should the firm order given that capacity limitation?

The remainder of this chapter is organized as follows. In Section 2.2, I review relevant literature on quantity discounts and the capacitated common resource problem. In Section 2.3, I introduce the baseline model and algorithm when facing all-units quantity discounts. This is followed in Section 2.4 by an extension of the baseline model to incorporate capacity *reduction* decisions (leading to potential capacity cost savings) along with capacity expansion decisions. The model and solution algorithm are then modified to handle incremental quantity discounts in Section 2.5. In Section 2.6, I describe numerical studies that test the performance of the algorithms along with sensitivity analysis that identifies the key parameters influencing the capacity expansion decision. Finally, in Section 2.7, I conclude the study and identify directions for future research.

2.2 Literature Review

Several hundred academic articles on quantity discounts have appeared. Benton and Park (1996) as well as Munson and Rosenblatt (1998) summarize the work published through the turn of the century and provide an overview of the landscape of the quantity discount literature. A steady stream of quantity discount papers has continued since then (e.g., Rubin and Benton 2003, Munson and Hu 2010, Manerba and Mansisi 2012, Hammani et al. 2014) and are summarized in an updated and expansive review of the quantity discount literature by Munson and Jackson

(2014). This chapter draws from the current quantity discount literature along with the capacitated common resource problem literature to determine both the correct common resource level and order quantities for each item.

Previously published models that address the common resource capacity problem have three key characteristics: pricing structure, ordering structure, and capacity flexibility. Table 2.1 provides a summary of prior literature and the characteristics of each article. There are two common pricing structures: fixed pricing (i.e., no quantity discounts) and variable pricing with quantity discounts. Hadley and Whitin (1963) and Johnson and Montgomery (1974) were among the first to analyze the fixed pricing (undiscounted) problem. They use an ordering structure whereby each product has an independent cycle length, i.e., time between orders. This type of ordering structure forces the firm to prepare for the worst-case scenario when all products are ordered at once and arrive simultaneously. Lagrangian relaxation is the most popular solution technique to solve the undiscounted common resource problem with independent cycle times. Rosenblatt (1981) and Rosenblatt and Rothblum (1990) modify the ordering structure to have every product on a fixed cycle length. The replenishment points are then phased within the fixed cycle length. This technique lowers the maximum inventory level by eliminating the possibility of the worst-case scenario that independent cycle lengths might produce.

There are several papers dedicated to solving the capacitated common resource problem with quantity discounts, but all of them assume that capacity is fixed. Pirkul and Aras (1985) wrote the seminal paper analyzing all-units quantity discounts, which solves the problem with each product having an independent cycle length. The authors introduce a Lagrangian relaxation on the capacity constraint to develop a lower bound on the original objective function. Once the Lagrangian problem is solved, a bisection method finds a near-optimal solution to the original

problem. Rubin and Benton (1993) extend the research area by introducing multiple constraints (e.g., budget and space). Güder and Zydiak (2000) combine the previous work done on the common resource-constrained problem with quantity discounts and the approach taken by Rosenblatt (1981) by examining the problem with a fixed cycle length for all products. All of the prior models incorporate the typical EOQ assumptions, one of which is constant and known demand for each product. Minner and Silver (2007) introduce stochastic demand into the resource-constrained problem, and Zhang (2010) expands their research by adding all-units quantity discounts. Shi and Zhang (2010) formulate the Zhang (2010) problem as a mixed integer nonlinear program and solve it using Lagrangian relaxation.

Table 2.1: Summary of prior literature.

Reference	Pricing Structure ^a	Ordering Structure ^b	Capacity Flexibility ^c
Güder et al. (1994)	I	I	F
Güder and Zydiak (1997)	A	NS	F
Güder and Zydiak (2000)	A	F	F
Hadley and Whitin (1963)	F	I	F
Haksever and Moussourakis (2005)	F	F,I	F
Haksever and Moussourakis (2008)	I	F	F
Hall (1988)	F	F	V
Johnson and Montgomery (1974)	F	I	F
Minner and Silver (2007)	F	I	F
Moussourakis and Haksever (2008)	A	F,I	F
Pirkul and Aras (1985)	A	I	F
Rosenblatt (1981)	F	F,I	F
Rosenblatt and Rothblum (1990)	F	F	V
Rubin and Benton (1993)	A	I	F
Rubin and Benton (2003)	I	I	F
Zhang (2010)	F	NV	F

^a A = all-units, F = fixed, I = incremental

^b F = fixed, I = independent, NS = non-stationary, NV = newsvendor

^c F = fixed, V = variable

Fewer articles have addressed the incremental quantity discount case. Güder et al. (1994) modify the approach from Pirkul and Aras (1985) for incremental discounts. Their heuristic assumes that each product has an independent cycle length, and it finds a near-optimal solution through the use of Lagrangian relaxation. Rubin and Benton (2003) extend prior research by allowing multiple constraints (i.e., financial and space limitations). Their methods are adopted from their earlier paper (Rubin and Benton 1993) that analyzes this problem when facing all-units discounts. Their incremental discount article also invokes a Lagrangian relaxation, coupled with partial enumeration, to solve for near-optimal order quantities.

Hall (1988) and Rosenblatt and Rothblum (1990) were among the first to analyze the common resource capacity as a decision variable. Recently, Ghiami et al. (2013) and Dye et al. (2007) have developed models that allow the opportunity to rent extra warehouse space when the inventory level exceeds the capacity of the original warehouse. To the best of my knowledge, no literature to date has introduced quantity discounts into the common resource capacity problem when the common resource capacity is a decision variable. This chapter fills that research gap.

I develop algorithms to determine efficient solutions for a constrained, multi-product inventory problem that incorporates quantity discounts (both all-units and incremental) and the resource capacity as decision variable. The main contribution of this chapter is the consideration of quantity discounts simultaneously with a dynamic inventory-based resource constraint. A second key contribution of this chapter is the ability of the solution algorithm to incorporate any functional form of capacity cost. Previous studies (e.g., Rosenblatt and Rothblum 1990, Hall 1988) have limitations on the functional form of capacity cost (e.g., convex or linear, respectively). This unrestricted capacity cost model allows, among other things, the capacity cost to incorporate annualized economies of scale benefits seen in larger expansions.

2.3 Model

The baseline model simultaneously solves for the appropriate common resource capacity level and the order quantity for each item given the capacity limitation when facing all-units quantity discounts (for incremental discounts, see Section 2.5). Recall that the common resource could be an inventory-based resource (e.g., warehouse space) or a financial-based resource (e.g., a credit line for inventory). In an unconstrained system, quantity discounts incentivize purchasing managers to purchase larger quantities, but that potentially requires a larger capacity. This model attempts to identify the ideal balance between the purchasing cost savings from quantity discounts and the capacity cost increase. I invoke a piecewise-linear approximation of capacity cost to determine efficient order quantities and the appropriate capacity level.

Consistent with prior literature, this model uses many traditional assumptions to assist with analytical tractability. I invoke the typical EOQ assumptions of deterministic and independent demand for each item, constant demand over time, no backorders, and instantaneous replenishment (i.e., infinite production rate and zero lead time). My model assumes that each item has an independent cycle length (or time between orders). Under this ordering structure, the capacity level must be able to accommodate the worst-case scenario where all items are ordered simultaneously, resulting in the highest possible inventory levels. I assume that the resource capacity is a decision variable that can be reduced (reducing capacity cost) or expanded (increasing capacity cost). Without loss of generality, from this point forward I assume that the constrained common resource is the space available in a warehouse (other shared resource constraints would be modeled similarly). Finally, I assume that the capacity cost function is monotonically non-decreasing with respect to capacity level.

2.3.1 Notation

Indices:

i item to be purchased, $i = 1, 2, \dots, I$

m segment of the piecewise-linear approximation of capacity cost, $m = 0, 1, 2, \dots, M$

Parameters:

D_i annual demand for item i

S_i setup (or ordering) cost for item i

h_i annual holding cost as a percentage of purchase price for item i

k_i amount of the common resource used per unit of item i

K_0 initial resource capacity level

y_m capacity breakpoint in the piecewise-linear approximation of capacity cost between segments $m - 1$ and m

V_m capacity cost associated with a capacity level of y_m

b_m marginal capacity cost for each additional unit of capacity in segment m

Decision Variables:

K resource capacity

Q_i order quantity for item i

Cost Equations:

$H(Q_i)$ annual holding cost for item i

$P(Q_i)$ annual purchasing cost for item i

$G(K)$ annual capacity cost associated with having capacity level K

2.3.2 Modeling Preliminaries

The goal is to determine the optimal combination of resource capacity level and order quantities to minimize annual setup, inventory holding, purchasing, and capacity costs when facing quantity discounts.

For an all-units quantity discount schedule with $J + 1$ prices, I define q_{ij} as the j^{th} price breakpoint in the price schedule for item i , $j = 0, 1, \dots, J$ ($q_{i0} = 0$, $q_{i(J+1)} = \infty$), and p_{ij}^A as the per-unit price for all units purchased when $q_{ij} \leq Q_i < q_{i(j+1)}$. The general objective function is:

$$Z = \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + G(K), \quad (2.1)$$

where $q_{ij} \leq Q_i < q_{i(j+1)}$, $H(Q_i) = (Q_i / 2)(h_i p_{ij}^A)$, and $P(Q_i) = p_{ij}^A D_i$. Note that the objective function (2.1) is not continuous due to the existence of all-units quantity discounts. If the problem did not include quantity discounts and if $G(K)$ were convex, then the objective function would be continuous and convex—hence solvable using first order conditions.

Rosenblatt and Rothblum (1990) and Hall (1988) assume that $G(K)$ is convex. However, the shape of $G(K)$ is dependent upon the interpretation of the resource capacity cost. Both articles define $G(K)$ as the yearly cost of maintaining a capacity level K . Hall (1988) assumes a special case where $G(K)$ is a linear function, whereas Rosenblatt and Rothblum (1990) assume a more general convex function. Alternatively, $G(K)$ could well represent the annualized cost associated with building, purchasing, or leasing extra warehouse space. Due to the prevalence of economies of scale, the cost of a capacity increase might easily be a *concave* function of the size of expansion (Nahmias 2009), as opposed to a convex function. Another potential feature of the capacity cost function would be the existence of a potentially large fixed cost associated with any capacity expansion. The potentially diverse nature of the capacity cost function leads to multiple analytical complications.

Let me begin with the general formulation of the problem:

$$\begin{aligned}
 & \min \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + G(K) \\
 & \text{s.t.} \\
 & \sum_{i=1}^I k_i Q_i \leq K \\
 & Q_i \geq 0 \quad \forall i \\
 & K \geq 0
 \end{aligned} \tag{2.2}$$

Since K is a decision variable and $G(K)$ is assumed to be monotonically non-decreasing in K , the capacity constraint will be binding at the optimal solution:

$$\sum_{i=1}^I k_i Q_i = K. \quad (2.3)$$

This result allows me to absorb (2.3) into the objective function (Rosenblatt and Rothblum 1990). Updating the optimization problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + G\left(\sum_{i=1}^I k_i Q_i\right) \\ \text{s. t.} \quad & Q_i \geq 0 \quad \forall i \end{aligned} \quad (2.4)$$

From here, I first analyze a simplified version of the problem with linear capacity cost and then extend to an approximation that can handle any functional form of $G(K)$.

2.3.3 Simplified Case: Linear Capacity Cost with No Initial Capacity

Define $G(K)$ as the annual cost of maintaining a capacity level K . Following Hall (1988), let me assume that $G(K)$ is a linear function of K . This simplified setup assumes there is no initial capacity K_0 . Let b represent the marginal cost of maintaining one additional unit of capacity per year. Updating the optimization problem:

$$\begin{aligned} \min \quad Z' = \quad & \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + b \sum_{i=1}^I k_i Q_i \\ \text{s. t.} \quad & Q_i \geq 0 \quad \forall i \end{aligned} \quad (2.5)$$

Due to the discontinuous objective function for each item, each price level must be considered individually to determine the optimal order quantity for that price. Consider the first and second derivatives of the objective function (Z') with respect to Q_i where $q_{ij} \leq Q_i < q_{i(j+1)}$:

$$\frac{\delta Z'}{\delta Q_i} = -\frac{D_i S_i}{Q_i^2} + \frac{h_i}{2} p_{ij}^A + b k_i, \quad (2.6)$$

$$\frac{\delta^2 Z'}{\delta Q_i^2} = \frac{2D_i S_i}{Q_i^3} > 0. \quad (2.7)$$

Note that the objective function is separable with respect to each item's order quantity. This allows me to optimize each item individually. As Z' is convex (from (2.7)) the first order conditions are sufficient to find the minimum point x_{ij} for price level j and item i :

$$x_{ij} = \sqrt{\frac{2D_i S_i}{h_i p_{ij}^A + 2bk_i}}. \quad (2.8)$$

However, x_{ij} may not be “feasible,” i.e., x_{ij} may not lie within price interval j . If $x_{ij} < q_{ij}$, then q_{ij} is the best order quantity for that interval. If $x_{ij} \geq q_{i(j+1)}$, then the global optimal order quantity will not lie within interval j . Adapting from Munson and Hu (2010), the optimal order quantity for item i is:

$$Q_i^* = \max(x_{ij^*}, q_{ij^*}), \quad (2.9)$$

where $j^* = \text{Argmin}_{\{j=0,1,\dots,J\}} Z_{ij}$, and

$$Z_{ij} = \left(\frac{D_i}{\max(x_{ij}, q_{ij})} \right) S_i + \left(\frac{\max(x_{ij}, q_{ij})}{2} \right) h_i p_{ij}^A + p_{ij}^A D_i + bk_i \max(x_{ij}, q_{ij}). \quad (2.10)$$

The value of Z_{ij^*} represents the total cost of Q_i^* .

Upon the completion of this process for each item, the optimal capacity (K^*) is:

$$K^* = \sum_{i=1}^I k_i Q_i^*. \quad (2.11)$$

While managers could implement the solution to the simplified case directly into spreadsheets, practical applications of linear capacity costs are limited. I now move to a more general solution procedure that is applicable regardless of the functional form of capacity cost.

2.3.4 General Case: General Capacity Cost with Initial Capacity

In the simplified case, I assumed that there is no initial capacity (e.g., building a new warehouse) and that $G(K)$ was the annualized linear cost of maintaining a capacity level K . Now let me consider an initial capacity level K_0 and the potential to expand the capacity through an investment that is non-decreasing in the expansion size. I convert the investment to an annualized payment to continue to express the objective function as the annual total cost. For now let me limit ourselves to capacity expansion opportunities (I extend this to incorporate capacity reduction in Section 2.4).

To eliminate restrictions on the functional form of the capacity cost, I introduce a piecewise-linear and monotonically non-decreasing approximation of $G(K)$. The piecewise-linear function can be broken into $M + 1$ segments. Obviously, more segments lead to a more accurate, but more computationally cumbersome, solution. When analyzing the piecewise-linear function segment by segment, each segment of the objective function (2.1) becomes convex regardless of the original functional form of capacity cost (i.e., convex or concave), allowing me to find approximate analytical solutions. The best solution is the feasible segment solution with the lowest total cost. Consider a segment m that goes from capacity level y_m to y_{m+1} and has a marginal cost of b_m for each additional unit of capacity added. Let V_m be the capacity cost of maintaining capacity level y_m (Chopra and Mindl 2010). Define $y_0 = 0$, $y_1 = K_0$, $y_{M+1} = \infty$, and $b_0 = 0$ to provide one segment in case the final capacity is less than or equal to the initial capacity level. This results in $V_0 = V_1 = 0$. I then calculate V_m for $2 \leq m \leq M$ as follows:

$$V_m = b_1(y_2 - y_1) + \dots + b_{m-1}(y_m - y_{m-1}). \quad (2.12)$$

Then:

$$G(K) = \begin{cases} 0 & \text{if } K < K_0 \\ V_m + b_m(K - y_m) & \text{if } y_m \leq K < y_{m+1}, \text{ for } m = 1, 2, \dots, M. \end{cases} \quad (2.13)$$

Note that if there is no initial capacity level, one can simply eliminate segment $m = 0$.

Additionally, capacity expansions could include a fixed upfront cost. Such a cost might incorporate a myriad of expenditures, e.g. a contractor's fixed fee or the cost of shutting the line down during production. Managers may also incorporate a theoretical fixed cost as a mechanism to prevent the algorithm from recommending insignificant changes in capacity. To include a fixed cost in (10), one can simply add a fixed cost F to $G(K)$ when $K \geq K_0$.

Updating the objective function Z for each segment m , where $m = 0, 1, \dots, M$:

$$Z^m = \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + \left[V_m + b_m \left(\sum_{i=1}^I k_i Q_i - y_m \right) \right]. \quad (2.15)$$

Similar to the simplified case, the approximate analytical minimum point (x_{ij}^m) for item i in price level j in segment m is found using first order conditions:

$$x_{ij}^m = \sqrt{\frac{2D_i S_i}{h_i p_{ij}^A + 2b_m k_i}}. \quad (2.16)$$

The order quantity for item i is:

$$Q_i^m = \max(x_{ij^*}^m, q_{ij^*}^m), \quad (2.16)$$

where $j^* = \text{Argmin}_{\{j=0,1,\dots,J\}} Z_{ij}^m$, and

$$Z_{ij}^m = \left(\frac{D_i}{\max(x_{ij}^m, q_{ij}^m)} \right) S_i + \left(\frac{\max(x_{ij}^m, q_{ij}^m)}{2} \right) h_i p_{ij}^A + p_{ij}^A D_i + b_m k_i \max(x_{ij}^m, q_{ij}^m). \quad (2.17)$$

Note that Z^m becomes:

$$Z^m = \sum_{i=1}^I Z_{ij^*}^m + V_m - b_m y_m. \quad (2.18)$$

Next, I must check that the resulting capacity falls into the m^{th} segment of the piecewise-linear approximation for the capacity cost. The adjustment to assure that the capacity level falls within the correct segment is difficult because the capacity level is dependent on I decision variables, namely, the order quantities for each item. Lemma 2.1 allows me to ignore certain segments based on a simple check of the segment solution's final capacity level.

Lemma 2.1 *If the solution provided by (13) for a particular segment \tilde{m} results in a capacity level $K_{\tilde{m}} < y_{\tilde{m}}$, then the overall optimal capacity level K^* will not uniquely lie in segment \tilde{m} .*

Proof. Assume the condition from Lemma 2.1 is true. Identify the segment w such that $y_w \leq K_{\tilde{m}} \leq y_{w+1}$, and let $\mathbf{Q}_i^w = \mathbf{Q}_i^{\tilde{m}}$, resulting in a feasible capacity level for segment w of $K_w = K_{\tilde{m}}$. Since the capacity cost is non-decreasing in capacity level, there exists a feasible solution in segment w with identical inventory-related costs and with capacity costs that are less than or equal to the capacity costs for segment \tilde{m} . \square

Algorithm 2.1 allows me to adjust the original solution for segment m to fall within the appropriate capacity interval. While this solution procedure does not guarantee an optimal solution to the common resource capacity decision problem when facing all-units quantity discounts, it does provide an efficient solution that is solvable regardless of the functional form of the cost of capacity expansion. In Step 3, I utilize Lemma 2.1 to only consider capacity adjustments where the final capacity level requires *reduction* to achieve feasibility.

Algorithm 2.1: All-Units Quantity Discounts

Step 1 For each segment m ($m = 0, 1, \dots, M$), calculate order quantities $(Q_1^m, Q_2^m, \dots, Q_I^m)$ from (2.16),

$$Z^m \text{ from (2.18), and } K_m = \sum_i k_i Q_i^m.$$

Step 2 Define segment $m^* = \min_m Z^m$.

Step 3 If $K_{m^*} \leq y_{m^*+1}$, then go to Step 5. Otherwise, go to Step 4.

Step 4 Let $m' = m^*$. Using a fixed capacity $y_{m'+1}$ for segment m' , solve for $Q_i^{m'}$ using Algorithm A.1 in Appendix A. Update $Z^{m'}$ using (2.18), and re-calculate $K_{m'} = \sum_i k_i Q_i^{m'}$. Go to Step 2.

Step 5 Set $(Q_1^*, Q_2^*, \dots, Q_I^*) = (Q_1^{m^*}, Q_2^{m^*}, \dots, Q_I^{m^*})$ and set $K^* = K_{m^*}$.

2.4 Capacity Reduction Option

I now relax the assumption that capacity can only expand from the initial capacity level, allowing firms to lease out excess capacity as another form of revenue. The solution technique is similar to the procedure described in Section 2.3.4, with minor modifications to (2.12) and (2.13) to incorporate capacity reductions. Let $r1, r2, \dots, rR$ be the segments of the piecewise-linear capacity cost approximation that result in capacity *reduction*, and let $e1, e2, \dots, eE$ be the segments associated with capacity *expansion*. Additionally, let $y_{K0} = y_{e1} = y_{r1} = K_0$ (thus $V_{K0} = V_{r1} = V_{e1} = 0$), $y_{r(R+1)} = 0$, $y_{e(E+1)} = \infty$, and remove the conditions from Section 2.3 that $V_0 = V_1 = 0$ and $b_0 = 0$. Updating (2.12) and (2.13):

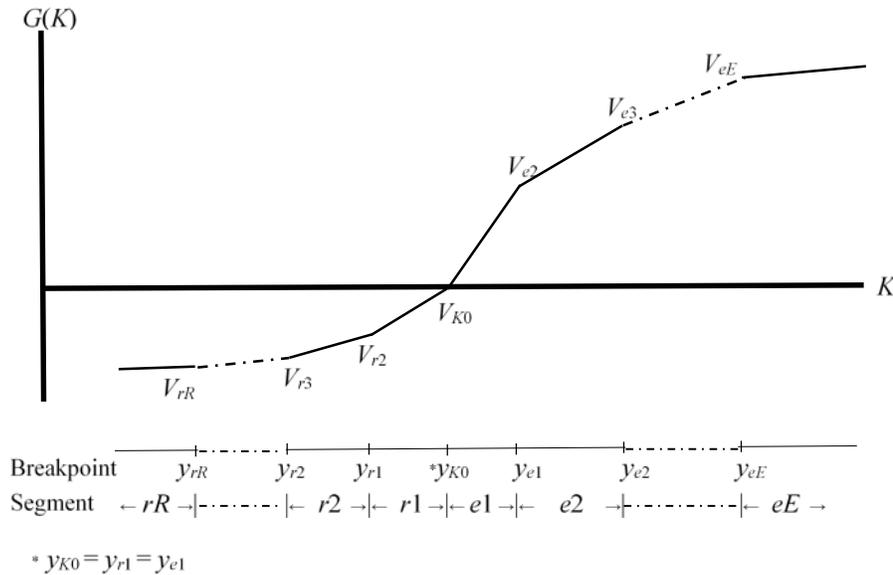
$$V_m = \begin{cases} b_{r1}(y_{r2} - y_{r1}) + b_{r2}(y_{r3} - y_{r2}) + \dots + b_{m-1}(y_m - y_{m-1}) & \text{for } m = r2, r3, \dots, rR \\ 0 & \text{for } m = r1, e1 \\ b_{e1}(y_{e2} - y_{e1}) + b_{e2}(y_{e3} - y_{e2}) + \dots + b_{m-1}(y_m - y_{m-1}) & \text{for } m = e2, e3, \dots, eE, \end{cases} \quad (2.19)$$

and

$$G(K) = \begin{cases} V_m + b_m(K - y_m) & \text{if } y_{m+1} \leq K < y_m \quad \text{for } m = r1, r2, \dots, rR \\ 0 & \text{if } K = K_0 \\ V_m + b_m(K - y_m) & \text{if } y_m \leq K < y_{m+1} \quad \text{for } m = e1, e2, \dots, eE, \end{cases} \quad (2.20)$$

which are illustrated in Figure 2.1. I maintain non-negativity for all the slopes (b_m) of the piecewise-linear approximation. To achieve the *reduction* in cost associated with a capacity reduction, the order of the segment endpoints is reversed for segments $r1, r2, \dots, rR$. Once the modifications are implemented, utilize Algorithm 2.1 to solve for the appropriate capacity level and order quantities.

Figure 2.1: Illustration of the piecewise-linear approximation for capacity cost when there is an option for capacity reduction.



2.5 Incremental Discounts

In this section, I explore the necessary changes to implement and solve the capacitated common resource problem when facing *incremental* as opposed to all-units quantity discounts. There are two distinct differences between all-units and incremental quantity discounts. First, with all-units quantity discounts, the optimal order quantity often falls on a price break quantity; therefore, I

calculate the order quantities in Section 2.3.3 as $Q_i^m = \max(x_{ij^*}^m, q_{ij^*})$. Alternatively, with incremental discounts the optimal order quantity never falls on a price break quantity.

Second, incremental discounts only apply each specified discount to any units that fall within that discount's respective quantity interval, whereas all-units discounts apply to all units in the order. Following standard convention, let R_{ij} represent the total purchasing cost associated with q_{ij} units when facing incremental discounts:

$$R_{ij} = p_{i0}^I q_{i1} + p_{i1}^I (q_{i2} - q_{i1}) + p_{i2}^I (q_{i3} - q_{i2}) + \cdots + p_{i(j-1)}^I (q_{ij} - q_{i(j-1)}), \quad (2.21)$$

where p_{ij}^I is the per-unit price for units $(Q_i - q_{ij})$ purchased when facing incremental quantity discounts and $q_{ij} \leq Q_i < q_{i(j+1)}$. Thus, the total purchasing cost associated with ordering Q_i units when facing incremental discounts and $q_{ij} \leq Q_i < q_{i(j+1)}$ is:

$$P(Q_i) = p_{ij}^I (Q_i - q_{ij}) + R_{ij}. \quad (2.22)$$

Given the two differences between all-units and incremental quantity discounts, a modification to the solution algorithm is required. Similar to the all-units case, I calculate the minimum point x_{ij}^m for item i in price level j in segment m using first order conditions:

$$x_{ij}^m = \sqrt{\frac{2D_i(S_i + R_{ij} - p_{ij}^I q_{ij})}{h_i p_{ij}^I + 2b_m k_i}}. \quad (2.23)$$

Adapting from Munson and Hu (2010), the optimal order quantity for item i among these candidate values x_{ij}^m is:

$$Q_i^m = x_{ij^*}^m, \quad (2.24)$$

where $j^* = \text{Argmin}_{\{j=0,1,\dots,J\}} Z_{ij}^m$, and

$$Z_{ij}^m = \left(\frac{D_i}{Q_i^m} \right) (S_i + R_{ij} - p_{ij}^I q_{ij}) + \left(\frac{h_i}{2} \right) (R_{ij} + p_{ij}^I (Q_i^m - q_{ij})) + p_{ij}^I D_i + b_m k_i Q_i^m. \quad (2.25)$$

Again, Z^m follows (2.18).

Algorithm 2.2 incorporates the necessary modifications to Algorithm 2.1 to handle incremental quantity discounts. Note that, due to the similarities between Algorithms 2.1 and 2.2, it is straightforward to combine them for a scenario where a firm's suppliers offer a mix of all-units and incremental quantity discounts. This is possible because the order quantity calculations in Algorithms A.1 and A.2 (see Appendix A) are separable by item. Thus, as long as those algorithms are solved simultaneously using the same λ , items with both forms of quantity discounts can be combined.

Algorithm 2.2: Incremental Quantity Discounts

Step 1 For each segment m ($m = 0, 1, \dots, M$), calculate order quantities $(Q_1^m, Q_2^m, \dots, Q_I^m)$ from (2.24),

$$Z^m \text{ from (2.18) and } K_m = \sum_i k_i Q_i^m.$$

Step 2 Define segment $m^* = \min_m Z^m$.

Step 3 If $K_{m^*} \leq y_{m^*+1}$, then go to Step 5. Otherwise, go to Step 4.

Step 4 Let $m' = m^*$. Using a fixed capacity $y_{m'+1}$ for segment m' , solve for $Q_i^{m'}$ using Algorithm A.2 in Appendix A. Update $Z^{m'}$ using (2.18), and re-calculate $K_{m'} = \sum_i k_i Q_i^{m'}$. Go to Step 2.

Step 5 Set $(Q_1^*, Q_2^*, \dots, Q_I^*) = (Q_1^{m^*}, Q_2^{m^*}, \dots, Q_I^{m^*})$ and set $K^* = K_{m^*}$.

2.6 Numerical Analysis

2.6.1 Algorithm Performance Analysis

The two key attributes that define the effectiveness and applicability of an algorithm are its accuracy and computational run-time. For these algorithms, the accuracy is difficult to measure for multiple reasons. First and foremost, when there are more than a handful of items, the true optimal solution becomes too computationally cumbersome to solve. Second, to the best of my knowledge, there are no other algorithms for comparison that allow for capacity expansions in the capacitated common resource problem when facing quantity discounts. The best alternative to test the accuracy of this algorithm is to “fix” capacity and compare these solutions to the fixed capacity problems from prior literature. Rubin and Benton (1993) and Moussourakis and Haksever (2008) solve the fixed capacity problem when facing all-units quantity discounts, and, similarly, Rubin and Benton (2003) and Haksever and Moussourakis (2008) solve the problem when facing incremental quantity discounts.

For the comparisons, I define an arbitrarily high capacity expansion cost to prevent the algorithms from considering capacity expansion. Rubin and Benton (1993) originally provided two examples with unique demands and cost structures. The first example, the “Benton Example,” has 10 items and 3 price levels, and it is incorporated into Studies 1 and 2 with different fixed capacity levels. Study 1 uses a capacity of 2350 square feet, whereas Study 2 uses a capacity of 1350 square feet. The second example, the “Fortune 500 Example,” has 15 items and 4 price levels, and it is utilized for Study 3 with a fixed capacity level of 35,000 square feet. All demand and cost information for the two examples is presented in Appendix B. In all six studies (three all-units and three incremental discounts), if an item has fewer than the maximum number of price levels, I introduce “dummy” price levels where the price does not change. Table 2.2 illustrates the competitiveness of this algorithm compared to the prior studies of Rubin and

Benton (1993) and Moussourakis and Haksever (2008). The algorithm penalty compares the total cost found here with the lowest cost found by the two prior algorithms. For example, in Study 2 my algorithm provides a total cost that is 0.005% higher than the best solution from prior literature. For each all-units quantity discount study, my algorithm performs comparably to the prior fixed capacity algorithms.

Table 2.2: All-units quantity discount algorithm performance compared to the best-performing algorithm between Rubin and Benton (1993) and Moussourakis and Haksever (2008).

Study	Total Cost Penalty
1	0%
2	0.005%
3	0.08%

Rubin and Benton (2003) and Haksever and Moussourakis (2008) analyze similar examples to those in Rubin and Benton (1993), except that they use incremental discounts and a slightly different quantity discount schedule for the Benton Example. Studies 4 and 5 incorporate the Benton Example with the incremental quantity discount schedule and fixed capacity levels of 2350 square feet and 1350 square feet, respectively. The incremental quantity discount schedule is presented in Appendix B. Study 6 utilizes the Fortune 500 Example with a fixed capacity level of 35,000 square feet. There is no adjustment in the quantity discount schedule for the Fortune 500 Example. Table 2.3 displays the competitiveness of my algorithm compared to Rubin and Benton (2003) and Haksever and Moussourakis (2008). Once again, my algorithm performs comparably with the prior algorithms.

I performed a computational run-time analysis to show that my algorithms have reasonable computational run-time requirements even for large problems and to understand the

key drivers that increase the algorithms' run-time. Following Nahmias (2009) I utilized a capacity cost function with the following functional form:

$$G(K) = r(K - K_0)^a, \tag{2.26}$$

where r is a constant of proportionality and a is a measure of the ratio between the incremental and average cost of an additional unit of capacity. This functional form of capacity cost allows for the function to be strictly concave ($a < 1$), linear ($a = 1$), or strictly convex ($a > 1$). Table 2.4 shows the structural input values considered in this study that I expected to potentially influence run time. For each combination, ten trials were run with randomly generated data for the other system parameters (Table 2.5). There are a total of 1920 trials split evenly between the all-units and incremental quantity discounts. The solution procedure was implemented in Microsoft's Excel[®] using VBA[®] and all trials were run on a PC with an Intel[®] Core™ 2 Duo CPU @ 3.00GHz. The computational times are reported in CPU time of this machine.

Table 2.3: Incremental quantity discount algorithm performance compared to the best-performing algorithm between Rubin and Benton (2003) and Haksever and Moussourakis (2008).

Study	Total Cost Penalty
4	0.11%
5	2.72%
6	0.08%

Traditionally, the primary driver of variability in computational run time for algorithms determining order quantities in a multi-item inventory system is the number of different items. Table 2.6 summarizes the run time as a function of the number of unique items for both all-units and incremental quantity discounts. The algorithms are able to efficiently solve every trial in under two minutes with up to 5000 items, five price levels, and ten segments in the piecewise-

linear approximation of capacity cost. The summary statistics include all 80 trials for each form of quantity discount and number of items combination.

Table 2.4: Parameter values used in the run-time analysis.

Parameter	Values
Number of items (I)	50, 100, 500, 1500, 3000, 5000
Number of price levels (J)	3, 5
Number of piecewise-linear segments ($M+1$)	5, 10
Capacity Cost Parameter (a) ^a	0.4, 2
Initial Capacity (K_0)	$0.8 \sum_{i \in I} EOQ_i, 1.25 \sum_{i \in I} EOQ_i$

^a The proportionality parameter (r) is adjusted appropriately for changes in a , the number of items, and the corresponding required capacity.

Table 2.5: Distributions used to generate data for other system parameters used in the run-time analysis.

$D_i \sim U(400, 2500)$	$S_i \sim U(10p_{i0}, 20p_{i0})$
$h_i \sim U(0.20, 0.40)$	$k_i \sim U(0.5, 1.5)$
$p_{i0} \sim U(1, 15)$	$q_{i0} = 0$
$p_{i1} = p_{i0} - \alpha_{i1}$, where $\alpha_{i1} \sim U(0.01, 0.30)$	$q_{i1} = (EOQ_{i0})\gamma_{i1}$, where $\gamma_{i1} \sim U(0.2, 2.0)$
$p_{ij} = p_{i(j-1)} - \beta_{ij}$, where $\beta_{ij} \sim U(0.00, 0.20)$	$q_{ij} = q_{i(j-1)} (1 + \delta_{ij})$, where $\delta_{ij} \sim U(0.5, 1.0)$

Table 2.6: Summary statistics on computational run time (in seconds) for each combination of number of items and form of quantity discounts.

# of Items	Average		Standard Deviation		Max	
	All-Units	Increm.	All-Units	Increm.	All-Units	Increm.
50	0.36	0.28	0.07	0.04	0.63	0.45
100	0.62	0.45	0.11	0.07	0.86	0.64
500	2.67	1.85	0.55	0.32	3.76	2.43
1500	8.77	6.16	1.74	1.30	11.97	7.50
3000	17.11	12.96	4.89	3.18	27.48	19.11
5000	43.02	30.64	18.00	8.92	93.08	51.19

After fixing the number of items, further analysis found that the number of segments $M + 1$ in the piecewise-linear approximation of capacity cost is another primary driver of variability

in the computational run time. Table 2.7 illustrates the impact, where on average I found a 39% increase in run time associated with increasing the number of segments from 5 to 10. The impact of the increase in the number of segments seems to be enhanced by an increase in the number of items. An increase in the number of segments in the piecewise-linear function increases the accuracy of the capacity cost approximation, but it significantly increases run-time.

Table 2.7: Impact on the number of piecewise-linear segments on run time (in seconds).

# of Items	All-Units Discounts		Incremental Discounts	
	5 Segments	10 Segments	5 Segments	10 Segments
50	0.31	0.40	0.25	0.31
100	0.53	0.71	0.41	0.50
500	2.22	3.13	1.57	2.13
1500	7.72	9.83	5.50	6.82
3000	13.35	20.86	10.27	15.64
5000	31.14	54.91	24.61	36.66

The number of price levels in the quantity discount schedule, the functional form of capacity cost (concave vs. convex), and the initial capacity level did not show consistent results in terms of their impact on the variability of computational run-time. Table 2.6 illustrates an interesting insight in that the form of quantity discount impacts the computational run time of the algorithms. It is apparent that all-units quantity discounts result in longer computational run times in comparison with incremental discounts. This result is counterintuitive in that traditional all-units quantity discount problems are typically easier to solve. I believe that this counterintuitive finding is a result of the reduced likelihood of an expansion with incremental discounts (see Section 2.6.2). Therefore the algorithm considers fewer segments in the cases where the capacity expansion costs outweigh the savings from the incremental quantity discounts.

To test the limitations of my algorithms with respect to the two primary factors causing an increase in run time, namely the number of items and the number of segments, I tested the algorithms' computational capabilities with scenarios containing up to 15,000 items and 500 segments. For this sub-experiment, I used all-units quantity discounts, five price levels in the quantity discount schedule, a tight (low) initial capacity level, and a concave capacity cost function. Table 2.8 shows that my algorithm can manage large-scale problems, as all solution times, even for 15,000 items, were well below one hour in length. Most importantly, the capacity cost function can be closely approximated by several hundred small segments, as desired.

Table 2.8: Computational time (in minutes) as a function of the number of segments and the number of items ($a = 0.4, r = 1000, J = 5, K_0 = 0.8 \sum EOQ_i$).

# of Items	Number of Segments				
	25	50	100	250	500
1500	0.50	0.92	1.57	3.27	6.27
3000	1.27	2.25	3.53	7.03	13.20
5000	2.60	4.53	6.58	12.75	22.53
10,000	3.53	6.30	9.20	17.90	27.22
15,000	6.80	11.80	16.52	29.65	40.95

2.6.2 Cost Analysis

As seen previously, multiple articles provide examples with solutions for the capacitated common resource problem when facing quantity discounts with a fixed capacity level. How much is the assumption of a fixed capacity level costing firms? Or in other words, what are the potential savings from allowing a variable capacity level? I analyze this question through the use of the Fortune 500 Example for both all-units and incremental quantity discounts. In the original examples, the fixed capacity level is 35,000 square feet. This constraint is not very restrictive in that the unconstrained solution when facing all-units quantity discounts results in an optimal

capacity level of less than 36,000 square feet. To fully illustrate the potential for capacity expansion, I calculated the unconstrained optimal capacity level with *no quantity discounts*. An undiscounted system requires roughly 18,000 square feet, so that represents a natural initial capacity level. This example illustrates the decisions facing managers who have recently been offered quantity discounts and are trying to make the appropriate order quantity and capacity level decisions. The example also illustrates the potential cost savings of using my algorithms over those previously seen in the literature.

To begin this example, I consider the following linear capacity cost function with an initial capacity level K_0 when $K \geq K_0$:

$$G(K) = 2(K - K_0). \quad (2.27)$$

This linear capacity cost function (also used by Hall (1988)) is the baseline model. Before beginning the analysis, I remove the overpowering effect of purchasing costs. Specifically, let me remove the *non-discretionary costs*, which are calculated as the minimum possible annual purchasing cost (i.e., a full year's demand at the minimum price per unit). For all-units quantity discounts, the yearly non-discretionary costs total \$2,446,654. From this point forward I will only refer to the *discretionary costs* that proper decisions on order quantities and capacity level can influence.

Prior literature limits the functional form of capacity cost to a linear or convex function. However, as mentioned previously, with the existence of economies of scale it is possible that the capacity cost is a concave function with respect to the size of the expansion. I present (2.28) as an example of a concave function factoring in economies of scale and the presence of an initial capacity level K_0 when $K \geq K_0$:

$$G(K) = 50(K - K_0)^{0.6}. \quad (2.28)$$

Note that (2.27) and (2.28) were constructed to be proportional to inventory costs and scaled to provide similar expansion costs despite the differences in functional form.

At this point I compare four solutions: (1) fixed capacity, (2) linear capacity using (2.27), (3) concave capacity using (2.28), and (4) unconstrained capacity. The inclusion of the unconstrained capacity solution provides a reference point for the analysis. For all subsequent analysis, I set $M+1 = 10$ and $K_0 = 18,000$ square feet. Table 2.9 compares the four solutions when facing all-units quantity discounts. It is clear that the ability of a firm to increase its capacity level when quantity discount opportunities exist may result in significant savings in its discretionary costs (\$17,106 or 12.48% when using the linear capacity solution). The flexibility of my algorithm to handle concave capacity functions provides firms even more benefit through a better approximation of their expansion costs when economies of scale are present, resulting in an additional cost reduction of \$5,031 (4.19%).

For incremental quantity discounts, I used the same setup as in the all-units quantity discount example. The *non-discretionary costs* for the incremental discounts total \$2,474,354. Table 2.10 provides the incremental quantity discount example results. Unlike the all-units quantity discount example, the incremental discounts had identical solutions for the linear and fixed capacities. Only the concave capacity cost function resulted in a capacity expansion of 3465 square feet. This capacity expansion led to a cost savings of \$2931 (1.58%).

To better understand the difference in the expansion decision when facing all-units and incremental quantity discounts, consider the following “bang for the buck” analysis. Specifically, consider the cost savings per square foot between the unconstrained capacity solution and the fixed capacity solution when facing all-units and incremental quantity discounts. For all-units quantity discounts, the average cost savings is \$1.99 per square foot. Meanwhile, for incremental

quantity discounts, the average cost savings is \$0.74 per square foot. This difference helps explain why a firm facing all-units quantity discounts is more likely to find an expansion worthwhile because it experiences a higher “bang for its buck.” The lower “bang for the buck” associated with incremental quantity discounts is a result of the smaller reductions in the average price per unit and the inability to experience a large total purchase price reduction at the price break quantities.

Table 2.9: Comparison of solution methods with all-units quantity discounts.

Capacity	Order Quantities														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Fixed	1500	1500	1500	900	567	582	362	285	503	1500	539	530	253	900	272
Linear	2500	1500	2500	1500	900	605	376	293	510	1500	1500	1500	262	1500	274
Concave	2500	1500	2500	2500	1500	876	528	360	565	1500	1500	1500	358	1500	293
Unconstrained	2500	1500	2500	2500	1500	1384	774	426	603	1500	2500	2500	900	1500	305
Solution	Total Discretionary Cost						Capacity Used (sq. ft.)								
Fixed Capacity	\$137,111						16,387								
Linear Capacity	\$120,005						25,266								
Concave Capacity	\$114,974						30,194								
Unconstrained Capacity	\$98,395						35,855								

Table 2.10: Comparison of solution methods with incremental quantity discounts.

Capacity	Order Quantities														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Fixed	2820	245	502	525	651	678	418	314	529	1912	621	622	289	1644	281
Linear	2820	245	502	525	651	678	418	314	529	1912	621	622	289	1644	281
Concave	3785	253	526	562	731	772	471	338	549	1934	699	1124	323	2178	288
Unconstrained	4486	277	3081	1912	1880	1384	774	426	603	1989	2899	4038	495	2850	305
Solution	Total Discretionary Cost						Capacity Used (sq. ft.)								
Fixed Capacity	\$185,004						17,253								
Linear Capacity	\$185,004						17,253								
Concave Capacity	\$182,073						21,465								
Unconstrained Capacity	\$166,452						42,442								

2.6.3 Sensitivity Analysis

In this section, I conduct sensitivity analysis to examine how the capacity expansion decision changes across various demand, cost, and quantity discount schedule characteristics to understand the key factors leading to, or preventing, capacity expansion. The results and insights come from a full-factorial experimental design resulting in 729 different runs (three levels of six characteristics, $3^6 = 729$). The analysis focuses on all-units quantity discounts based on the findings in Section 2.6.2 that there is more variability in the capacity expansion decision compared to incremental discounts. The baseline values come from the Fortune 500 Example with an initial capacity level of 18,000 square feet and a concave capacity cost function:

$$G(K) = 100(K - K_0)^{0.6}. \quad (2.29)$$

Starting with the baseline model, I varied the annual demand, setup cost, holding cost percentage, square footage required per unit of each item, size of the discounts, and quantities required to achieve the discounts (price break quantities). For each parameter, I considered a “high” value at 15% higher than the baseline value, as well as a “low” value at 15% lower than the baseline value.

Table 2.11: Percentage of the 243 runs for each level of each characteristic in the sensitivity analysis study that results in capacity expansion.

Level	Characteristic					
	D_i	S_i	h_i	Discount Size	q_{ij}	k_i
Low	49%	54%	60%	0%	54%	47%
Baseline	61%	58%	57%	74%	60%	61%
High	64%	61%	56%	99.6%	59%	65%

The algorithm recommended capacity expansion for 421 (57.8%) of the 729 total runs. Table 2.11 provides a breakdown of the percentage of runs that led to a capacity expansion for

each level (low, baseline, and high) of each characteristic. For example, of the 243 runs with low annual demand, the algorithm recommended capacity expansion in 49% of those runs. The most influential parameter was the discount size. None of the runs with a low or small discount size resulted in capacity expansion, whereby 99.6% of the runs with a high or large discount size resulted in capacity expansion. On the other extreme, variability in the setup cost, holding cost percentage, and price break quantities had minimal to no effect on the likelihood of a capacity expansion. Although not as impactful as the discount size, the annual demand per item and the required capacity for each unit did play a role in the capacity expansion decision. Specifically, as those two characteristics increased, the likelihood that the algorithm recommended a capacity expansion rose.

To better understand the factors driving capacity expansion, let me define a “large” expansion as an expansion resulting in a final capacity level of at least 30,000 square feet (a 67% increase). Of the 421 runs resulting in capacity expansion, 113 runs resulted in a large expansion. Examining only those 113 runs, it is interesting to note the changes in key characteristics that influenced the capacity expansion decision. Table 2.12 shows a breakdown of the percentage of the 113 runs resulting in a large expansion for each level of each characteristic. For example, 22% of the 113 runs resulting in a large expansion had low annual demand. The discount size continued to be a major factor in the capacity expansion decision, as a low discount size never resulted in a large expansion. Two new key factors emerged when considering a large expansion, namely the price break quantities and the required capacity of each unit. Specifically, large expansions primarily occurred when the price break quantities were high and the capacity per unit was high.

Table 2.12: Percentage of the 113 runs in the sensitivity analysis study that led to a “large” ($\geq 67\%$) capacity expansion.

Level	Characteristic					
	D_i	S_i	h_i	Discount Size	q_{ij}	k_i
Low	22%	31%	40%	0%	8%	0%
Baseline	34%	32%	34%	22%	30%	24%
High	44%	37%	26%	78%	62%	76%
Total	100%	100%	100%	100%	100%	100%

When facing all-units quantity discounts, it is well known that the order quantity often falls on a price break quantity (Hadley and Whitin 1963). The sensitivity analysis found 6506 order quantities across the 729 runs that took advantage of quantity discounts. Of the 6506 order quantities greater than or equal to the first price break quantity, 5788 (89%) fell on a price break quantity in the discount schedule. And the 718 (11%) that did not fall on a price break quantity fell exclusively inside of the first (lowest) discount level (instead of a higher discount level). This is an important insight for managers who face all-units quantity discounts with capacity limitations but may not wish to code and run the full algorithm. This insight suggests a simple heuristic rule for all-units discounts with capacity constraints that would significantly reduce the problem complexity: only consider order sizes at price-break quantities. Alternatively, such a heuristic would not work with incremental quantity discounts, as it remains suboptimal to order at a price break quantity under that pricing scheme.

2.7 Conclusions

This study provides an approximation technique to determine efficient solutions for a constrained, multi-product lot-sizing problem that incorporates all-units and/or incremental quantity discounts and considers resource capacity as a decision variable. To the best of my

knowledge, this study represents the first to consider quantity discounts while simultaneously allowing capacity to be a decision variable. A second key contribution of this study is the ability of the solution algorithm to incorporate any functional form of capacity cost. Previous models in the literature restrict the functional form of capacity cost (e.g., convex or linear), which in turn limits the model's applicability. These algorithms allow for any form of capacity cost function (e.g., concave, convex-concave, concave-convex, etc.). Concave functions, in particular, can incorporate annualized economies of scale benefits often seen in larger expansions. A third contribution is the ability of my algorithms to incorporate a mix of all-units and incremental quantity discounts on the items being purchased.

My algorithms can accommodate practical problems incorporating simultaneous lot-sizing decisions for thousands of items in real time. Additionally, the capacity cost functions can be parsed into several hundred piecewise-linear segments without seriously impacting the run time for decision makers.

The numerical studies led to some important managerial insights. First, the ability to expand capacity can provide significant savings to the firm. Moreover, the ability to accurately model a concave capacity cost function can reduce annual costs above and beyond the savings seen from implementing a dynamic capacity level. Second, when facing all-units quantity discounts, 89% of the order quantities in this study that eclipsed the initial price break quantity equaled one of the price break quantities in the discount schedule. Managers may use this knowledge to develop heuristics when facing all-units quantity discounts with capacity constraints. All-units quantity discounts also lend themselves to capacity expansion more so than incremental discounts due to the higher "bang for the buck" associated with expansion opportunities. Finally, the primary characteristic that influences the capacity expansion decision

is the size of the quantity discount offered, but variability in demand and price break quantities influence the expansion decision as well.

As with all models, there are limiting assumptions required to achieve analytical tractability. Future researchers can extend this work by removing some of the typical EOQ assumptions such as deterministic demand, infinite production rate, zero lead time, and no backorders. There are two other natural areas of extension to explore. First, “What if the form of quantity discount was changed to business volume discounts?” Second, “What if the order timing was changed to having all products on a fixed cycle length?” Prior literature considers these extensions in isolation, but not along with quantity discounts and a flexible common resource capacity.

CHAPTER THREE
COMMON REPLENISHMENT CYCLE ORDER POLICIES FOR MULTIPLE
PRODUCTS WITH CAPACITY EXPANSION OPPORTUNITIES
AND QUANTITY DISCOUNTS

Abstract

I extend the traditional shared resource capacity allocation problem to incorporate all-units quantity discounts and the ability to integrate the capacity level as a decision variable. I develop heuristics to solve the problem that utilize the *common replenishment cycle* approach, and I introduce a refinement heuristic that allows each item to be replenished every n cycles. This refinement policy allows for more flexibility in a firm's procurement policy, which in turn reduces annual cost. I analyze multiple numerical examples to better understand the advantages and disadvantages of the two common approaches to the shared resource capacity allocation problem: the common replenishment cycle approach and the independent cycle approach.

3.1 Introduction

Similar to Chapter Two, this chapter addresses the inventory-based problem of simultaneously determining the order quantity of each item as well as the required shared resource capacity level in the presence of quantity discounts. The necessary capacity level of the shared resource is heavily dependent on the *size* and *timing* of each item's order. Historically, much of the academic literature (including Chapter Two) has focused on an ordering structure in which all items have independent cycle lengths, or time between orders. In such an ordering structure, the firm must maintain a capacity level large enough to handle the worst-case scenario where all items are ordered simultaneously. In a fixed-capacity environment with no price breaks, Rosenblatt (1981) and others have sought way to reduce the impact of preparing for the worst-case scenario.

In this chapter, I build upon Chapter Two by considering an alternative ordering structure that places all items on a common replenishment cycle and phases the procurement of the items throughout the cycle. This approach is beneficial as it reduces the required capacity level, but it also hurts the firm by not allowing each item to be ordered optimally with regard to minimizing the traditional inventory-related costs. I develop heuristics to solve the common replenishment cycle problem with multiple products and all-units quantity discounts and compare that with the results from Chapter Two to see when it is advantageous to use independent replenishment cycles vs. a common replenishment cycle. Additionally, I introduce a refinement policy that allows each products more flexibility in the size and timing of its order.

To help visualize the difference between the independent cycle ordering structure and the common cycle ordering structure, consider the following example with three products. The annual demands for items 1, 2, and 3 are 1200, 2400, and 3600 units, respectively. Assume that

there are 360 working days in a year. The independent cycle solution technique then finds the optimal order quantities: $Q_1 = 24$, $Q_2 = 75$, and $Q_3 = 100$ units. That implies that item 1 is ordered every 7.2 days (daily demand is 3.333 units, therefore the 24 unit order quantity lasts 7.2 days ($24/3.333$)). Similarly, item 2 is ordered every 11.25 days, and item 3 is ordered every 10 days. This solution minimizes the setup, holding, and purchasing costs of each item individually, but it does not attempt to synchronize the orders (neither for capacity reduction nor for managerial convenience). Next consider the common cycle approach. The solution technique computes the optimal common cycle length to be 7 days, implying that each item is ordered once per week. Therefore, the order quantities must satisfy one week (7 days) worth of demand: $Q_1 = 23$, $Q_2 = 47$, and $Q_3 = 75$ units. For example, item 1 has daily demand of 3.333 units, therefore weekly demand is $3.333 * 7 = 23$ units. To avoid a situation where all items are ordered simultaneously, the order points are phased within the week. For example, item 1 could be ordered every Monday, item 2 every Tuesday, and item 3 every Friday. This solution technique provides a consistent and convenient schedule for managers. Additionally, it reduces the decision variables from I (an order quantity for each item) to 1 (the common cycle length).

The remainder of this chapter is organized as follows. In Section 3.2, I review relevant literature on quantity discounts and the shared resource capacity allocation problem. In Section 3.3, I introduce the model and heuristics to solve the common replenishment cycle problem when facing all-units quantity discounts and capacity decisions. This is followed in Section 3.4 by an extension of the baseline model and heuristics to incorporate a refinement policy to allow for more flexibility in the procurement policy. In Section 3.5, I describe numerical studies that test the performance of the heuristics along with sensitivity analysis that identifies the key parameters influencing the preference between independent replenishment cycles and a common

replenishment cycle. Finally, in Section 3.6, I conclude the study and identify directions for future research.

3.2 Literature Review

Much of the relevant literature is reviewed in Chapter Two. In addition, there are a few points of emphasis in the shared resource capacity allocation problem literature that are particularly useful for this chapter.

Page and Paul (1976) and Rosenblatt (1981) were among the first to consider an alternative approach to the ordering structure of the common resource capacity problem, specifically a common replenishment cycle for all products. The order points for each product are then phased within the common cycle to minimize the maximum inventory level. Both articles compare the independent cycle and common cycle approaches in an undiscounted pricing structure. Güder and Zydiak (2000) develop a heuristic to solve the shared resource capacity allocation problem with a common cycle length and all-units quantity discounts. Haksever and Moussourakis (2005) and Moussourakis and Haksever (2008) incorporated the ordering structure decision into their mixed integer program when facing all-units quantity discounts and multiple resource constraints.

Similar to Chapter Two, to the best of my knowledge, no previous researchers have examined the shared resource capacity allocation problem utilizing a common cycle ordering structure in the presence of all-units quantity discounts and with the ability to alter the capacity of the common resource. Additionally, I introduce a refinement policy that allows for some items to skip replenishment on certain periodic cycles. This allows for more flexibility in order size

and timing, especially when there are a large number of items and the common replenishment cycle may be far from individually optimal for a portion of those items.

3.3 The Model

Much like Chapter Two, the goal of this chapter is to develop a model to simultaneously solve for the appropriate shared resource capacity level and the order quantity for each item when facing all-units quantity discounts. Section 3.3.2 begins with the “base case” model with no capacity limitation and a linearly increasing capacity cost. Then in Section 3.3.3, I generalize the model to incorporate an initial capacity level and allow for any functional form of capacity cost (i.e., concave, convex, concave-convex, etc.) through the use of a piecewise-linear approximation of capacity cost.

Much of the notation and most of the assumptions from Chapter Two hold true for this chapter with a few exceptions detailed below. I utilize a common replenishment cycle ordering structure in this chapter, compared to the independent cycle ordering structure from Chapter Two. This new ordering structure requires a few new pieces of notation:

Parameters:

t_1 interval from the replenishment of item I to the replenishment of item 1
 t_i interval from the replenishment of item $i - 1$ to the replenishment of item i
 Q_i order quantity for item i , where $Q_i = TD_i$

Decision Variable:

T common replenishment cycle for all items (in years)

The common cycle ordering structure solution specifies the length of the common cycle T . Once T is determined, to minimize the maximum inventory level, each item’s replenishment

point is phased within the cycle. Adapting from Güder and Zydiak (2000), the phasing of the replenishment point for item i is as follows:

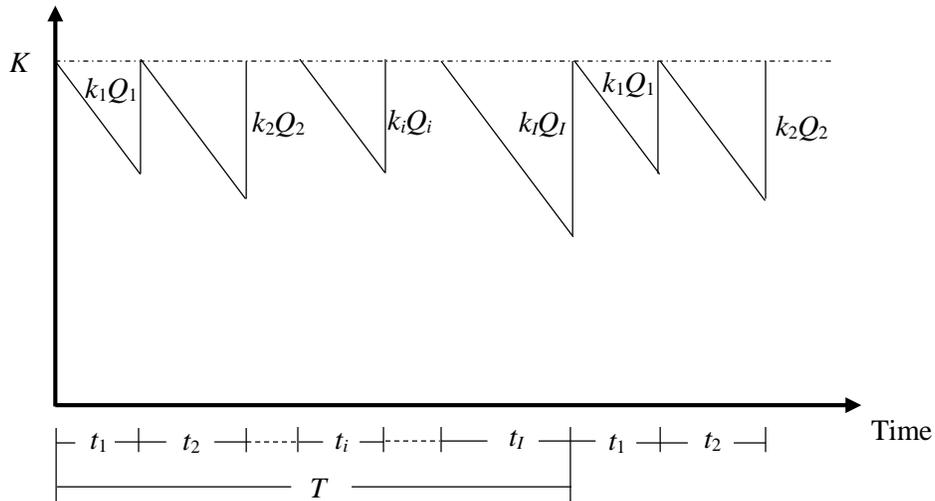
$$t_i = \frac{Tk_i D_i}{\sum_{l=1}^I k_l D_l}, \forall i. \quad (3.1)$$

The t_i intervals are determined in a way such that after each replenishment point, the capacity level reaches a value K . Figure 3.1 illustrates the inventory level over time. With that in mind, the warehouse must satisfy this minimum capacity level defined as:

$$K = \frac{1}{2}T \left[\sum_{i=1}^I k_i D_i + \frac{\sum_{i=1}^I k_i^2 D_i^2}{\sum_{i=1}^I k_i D_i} \right]. \quad (3.2)$$

A more detailed explanation of (3.2) can be found in Page and Paul (1976).

Figure 3.1: Illustration of the common replenishment cycle ordering structure.



3.3.1 Modeling Preliminaries

Recall the general objective function (2.1) from Chapter Two. It is restated below for convenience:

$$Z = \sum_{i=1}^I \left[\left(\frac{D_i}{Q_i} \right) S_i + H(Q_i) + P(Q_i) \right] + G(K), \quad (2.1)$$

where $q_{ij} \leq Q_i < q_{i(j+1)}$, $H(Q_i) = (Q_i / 2)(h_i p_{ij})$, and $P(Q_i) = p_{ij} D_i$. This objective function is a function of the order quantity of each item and the capacity level, or the decision variables from Chapter Two. With the common replenishment cycle order structure, there are only two variables: the common replenishment cycle T and the capacity level K . Therefore to convert (2.1) to the new objective function, replace Q_i with TD_i for all items. After updating the objective function, the general formulation of the problem becomes:

$$\begin{aligned} \min \quad & \sum_{i=1}^I \left[\left(\frac{S_i}{T} \right) + \frac{TD_i}{2} h_i p_{ij} + p_{ij} D_i \right] + G(K) \\ \text{s.t.} \quad & \\ & \frac{1}{2} T \left[\sum_{i=1}^I k_i D_i + \frac{\sum_{i=1}^I k_i^2 D_i^2}{\sum_{i=1}^I k_i D_i} \right] \leq K \\ & T \geq 0 \\ & K \geq 0 \end{aligned} \quad (3.3)$$

Since K is a decision variable and $G(K)$ is assumed to be monotonically non-decreasing in K , the capacity constraint will be binding at the optimal solution:

$$\frac{1}{2} T \left[\sum_{i=1}^I k_i D_i + \frac{\sum_{i=1}^I k_i^2 D_i^2}{\sum_{i=1}^I k_i D_i} \right] = K. \quad (3.4)$$

This result allows me to absorb (3.4) into the objective function (Rosenblatt and Rothblum 1990). Updating the optimization problem:

$$\min \sum_{i=1}^I \left[\left(\frac{S_i}{T} \right) + \frac{TD_i}{2} h_i p_{ij} + p_{ij} D_i \right] + G \left(\frac{1}{2} T \left[\sum_{i=1}^I k_i D_i + \frac{\sum_{i=1}^I k_i^2 D_i^2}{\sum_{i=1}^I k_i D_i} \right] \right) \quad (3.5)$$

s.t.
 $T \geq 0$

From here, I first analyze a simplified version of the problem with linear capacity cost and then extend to an approximation that can handle any functional form of $G(K)$.

3.3.2 Simplified Case: Linear Capacity Cost with No Initial Capacity

For the base case, I again borrow the interpretation and functional form of capacity cost from Hall (1988). More specifically, I use the same capacity cost function from the base case in Chapter Two: $G(K) = bK$. Updating the optimization problem:

$$\min Z' = \sum_{i=1}^I \left[\left(\frac{S_i}{T} \right) + \frac{TD_i}{2} h_i p_{ij} + p_{ij} D_i \right] + bT \sum_{i=1}^I k_i D_i \quad (3.6)$$

s.t.
 $T \geq 0$

There are numerous discontinuities in the objective function due to the presence of an all-units quantity discount schedule for each item. This prevents a simple analytical solution, and instead forces me to consider heuristics to find near-optimal solution.

Güder and Zydiak (2000) utilize a fixed-point algorithm to determine an efficient solution to the problem with a common replenishment cycle, fixed capacity, and all-units quantity discounts. Algorithm 3.1 adapts their fixed-point algorithm to the base case with a linear capacity cost and no initial capacity level. Due to the iterative process of the algorithm, it requires some additional notation. Let T_n , Q_{in} , p_{ijn} , and j_{in} be the common replenishment cycle, order quantity for item i , per-unit price for item i , and price level for item i , respectively, for the n th iteration of Algorithm 3.1. Step 0 initializes the fixed-point aspect of the algorithm, and then Steps 1 and 2

iterate until the solution converges. The convergence is certain because the prices are non-increasing throughout iterations, resulting in a non-decreasing common cycle length, and there are a finite number of price changes. This initial solution *ignores* the well-known commonality of the optimal order quantity for an individual item occurring at a price breakpoint with all-units quantity discounts (Hadley and Whitin 1963). Step 3 considers each price breakpoint for each item to check if increasing the common replenishment cycle T is beneficial to allow another item to achieve a lower price per unit. The price breakpoints are considered sequentially in increasing order of the required common replenishment cycle. Define i_{up} as the item with the “closest” price breakpoint (i.e., has the smallest $T_{up} > T^*$). Similarly, define T_{up} as the common replenishment cycle associated with ordering item i_{up} at its next highest price breakpoint.

Algorithm 3.1

Step 0 Initialize $n = 0$ and $j_{i0} = 0$ ($\forall i$) (no discount on all items). Calculate:

$$T_0 = \sqrt{\frac{2 \sum_{i=1}^I S_i}{\sum_{i=1}^I h_i p_{i00} D_i + 2b \sum_{i=1}^I k_i D_i}}.$$

Step 1 Calculate $Q_{in} = T_n D_i$ ($\forall i$), and define $p_{ij(n+1)}$ as the per-unit price for all units

when $q_{ij} \leq Q_{in} < q_{i(j+1)}$ ($\forall i$). Calculate:

$$T_{n+1} = \sqrt{\frac{2 \sum_{i=1}^I S_i}{\sum_{i=1}^I h_i p_{ij(n+1)} D_i + 2b \sum_{i=1}^I k_i D_i}}.$$

Step 2 If $T_{n+1} = T_n$, then $T^* = T_n$, $Z^* = Z'(T_n)$, $Q_i^* = T^* D_i$ and $j_i^* = j_{in}$ ($\forall i$), and go to Step 3. Else, set $n = n + 1$, and go to Step 1.

Step 3 Let $j_i' = j_i^*$ ($\forall i$).

Step 4 Calculate:

$$i_{up} = \arg \min_i \frac{q_{i(j_i'+1)}}{D_i} \text{ and } T_{up} = \min_i \frac{q_{i(j_i'+1)}}{D_i}.$$

If $Z'(T_{up}) < Z^*$, then $T^* = T_{up}$, $Z^* = Z'(T_{up})$, $Q_{i_{up}}^* = T_{up} D_i$ ($\forall i$) and $j_{i_{up}}^* = j_{i_{up}}' + 1$.

If $\sum_{i=1}^I j_i^* = IJ$, then STOP, else $j_{i_{up}}^* = j_{i_{up}}' + 1$, and go to Step 4.

3.3.3 General Case: General Capacity Cost with Initial Capacity

Previously, I assumed that there was no initial capacity (e.g., building a new warehouse) and that $G(K)$ was the linear annualized cost of maintaining a capacity level K . In this section, I reconsider the base case by adding an initial capacity level K_0 and the potential to increase the capacity through an investment. I transform the investment to an annualized payment to continue to express the objective function as the total annual cost. Without loss of generality, let me only consider capacity *expansion* opportunities (capacity *reduction* opportunities would be modeled similarly, see Chapter Two). The solution technique for a general, unrestricted functional form of capacity cost follows Chapter Two through the introduction of a piecewise-linear approximation.

Updating the objective function Z to include the piecewise-linear approximation for capacity cost for each segment m , where $m = 0, 1, \dots, M$:

$$Z^m = \sum_{i=1}^I \left[\left(\frac{S_i}{T} \right) + \frac{TD_i}{2} h_i p_{ij} + p_{ij} D_i \right] + \left[V_m + \frac{1}{2} b_m T \left(\sum_{i=1}^I k_i D_i + \frac{\sum_{i=1}^I k_i^2 D_i^2}{\sum_{i=1}^I k_i D_i} - y_m \right) \right]. \quad (3.7)$$

The only change in (3.7) from the objective function in (3.6) is the last bracketed term (capacity cost). Algorithm 3.2 follows similar logic as the base case, but there are a couple of new features. First, because the analysis of the piecewise-linear approximation is segment by segment, I need to ensure that the solution for segment m falls within the appropriate capacity range. To quantify this, define the largest common replenishment cycle T_{\max}^m that results in a feasible final capacity level for segment m . This result comes from substituting the maximum feasible capacity for segment m , y_{m+1} into (3.2) for K . Then solve for T :

$$T_{\max}^m = \frac{2y_{m+1}}{\sum_{i=1}^I k_i D_i + \sum_{i=1}^I k_i^2 D_i^2 / \sum_{i=1}^I k_i D_i}. \quad (3.8)$$

Let me introduce a few minor modifications to the notation to incorporate the solution for each segment. Define T_n , Q_{in} , p_{ijn} , j_{in} as before, but add a superscript m denoting the segment. Therefore, I have: T_n^m , Q_{in}^m , p_{ijn}^m , j_{in}^m for each segment m . Algorithm 3.2 is a modification of Steps 0 – 2 in Algorithm 3.1. There are two primary modifications. The first is the introduction of m different solutions, one for each segment. The second is the check to ensure that the segment solution is feasible, i.e., that the common replenishment cycle solution falls under the maximum replenishment cycle for that segment. From Chapter Two, recall that any segment with a solution recommendation below the minimum segment capacity level will not be optimal, and thus those solutions can be ignored.

Algorithm 3.2

Step 0 Initialize $n = 0$ and $j_{i0}^m = 0$ ($\forall i$) (no discount on all items). Calculate:

$$T_0^m = \sqrt{\frac{2 \sum_{i=1}^I S_i}{\sum_{i=1}^I h_i p_{i00}^m D_i + 2b_m \sum_{i=1}^I k_i D_i}} \quad \text{and} \quad T_{\max}^m = \frac{2y_{m+1}}{\sum_{i=1}^I k_i D_i + \sum_{i=1}^I k_i^2 D_i^2 / \sum_{i=1}^I k_i D_i}.$$

If $T_0^m \geq T_{\max}^m$, then $T^m = T_{\max}^m$, $Q_i^m = T^m D_i$ ($\forall i$), and STOP.

Step 1 Calculate $Q_{in}^m = T_n^m D_i$ ($\forall i$), and let $p_{ij(n+1)}^m$ be the per-unit price for all units when

$q_{ij} \leq Q_{in}^m < q_{i(j+1)}$ ($\forall i$). Calculate:

$$T_{n+1}^m = \sqrt{\frac{2 \sum_{i=1}^I S_i}{\sum_{i=1}^I h_i p_{ij(n+1)}^m D_i + 2b_m \sum_{i=1}^I k_i D_i}}.$$

Step 2 If $T_{n+1}^m = T_n^m$, then $T^m = T_n^m$, $Z^m = Z'(T_n^m)$, $Q_i^m = T^m D_i$ and $j_i^m = j_{in}^m$ ($\forall i$), and

STOP. Else if $T_n^m \geq T_{\max}^m$, then $T^m = T_{\max}^m$, $Q_i^m = T^m D_i$ ($\forall i$), and STOP. Else, set

$n = n + 1$, and go to Step 1.

Note that Algorithm 3.2 does not provide a final solution as seen in Algorithm 3.1.

Algorithm 3.2 is the initial setup for the three heuristics presented in the subsequent sub-sections.

3.3.4 Full Enumeration Heuristic

Recall that Step 3 of Algorithm 3.1 searches the price breakpoints for each item to find the best common replenishment cycle. Algorithm 3.3 is very similar in nature. It takes the segment solutions from Algorithm 3.2, finds the best segment solution, and then checks that against the

price breakpoints for each item. The full enumeration of price breakpoints ensures that the best price breakpoint is found. For larger problems, checking *every* price breakpoint can become computationally cumbersome. In Chapter Two I tested problems with up to 15,000 items and 5 price levels. Apply a problem of that size to the Full Enumeration Heuristic, and it can result in up to 60,000 price breakpoints analyzed (15,000 items \times 4 price breakpoints (the first price level has a “breakpoint” of 0 units, and therefore is not considered)). Of the three heuristics presented, the Full Enumeration Heuristic guarantees a solution no worse than the other two, but it will also have the longest computation time. Section 3.5 provides detailed comparisons of the three heuristics presented in terms of accuracy and computation time.

Algorithm 3.3: Full Enumeration Heuristic

Step 1 For all $m = 0, 1, \dots, M - 1$

Run Algorithm 3.2.

Calculate:

$$Z^m(T^m) = \sum_{i=1}^I \left[\left(\frac{S_i}{T^m} \right) + \frac{T^m D_i}{2} h_i p_{ij}^m + p_{ij}^m D_i \right] + \left[V_m + b_m T^m \left(\sum_{i=1}^I k_i D_i - y_m \right) \right].$$

Step 2 Let $m' = \arg \min_m Z^m(T_m)$.

Then $T^* = T^{m'}$, $Z^* = Z^m(T^{m'})$, $Q_i^* = T^{m'} D_i$ ($\forall i$), and $j_i^* = j_i^{m'}$ ($\forall i$).

Step 3 Let $j_i' = j_i^* \forall i$.

Step 4 Calculate:

$$i_{up} = \arg \min_i \frac{q_{i(j_i'+1)}}{D_i} \text{ and } T_{up} = \min_i \frac{q_{i(j_i'+1)}}{D_i}.$$

Let m'' be the segment such that $T_{\max}^{m''-1} < T_{up} \leq T_{\max}^{m''}$. If $Z^{m''}(T_{up}) < Z^*$, then $T^* = T_{up}$,

$$Z^* = Z^{m''}(T_{up}), Q_i^* = T_{up} D_i (\forall i), j_{i_{up}}^* = j_{i_{up}}' + 1, \text{ and } j_i^* = j_i' (\forall i \neq i_{up}).$$

$$\text{Set } j_{i_{up}}' = j_{i_{up}}^* + 1.$$

Step 5 If $\sum_{i=1}^I j_i' = IJ$, then STOP. Otherwise, go to Step 4.

3.3.5 Sequential Heuristic

The Sequential Heuristic aims to improve on the computation time of the Full Enumeration Heuristic by not analyzing *every* price breakpoint, but instead beginning with the price breakpoint that results in the smallest common replenishment cycle and sequentially checking the price breakpoints, ordered based on the required common replenishment cycle, until the cost increases. Intuitively, a price breakpoint requiring a large T will result in a large capacity cost, which likely outweighs the purchasing cost savings from quantity discounts. Therefore, ignoring the larger price breakpoints should not significantly hurt the performance of the heuristic. The Sequential Heuristic is identical to the Full Enumeration Heuristic except for Step 4. Algorithm 3.3a shows the appropriate Step 4 for the Sequential Heuristic. It adds a stopping criteria if the cost does not decrease at the next highest price breakpoint, whereas the Full Enumeration Heuristic does not stop until all price breakpoints have been considered.

Algorithm 3.3a: Sequential Heuristic

Step 4 Calculate:

$$i_{up} = \arg \min_i \frac{q_{i(j_i+1)}}{D_i} \text{ and } T_{up} = \min_i \frac{q_{i(j_i+1)}}{D_i}.$$

Let m'' be the segment such that $T_{\max}^{m''-1} < T_{up} \leq T_{\max}^{m''}$. If $Z^{m''}(T_{up}) < Z^*$, then $T^* = T_{up}$,

$$Z^* = Z^{m''}(T_{up}), Q_i^* = T_{up} D_i (\forall i), j_{i_{up}}^* = j_{i_{up}}' + 1, j_i^* = j_i' (\forall i \neq i_{up}), \text{ set } j_{i_{up}}' = j_{i_{up}}^* + 1,$$

and go to Step 5. Otherwise, STOP.

3.3.6 Bang-for-the-Buck (BFB) Heuristic

As an alternative to the Sequential Heuristic, the BFB Heuristic re-organizes the order in which the price breakpoints are considered. Instead of sequentially checking them as a function of T , this heuristic considers the “bang-for-the-buck” or the price savings per unit relative to the size of the required common replenishment cycle. Then the price breakpoints are checked from the largest bang-for-the-buck to the smallest. Algorithm 3.3b provides the modified Step 4 for the BFB Heuristic. All other steps are identical to the steps seen in Algorithm 3.3. This heuristic avoids any potential pitfalls of the Sequential Heuristic by looking for the largest potential savings in the quantity discount schedule instead of just looking at the next closest price breakpoint. In a way, it provides a “look-ahead enhancement” to see if there are large discounts on the horizon. It can be especially useful if some items have “dummy” price breakpoints where the price does not change. This occurs if the item has fewer price breakpoints in its quantity discount schedule in comparison to other items, or if it has no quantity discounts.

For Algorithm 3.3b, there are two new pieces of notation. First, define BFB_{qij} as the bang-for-the-buck associated with price breakpoint q_{ij} . Second, define T_{qij} as the fixed

replenishment cycle associated with price breakpoint q_{ij} . Again, the primary difference in Algorithm 3.3b comes from the stopping condition in Step 4.

Algorithm 3.3b: BFB Heuristic

Step 4 For $j \in [j'_i + 1, J]$ and $\forall i$, calculate:

$$BFB_{q_{ij}} = \frac{P_{i(j-1)} - P_{ij}}{T_{q_{ij}}}.$$

Let $(i_{up}, j_{up}) = \arg \max_{(i,j)} BFB_{q_{ij}}$ and $T_{up} = T_{q(i_{up}, j_{up})}$.

Let m'' be the segment such that $T_{max}^{m''-1} < T_{up} \leq T_{max}^{m''}$. If $Z^{m''}(T_{up}) < Z^*$, then $T^* = T_{up}$,

$Z^* = Z^{m''}(T_{up})$, $Q_i^* = T_{up} D_i (\forall i)$, $j_{i_{up}}^* = j'_{i_{up}} + 1$, $j_i^* = j'_i (\forall i \neq i_{up})$, set $j'_{i_{up}} = j_{i_{up}}^* + 1$,

and go to Step 5. Otherwise, STOP.

3.4 Refinement Policy

The heuristics in Section 3.3 provide a procurement policy such that the replenishment of all items occurs every T years. Inevitably, as the number of items grows, the likelihood increases that ordering every T years significantly increases the holding and setup cost for at least one item. This refinement policy provides more flexibility in the common replenishment cycle procurement policy. Define n_i as a positive integer multiple such that the replenishment of item i occurs every $n_i T$ years. Or in other words, item i is replenished every n_i cycles. Additionally, alter the definition of T such that T is the shortest common replenishment cycle where at least one item has $n_i = 1$.

With the introduction of the multipliers, the updated objective function is:

$$Z = \sum_{i=1}^I \left[\left(\frac{S_i}{n_i T} \right) + \frac{n_i T D_i}{2} h_i p_{ij} + p_{ij} D_i \right] + G(K), \quad (3.9)$$

where $q_{ij} \leq n_i Q_i < q_{i(j+1)}$. Due to the presence of multiple price levels for each item and multiple segments in the piecewise-linear approximation of capacity cost, it is quite difficult to find analytical solutions for n_i . Instead, I consider a heuristic to search only “smart” multipliers based on each item’s EOQ and price breakpoints. The intuition for these possible values of n_i comes from the knowledge that the EOQ minimizes setup and holding costs, and the price breakpoints provide significant purchasing cost savings when facing all-units quantity discounts. The original Section 3.3 algorithms did not recommend these increases in order quantity because it would require a proportional increase for *all items*. This type of increase would have a significant impact on capacity; however with the refinement policy, I only adjust the order quantity of a single item. Therefore, the impact on capacity is not as severe.

Outside of its impact on the objective function, any refinement policy will also impact the required capacity levels. The refinement policy does not change the timing of the orders within the cycle (i.e., the t_i values remain fixed), but instead each item may not be ordered every cycle. Therefore, each cycle will have a different peak required capacity level. To determine the required capacity level, consider a cycle in which every item is replenished. In that cycle, or the subsequent cycle, a maximum inventory level is achieved. Because of the uncertainty and inconsistency of the inventory levels with the introduction of the multipliers, I must consider a variety of points. Each potential point of maximum inventory corresponds with the replenishment point of an item. The difficulty comes into play in terms of determining the inventory levels for each other item at these different re-order points.

Define $SPACE_{ig}$ as the warehouse space occupied by item i when ordering item g in the cycle for which every item is ordered. Similarly, define $SPACE_{ig}^s$ as the warehouse space occupied by item i when at the replenishment point for item g in the cycle subsequent to the cycle for which all items were ordered. Notice the slight change in definition. In the subsequent period, each item may reorder if $n_i = 1$, but it will not reorder if $n_i > 1$. An implicit assumption in the ordering of the items is that within a cycle, the first item to order is item 1, the second is item 2, etc. It is important to differentiate the calculations based on where each item is relative to the item being reordered. Additionally, let I_i be an indicator variable for item i equal to 1 if $n_i = 1$, and 0 otherwise. In the cycle where every item is ordered, calculate the required space for item i when item g is ordered as:

$$SPACE_{ig} = \begin{cases} (n_i T D_i) k_i - \left(\sum_{l=i+1}^g t_l \right) k_i D_i & \text{if } i < g \\ (n_i T D_i) k_i & \text{if } i = g \\ \left(\sum_{l=g+1}^i t_l \right) k_i D_i & \text{if } i > g. \end{cases} \quad (3.10)$$

Each item i such that $i < g$ was replenished earlier in this cycle; thus, the first term represents the required capacity for that replenishment. The second term subtracts the units sold during the interval between the replenishment of item i and the replenishment of item g . For an item i such that $i = g$, at the instant before replenishing, there are no units of item i in inventory; therefore, the only capacity comes from the incoming order. Finally, each item i such that $i > g$ is due to replenish in this cycle; therefore, the only remaining units are the units necessary to satisfy demand until item i replenishes.

Next, for the subsequent cycle, calculate the required space for item i when item g has the opportunity to replenish as:

$$SPACE_{ig}^s = \begin{cases} [(n_i - 1 + I_i)TD_i]k_i - \left(\sum_{l=i+1}^g t_l\right)k_i D_i & \text{if } i < g \\ [(n_i - 1 + I_i)TD_i]k_i & \text{if } i = g \\ [(n_i - 1)TD_i]k_i + \left(\sum_{l=g+1}^i t_l\right)k_i D_i & \text{if } i > g. \end{cases} \quad (3.11)$$

Recall that $SPACE_{ig}^s$ deals with the subsequent cycle after every item orders. The indicator function I_i indicates if item i is replenished again in this cycle (i.e., $n_i = 1$). If $n_i = 1$, then the first line in $SPACE_{ig}^s$ is equivalent to the first line in $SPACE_{ig}$. Otherwise, the first term represents the remaining capacity from the order in the previous cycle (the previous cycle sold TD_i units making up $(TD_i)k_i$ square feet). Then subtract the units sold from the replenishment opportunity of item i to the current replenishment point for item g . The second line again utilizes the indicator function. If $I_i = 1$, indicating a reorder for item i , then the second line in $SPACE_{ig}^s$ is equivalent to the second line in $SPACE_{ig}$. Otherwise, just factor in the units sold in the previous cycle. Finally, each item i such that $i > g$ has not had the opportunity to replenish; therefore, only factor in the units sold from the replenishment of item i to the current replenishment point for item g .

Now, define CAP_g as the required capacity at the replenishment point for item g in the cycle for which every item is ordered. Similarly, define CAP_g^s as the required capacity at the replenishment point for item g in the subsequent cycle. Then:

$$CAP_g = \sum_{i=1}^I SPACE_{ig}, \quad (3.12)$$

$$CAP_g^s = \sum_{i=1}^I SPACE_{ig}^s. \quad (3.13)$$

At this point, there are two potential candidates for the maximum required capacity at each replenishment point. This allows me to find the overall required capacity as:

$$K = \max_g \left(\max \left(CAP_g, CAP_g^s \right) \right). \quad (3.12)$$

Algorithm 3.4 provides the details for the Refinement Heuristic. Note that the solution in Section 3.3 implicitly defines $n_i = 1$ for all items. There is one new piece of notation in Step 2 of Algorithm 3.4: EOQ_i is the economic order quantity (EOQ) for item i with no discount.

Algorithm 3.4: Refinement Heuristic

Step 0 Initialize $n_i^R = 1$ ($\forall i$), and set K , Q_i^R and $p_{ij_i^R}^R$ ($\forall i$) according to the solution to a heuristic from Section 3.

Step 1 Calculate Z^R :

$$Z^R = \sum_{i=1}^I \left[\left(\frac{D_i}{n_i^R Q_i^R} \right) S_i + \frac{n_i^R Q_i^R}{2} h_i p_{ij_i^R}^R + p_{ij_i^R}^R D_i \right] + G(K).$$

Step 2 Calculate possible values of n_i for all i :

$$n_i = \left\{ \left\lceil \frac{EOQ_i}{Q_i^R} \right\rceil, \left\lceil \frac{q_{i(j_i^R+1)}}{Q_i^R} \right\rceil \text{ for } j_i = j_i^R, \dots, J \right\}.$$

Step 3 For each item, calculate Z' using (3.9) for each value of n_i found in Step 2. If $Z' < Z^R$ for some n_i , then set $n_i^R = n_i$, $Z^R = Z'$, $Q_i^R = n_i T D_i$, and j_i^R is such that

$$q_{ij_i^R}^R \leq Q_i^R < q_{i(j_i^R+1)}^R.$$

Initially, I set the Refinement Heuristic to only run after a heuristic from Section 3.3, but after running initial tests, I found that it limited the effectiveness of the refinement. Instead, I update the heuristics from Section 3.3 to include refinement checks throughout the procedure. Algorithms 3.5, 3.5a, and 3.5b illustrate the updated heuristics from Section 3.3 with

opportunities to check for refinement at multiple stages of the heuristic. This change allows for the solution technique to use both the multipliers and the common replenishment cycle to find the best solution. In the Refinement Heuristic, the multipliers were not considered until the T is determined and fixed. In such cases, it may have been beneficial to keep the T smaller and introduce multipliers for certain items to minimize total cost. These new algorithms incorporate these opportunities.

I incorporate the Refinement Heuristic into the subsequent algorithms, but it could also run independently. For example, consider a situation where a company fixes (or limits) T to convenient lengths of time (e.g., daily, weekly, monthly, etc.). With the fixed (or limited) common replenishment cycle, the only lever a purchasing manager can use is the multipliers to determine the number of cycles between orders for each particular item. In such a situation, Algorithm 3.4 can be used to determine the appropriate multipliers for a given T or perform sensitivity analysis on possible common replenishment cycle lengths.

Similar to Section 3.3, the Sequential Heuristic with Refinement (Algorithm 3.5a) and the BFB Heuristic with Refinement (Algorithm 3.5b) only change one step off the Full Enumeration Heuristic with Refinement (Algorithm 3.5). It again deals with the stopping condition of each heuristic (Step 5).

Algorithm 3.5: Full Enumeration Heuristic with Refinement

Step 1 For all $m = 0, 1, \dots, M - 1$

Run Algorithm 3.2.

Calculate:

$$Z^m(T^m) = \sum_{i=1}^I \left[\left(\frac{S_i}{T^m} \right) + \frac{T^m D_i}{2} h_i p_{ij}^m + p_{ij}^m D_i \right] + \left[V_m + b_m T^m \left(\sum_{i=1}^I k_i D_i - y_m \right) \right].$$

Step 2 Let $m' = \arg \min_m Z^m(T_m)$.

Then $T' = T^{m'}$, $Z' = Z^m(T^{m'})$, $Q_i' = T^{m'} D_i$ ($\forall i$), and $j_i' = j_i^{m'}$ ($\forall i$).

Step 3 Set $n_i^R = 1$ ($\forall i$), $T^R = T'$, $Z^R = Z'$, $Q_i^R = Q_i'$ ($\forall i$), and $j_i^R = j_i'$ ($\forall i$).

Run Refinement Heuristic.

If $Z^R < Z'$, then $n_i^* = n_i^R$, $T^* = T^R$, $Z^* = Z^R$, $Q_i^* = n_i^R T^* D_i$ ($\forall i$), and $j_i^* = j_i^R$ ($\forall i$).

Otherwise, $n_i^* = 1$ ($\forall i$), $T^* = T'$, $Z^* = Z'$, $Q_i^* = Q_i'$ ($\forall i$), and $j_i^* = j_i'$ ($\forall i$).

Step 4 Set $Z^B = Z'$.

Step 5 Calculate:

$$i_{up} = \arg \min_i \frac{q_{i(j_i'+1)}}{D_i} \text{ and } T_{up} = \min_i \frac{q_{i(j_i'+1)}}{D_i}.$$

Let m'' be the segment such that $T_{max}^{m''-1} < T_{up} \leq T_{max}^{m''}$. If $Z^{m''}(T_{up}) < Z^B$, then $T^* = T_{up}$,

$Z^B = Z^{m''}(T_{up})$, $Q_i^B = T_{up} D_i$ ($\forall i$), $j_{i_{up}}^B = j_{i_{up}}' + 1$, and $j_i^B = j_i'$ ($\forall i \neq i_{up}$).

Set $j_{i_{up}}' = j_{i_{up}}^B + 1$.

Step 6 Set $n_i^R = 1$ ($\forall i$), $T^R = T^B$, $Z^R = Z^B$, $Q_i^R = Q_i^B$ ($\forall i$), and $j_i^R = j_i^B$ ($\forall i$).

Run Refinement Heuristic.

If $Z^R < Z^*$, then

$n_i^* = n_i^R$ ($\forall i$), $T^* = T^R$, $Z^* = Z^R$, $Q_i^* = n_i^R T^* D_i$ ($\forall i$), and $j_i^* = j_i^R$ ($\forall i$).

Step 7 If $\sum_{i=1}^I j_i^* = IJ$, then STOP. Otherwise, go to Step 5.

Algorithm 3.5a: Sequential Heuristic with Refinement

Step 5 Calculate:

$$i_{up} = \arg \min_i \frac{q_{i(j_i^*+1)}}{D_i} \text{ and } T_{up} = \min_i \frac{q_{i(j_i^*+1)}}{D_i}.$$

Let m'' be the segment such that $T_{\max}^{m''-1} < T_{up} \leq T_{\max}^{m''}$. If $Z^{m''}(T_{up}) < Z^B$, then $T^* = T_{up}$,

$Z^B = Z^{m''}(T_{up})$, $Q_i^B = T_{up} D_i$ ($\forall i$), $j_{i_{up}}^B = j_{i_{up}}^* + 1$, $j_i^B = j_i^*$ ($\forall i \neq i_{up}$), and go to Step 6.

Otherwise, STOP.

Set $j_{i_{up}}^* = j_{i_{up}}^* + 1$.

Algorithm 3.5b: BFB Heuristic with Refinement

Step 5 For $j \in [j_i' + 1, J]$ and $\forall i$, calculate:

$$BFB_{qij} = \frac{P_{i(j-1)} - P_{ij}}{T_{qij}}.$$

Let $(i_{up}, j_{up}) = \arg \max_{(i,j)} BFB_{qij}$ and $T_{up} = T_{q(i_{up}, j_{up})}$.

Let m'' be the segment such that $T_{max}^{m''-1} < T_{up} \leq T_{max}^{m''}$. If $Z^{m''}(T_{up}) < Z^B$, then $T^* = T_{up}$,

$Z^B = Z^{m''}(T_{up})$, $Q_i^B = T_{up} D_i$ ($\forall i$), $j_{i_{up}}^B = j_{i_{up}}' + 1$, $j_i^B = j_i'$ ($\forall i \neq i_{up}$), and go to Step 6.

Otherwise, STOP.

Set $j_{i_{up}}' = j_{i_{up}}^B + 1$.

3.5 Numerical Analysis

To justify the benefits of the heuristics, it is critical to confirm that each heuristic is accurate and computationally efficient. Oftentimes the development of a heuristic stems from a problem for which the optimal solution cannot be determined, or it is computationally cumbersome to do so, even for small-scale problems. That is the case for this problem, and that makes it difficult to judge the accuracy of the heuristics. To validate the accuracy of each heuristic, I compare them against previously published solutions to the shared resource capacity allocation problem. As mentioned previously, this research is split into two areas based on the ordering structure. The first option, which I utilize, is a common replenishment cycle for all items. Alternatively, each item can be on an independent replenishment cycle (see Chapter Two). The numerical analysis will compare my solutions against solutions from not only other common replenishment cycle research, but also against independent replenishment cycle research.

For the computational efficiency of the heuristics, I explore the performance limitations of the heuristics as the size of the problem increases. Following the numerical analysis of Chapter Two, the two most influential characteristics of the problem that drive computation time are: (1) the number of items and (2) the number of segments in the piecewise-linear approximation. Therefore, I analyze each heuristic as those two characteristics grow in size to see how each heuristic performs.

3.5.1 Accuracy Check

For the accuracy check, I incorporate four examples. Three of them are equivalent to those seen in Chapter Two, and the fourth is from Güder and Zydiak (2000). The Güder and Zydiak (2000) example has four items, each of which has three price levels in its quantity discount schedule. Table 3.1 shows the solution to the Güder and Zydiak (2000) problem provided in that paper, as well as those provided by my three heuristics. For this particular example, the refinement policy does not change the solution. After identifying a price breakpoint solution, Güder and Zydiak (2000) allow for a modification to T . This results in their capacity and order quantities being slightly higher than mine. This change has negligible impact on the annual cost (my heuristics have costs that are mere pennies higher).

Solution	Z	K	Q_1	Q_2	Q_3	Q_4
GZ(2000)	\$37,823	2011	268	503	1274	1006
Full Enumeration	\$37,823	1999	267	500	1267	1000
Sequential	\$37,823	1999	267	500	1267	1000
BFB	\$37,823	1999	267	500	1267	1000

Moussourakis and Haksever (2008) test this example using their independent ordering structure algorithm and are able to duplicate the results from Güder and Zydiak (2000).

Additionally, they find that the common replenishment ordering structure does not perform well against the independent cycle ordering structure with fixed capacity, quantity discounts, and a large number of items. In fact, they found no cases where the common replenishment cycle ordering structure outperformed the independent cycle ordering structure with over thirty items. Naturally, this brings up the question of how the refinement policy compares against the two other ordering structures (independent cycle vs. common cycle).

Rubin and Benton (1993) and Moussourakis and Haksever (2008) explore three fixed capacity problems originally provided in Rubin and Benton (1993). The first and second examples have ten items and three price levels. The two examples are differentiated by the fixed capacity level (details provided in Appendix B). Example 1 has a fixed capacity level of 2,350 square feet and Example 2 has a fixed capacity level of 1,350 square feet. The third example has 15 items and four price levels (details provided in Appendix B). Table 3.2 shows the annual discretionary cost Z_D and capacity level K for each type of solution for each example. I define the non-discretionary cost Z_{ND} as the minimum purchasing cost if all the units were ordered at once, therefore achieving the lowest price per unit. Consequently, Z_D is the cost for which proper inventory and capacity management can impact. Note that $Z = Z_D + Z_{ND}$. For Examples 1 and 2, the non-discretionary cost totals \$105,809. Similarly for Example 3, the non-discretionary cost totals \$2,446,554. The independent cycle solution comes from Rubin and Benton (1993), the common cycle solution comes from the Full Enumeration Heuristic, and the refinement policy solution comes from the Full Enumeration with Refinement Heuristic.

Table 3.2: Comparison of the independent cycle solution, the common cycle solution, and the refinement policy solution.

Example	Independent Cycle		Common Cycle		Refinement Policy	
	Z_D	K	Z_D	K	Z_D	K
1	\$3421	1482	\$8402	2266	\$3852	1705
2	\$3450	1350	\$10,306	1133	\$9601	1323
3	\$140,373	34,992	\$248,040	16,462	\$212,625	21,257

As found by Moussourakis and Haksever (2008), the independent cycle solution outperforms the common cycle solution for these three examples. The refinement policy significantly helps the common cycle solution to reduce the cost gap between the two solutions, but it cannot fully overcome the difference. It is interesting to note that in Example 3, the required capacity with the refinement policy is significantly less than the required capacity with the independent cycle ordering structure. The reduction in capacity is one of the major benefits of a common replenishment cycle approach, but in turn the capacity reduction has minimal impact on cost for these three examples.

For all three examples, the performance of the Sequential Heuristic and BFB Heuristic is similar to the Full Enumeration Heuristic. In Example 1, the original solutions and refined solutions for each heuristic are identical. For Example 2, the Full Enumeration and Sequential Heuristics have identical original solutions, with the BFB Heuristic performing slightly worse. All three have identical refined solutions for Example 2. Finally, in Example 3, the Full Enumeration and BFB Heuristics have identical original solutions, but the Full Enumeration and Sequential Heuristics have identical refined solutions. The Example 3 result is interesting, but in general the BFB performs worst with the refinement. This likely is due to the BFB Heuristic favoring higher T values as it seeks out the largest discount. In the refinement heuristic, it is

beneficial to keep T low, and instead raise the multipliers n_i as necessary for a more tailored procurement policy.

The sub-optimal performance of the common replenishment cycle ordering structure in the previous examples brings up an interesting question: “Under what circumstances does the common replenishment cycle ordering structure outperform the independent replenishment cycle ordering structure?” Moussourakis and Haksever (2008) indicate that the independent cycle approach outperforms the common cycle approach with a fixed capacity level and all-units quantity discounts for all examples with over 30 items. Intuitively, the common cycle approach performs best with significant capacity limitations. This can come in two forms: a very restrictive fixed capacity or a very high capacity expansion cost coupled with a low (or no) initial capacity level. In fact, in my numerical analysis I found multiple examples with up to 200 items for which the common cycle approach outperforms the independent cycle approach by up to 5%. The shared characteristic of all these problems is a very restrictive (or expensive) capacity cost.

Given this insight, I believe there are two valuable situation where the common cycle ordering structure and its refinement policy are favored over the independent cycle ordering structure. The first is when there are serious restrictions on capacity cost (<50% of ideal, unconstrained capacity) or when there are significant costs associated with adding or maintaining extra capacity. The second is when a firm currently utilizes a common replenishment ordering structure (i.e., order daily, weekly, bi-weekly, etc.). Algorithm 3.4 makes it easy to find the appropriate multipliers for a given T , or perhaps run sensitivity analysis on potential common cycle lengths. Such a situation aligns nicely with a periodic review inventory management system (Heizer and Render, 2014). A primary justification for the periodic review system over a

continuous review system is convenience. Therefore that “cost” needs to be considered when comparing the common cycle approach to the independent cycle approach.

3.5.2 Computational Efficiency Check

The computation time for the independent cycle approach to the shared resource capacity allocation problem seen in Chapter Two is heavily tied to the number of items in the inventory system and the number of segments in the piecewise-linear approximation. To analyze the computational efficiency, I use the same data set from Chapter Two. This sub-experiment uses randomly generated data for up to 15,000 items, where each item has five price levels in its respective quantity discount schedule. The initial capacity level is a fraction of the sum of the unconstrained EOQ capacities for each item, and capacity expansion is allowed using: $G(K) = 1000(K - K_0)^{0.6}$ if $K > K_0$. Table 3.3 provides the distributions for which I randomly generated the necessary data.

Table 3.3: Distributions used to generate data for other system parameters used in the computation time analysis.

$D_i \sim U(400, 2500)$	$S_i \sim U(10p_{i0}, 20p_{i0})$
$h_i \sim U(0.20, 0.40)$	$k_i \sim U(0.5, 1.5)$
$p_{i0} \sim U(1, 15)$	$q_{i0} = 0$
$p_{i1} = p_{i0} - \alpha_{i1}$, where $\alpha_{i1} \sim U(0.01, 0.30)$	$q_{i1} = (EOQ_{i0})\gamma_{i1}$, where $\gamma_{i1} \sim U(0.2, 2.0)$
$p_{ij} = p_{i(j-1)} - \beta_{ij}$, where $\beta_{ij} \sim U(0.00, 0.20)$	$q_{ij} = q_{i(j-1)}(1 + \delta_{ij})$, where $\delta_{ij} \sim U(0.5, 1.0)$

The results in Table 3.4 illustrate that the original heuristics (without refinement) are very efficient in terms of computation time—even faster than the independent cycle approach from Chapter Two. Unfortunately, this quick technique also provides the worst total cost. As expected, the Full Enumeration Heuristic has the longest computation time, but with that being said, it is capable of solving problems with 15,000 items and 500 piecewise-linear segments in under thirty

minutes. Both the Sequential Heuristic and BFB Heuristic perform very well in terms of computation time. Each heuristic can solve large-scale problems in just over a minute. Across all three heuristics it is interesting to note that the number of piecewise-linear segments has minimal impact on the computation time of each heuristic. This differs from what we observed in Chapter Two, and the primary reason is the way in which the algorithms work. In Chapter Two, the algorithm checks a portion of the segments for possible alternative solutions, whereas these heuristics are not searching through the segments as much as they are through the items. Therefore the number of segments has minimal impact on the computation time, primarily coming through the determination of the initial solution (before considering price breakpoints).

Table 3.4: Computation time (in minutes) as a function of the number of segments and the number of items ($a = 0.4$, $r = 1000$, $J = 5$, $K_0 = 0.8 \sum EOQ_i$).

Number of Items	Number of Segments					
	Full Enumeration		Sequential		BFB	
	25	500	25	500	25	500
1,500	0.35	0.47	0.02	0.14	0.02	0.14
3,000	1.24	1.55	0.04	0.30	0.04	0.29
5,000	3.29	3.82	0.08	0.51	0.06	0.49
10,000	12.18	13.39	0.10	0.97	0.14	1.01
15,000	25.73	27.35	0.14	1.39	0.23	1.49

Table 3.4 intentionally does not consider the three heuristics with refinement. The introduction of the refinement policy significantly increases the computation time. To measure the benefit of the refinement policy, Table 3.5 shows the original solution and refined solution using the Full Enumeration and Full Enumeration with Refinement Heuristics, respectively. In Table 3.5, I compare the computation time as well as the savings through the introduction of the refinement policy for multiple sized problems. Each problem utilizes 25 piecewise-linear segments and five price levels for each item. The original solutions are all solved in under a

minute, while the refinement solution for 1500 items takes roughly 3.5 hours. The cost savings through the introduction of the refinement policy are significant dollar figures, but are all less than 5% of total cost.

Table 3.5: Computation time (in minutes) of the Full Enumeration Heuristic and the Full Enumeration Heuristic with refinement. The cost savings are from the refinement policy.

Number of Items	Full Enumeration	Full Enumeration w/ Refinement	Cost Savings
250	0.01	1.04	\$2238
500	0.04	8.17	\$8668
1000	0.15	63.93	\$24,212
1500	0.32	214.80	\$35,815

3.5.3 Expansion Opportunities

Similar to Chapter Two, a primary benefit of this solution technique is the ability of the algorithms to incorporate opportunities to include the warehouse capacity as a decision variable. Up to this point, I only consider fixed capacity examples from prior literature, and therefore have not fully explored the expansion opportunities as they relate to the common replenishment cycle approach. Let me consider the expansion example from Chapter Two. Namely, utilizing Example 3 from above, but changing the initial capacity level to 18,000 square feet (this provides a more restrictive capacity level than the one in Example 3). Additionally, I consider four options for capacity cost: (1) fixed capacity (FC), (2) linear cost (LC), (3) concave cost (CC), and unconstrained capacity (UC). The fixed capacity solution provides a baseline to compare the expansion opportunities against. The linear cost function is:

$$G(K) = \begin{cases} 2(K - K_0) & \text{if } K \geq K_0 \\ 0 & \text{otherwise.} \end{cases} \quad (3.15)$$

Similarly, the concave cost function is:

$$G(K) = \begin{cases} 50(K - K_0)^{0.6} & \text{if } K \geq K_0 \\ 0 & \text{otherwise.} \end{cases} \quad (3.16)$$

Finally, the unconstrained capacity solution eliminates capacity cost to see what the maximum capacity level would be for that inventory system with the common replenishment cycle approach. This setup is identical to that of Chapter Two, which will provide some unique comparisons between the two approaches when there are opportunities for capacity expansion.

Table 3.6: Comparing expansion opportunities between the independent cycles approach, the common cycle approach, and the common cycle approach with refinement.

Solution	Independent Cycles		Common Cycle		Refinement Policy	
	Z_D	K	Z_D	K	Z_D	K
FC	\$137,311	16,387	\$248,040	16,462	\$215,115	17,909
LC	\$120,005	25,266	\$248,040	16,462	\$214,513	18,558
CC	\$114,974	30,194	\$248,040	16,462	\$213,951	18,558
UC	\$98,395	35,855	\$248,040	16,462	\$212,625	21,257

Table 3.6 compares the two approaches for all four solution types. The independent replenishment cycles approach outperforms the common replenishment cycle approach regardless of the functional form of capacity cost. Additionally, it is interesting to note that the original common replenishment cycle solution *never* recommends capacity expansion for this particular problem; meanwhile, when available, the refined solution *always* recommends capacity expansion. Despite the constant recommendation for capacity expansion for the refined solutions, the expansion size never exceeds a 2,000 square foot expansion. On the contrary, the independent cycle approach recommends expansions of 7,000 – 12,000 square feet with the same capacity cost functions. Even with an unconstrained capacity cost, the refined common replenishment cycle solution does not exceed a 3,257 square foot expansion. The savings in capacity expansion size do not correspond to lower costs. The fixed capacity independent cycle

solution is cheaper than any expansion recommended by the common replenishment cycle approach.

The common cycle solutions and refined solutions in Table 3.6 come from the Full Enumeration Heuristic and Full Enumeration with Refinement Heuristic, respectively. Similar to the examples seen previously, the other heuristics performed similarly. The Sequential Heuristic with Refinement duplicates the refined solution, but the original solution performs slightly worse. The BFB Heuristic duplicates the original solution, but the refinement solution is slightly worse.

Further analysis of the common replenishment cycle approach with refinement shows that the 3,257 square foot expansion (with no capacity cost) only results in savings of roughly \$2,500. Therefore, the utilization of a common replenishment cycle (even with refinement) leads to a reduced likelihood of expanding capacity. Despite the decreased likelihood of needing an expansion, this solution technique is still quite valuable for the potential capacity reduction that could be possible through the use of a common replenishment cycle (see discussion in Section 3.5.2 on the beneficial scenarios for the common cycle approach). Additionally, having the capacity as a decision variable adds another lever to a manager's toolbox to optimize his or her business.

3.6 Conclusions

The objective of this chapter began as an alternative solution technique to solve the shared resource capacity allocation problem utilizing a common replenishment cycle approach. The goal was to find a way to avoid the worst-case scenario associated with the independent cycle approach to the problem where a firm must plan for a situation where all items are ordered

simultaneously. The common replenishment cycle approach eliminates that situation by phasing the replenishment of each item throughout the common cycle. Despite this benefit, the inventory-related costs grow significantly with this technique as many items are not replenished near their optimal levels as if they were considered independently. The introduction of the refinement policy does help by adding more flexibility to the procurement policy, but causes a significant increase in computation time. The common cycle approach thrives in situations with a very restrictive fixed capacity and expensive expansion opportunities for the shared resource capacity.

My model and solution heuristics contribute to the literature in multiple ways. First, no prior research simultaneously considered the common replenishment cycle approach to this problem while allowing capacity as a decision variable in the presence of all-units quantity discounts. This research found that this approach is less likely to consider a capacity expansion, but it may very well take advantage of a capacity reduction (which this model can easily incorporate). Second, there is high practicality of this approach given the convenience factor that cannot be overstated. Firms continue to use the periodic review inventory management system, and this approach mirrors the logic and convenience of that inventory management system. Utilizing the independent cycle approach with an inventory system of thousands of items can result in a purchasing manager making purchasing decisions constantly, instead of setting up a convenient, logical, and repeatable system. The ability to utilize the refinement procedure for a fixed cycle length T can be valuable for firms that have a policy in place that fixes T or that may want to run sensitivity analysis on potential values of T . In either situation, they can find the appropriate multipliers for each item.

CHAPTER FOUR
MANUFACTURERS' CHANNEL STRUCTURES WHEN SELLING ASYMMETRIC
COMPETING PRODUCTS

Abstract

Manufacturers often face the fundamental channel structure decision, namely, whether to sell their products directly to consumers or indirectly through an intermediary. To address this issue, extensive research has analyzed equilibrium channel structures for competing, equally substitutable products. However, in reality, many competing products are asymmetric in both substitutability and brand equity. In this chapter, I study how these two asymmetric characteristics affect the equilibrium channel structures of manufacturers who sell competing products. Following some recent studies, I adopt a refined consumer demand model derived from the representative consumer utility function. Based on this model, I examine three possible types of competition between the two supply chains: Cournot (quantity) competition, Bertrand (price) competition, and Bertrand-Cournot competition. The results show that brand equity, substitutability, as well as the type of competition play important roles in determining the equilibrium channel structures. Specifically, in equilibrium a manufacturer always sells directly when its rival competes on quantity. Moreover, when there is sufficient asymmetry in either brand equity or substitutability, manufacturers tend to sell directly. These results demonstrate the benefits of selling indirectly shown in previous studies depend critically on the assumptions of equally substitutable products and Bertrand competition.

4.1 Introduction

The rise of internet-based commerce has increased the importance of understanding when selling directly benefits a manufacturer and its supply chain. This is the classic channel structure decision with two alternatives: selling directly, or selling indirectly through an intermediary such as an independent retailer. Intuition leads us to believe that selling directly helps improve profitability as it eliminates double marginalization. But, research shows that under certain scenarios, selling indirectly is the equilibrium channel structure (e.g., see McGuire and Staelin 1983).

The oil industry is currently deliberating on this channel structure or vertical integration decision. Historically, oil companies followed John Rockefeller's business model of complete vertical integration for Standard Oil. The oil giants desired control of the entire operation from the oil fields all the way to the pump. In the last five years, their business models have begun to transform. ConocoPhillips recently split off from their downstream operations of refining and marketing. ExxonMobil is considering a similar business model. The downstream operations are low-margin operations for the oil giants due to "the high cost of crude and stiff competition at the pump" (Jelter 2012). The fierce downstream competition is forcing large oil companies to reconsider their previous vertically integrated business model.

Most existing literature on channel structure decisions makes two important assumptions: the manufacturers' products are equally substitutable, and the manufacturers compete on price. When comparing two manufacturers and their products, inevitably there are differences or asymmetries between the two products. These asymmetries can occur in a variety of forms, and for a variety of reasons. In this chapter, I focus on two types of asymmetries: brand equity (recognition) and product substitutability. In 2012, *Time Magazine (online)* featured an article discussing the effect of brand premiums on consumers, "No matter if we're talking about cereal,

cough syrup or batteries, products featuring nationally recognized name brands tend to cost more than their generic store-brand counterparts” (Tuttle 2012). In other words, consumers often pay a premium for brands that they recognize due to their national branding or advertising. Therefore, a natural question to ask is, “As the national brand (local brand) manufacturer, does your superior (inferior) brand equity or recognition affect your channel structure decision?”

Two competing products with some common functionality often have some level of substitutability. They can be perfect substitutes, but typically they have features that distinguish them. Historically, academic models assume this substitutability is symmetric from product A to product B, and vice versa. More realistically, due to the characteristics and functionality of the two competing products, consumers will have asymmetric likelihoods of substituting their preferred product for its competitor. Thus asymmetric product substitution is a very realistic and likely possibility. The asymmetric substitution can stem from the purpose of the product (all-in-one product vs. special purpose product) or features of the product (De Jaegher 2009, Kim and Bell 2011).

For example, Microsoft’s Xbox One and Sony’s PlayStation 4 are the two new generation video game consoles. These two consoles are substitutable, but have asymmetric characteristics in purpose and features. Microsoft markets Xbox One as the ultimate entertainment console, or all-in-one product, whereas Sony markets PlayStation 4 as a more game-focused console, or specific purpose. Similarly, for features, Xbox One has a superior motion-based gaming platform and PlayStation 4 allows for users to access their system through remote connection (Loveridge 2014). Despite these differences the two products are undeniably substitutable, but a consumer’s willingness to substitute is highly variable based on their interest in the purpose and features of their preferred product.

In economics and operations management research, duopoly competition traditionally comes in two forms: Bertrand (price) and Cournot (quantity). Firms often choose to set prices, not quantities, and hence Bertrand competition is more common (Nicholson and Snyder 2008). Despite the popularity of Bertrand competition, Cournot competition should not be ignored as it provides unique equilibriums, and is used when there are capacity limitations, or more generally, decreasing returns to scale (Mas-Colell et al. 1995). A third mixed form of competition is Bertrand-Cournot competition (e.g., see Tremblay et al. 2010). In Bertrand-Cournot competition, one firm (supply chain) competes on price, while the other competes on quantity. A motivating example is in the automobile industry, “Saturn and Scion dealers set prices and Honda and Subaru dealers set quantities” (Tremblay et al. 2010). In this particular case, the stimulus for this new form of competition is the available space on the car lot. Saturn and Scion often utilize a smaller car lot, and consequently have changed their form of competition when competing against a larger dealer such as Honda or Subaru.

This chapter makes the following contributions. First, while previous research typically focuses on equally substitutable products, I study competing products that are asymmetric in both substitutability and brand equity. This study’s analysis shows that with sufficient asymmetry in either substitutability or brand equity, the incentives to sell indirectly are removed, and manufacturers tend to sell directly. Second, while most of the prior literature analyzes only Bertrand competition, this study investigates three possible types of competition: Bertrand, Cournot, as well as Bertrand-Cournot. The subsequent analysis shows that a manufacturer always sells directly when their rival competes on quantity. In conclusion, these results demonstrate that the assumptions of equal substitutability and Bertrand competition critically limit our understanding of the equilibrium channel structures.

4.2 Literature Review

McGuire and Staelin (1983) wrote a seminal piece on channel structure considering a duopoly where two exclusive channels sell two products which are equally substitutable. Their primary objective is to determine the equilibrium channel structure (selling directly or indirectly) under Bertrand competition with varying levels of (symmetric) product substitutability. One of their key results is the counterintuitive finding that selling indirectly is beneficial for competing manufacturers (and supply chains) when the product substitutability is high (the competition is fierce). This allows the manufacturers to remove themselves from the intense competition at the retail level and profit via a wholesale price offered to the retailer. This interesting result led several authors to do extensions off of this fundamental piece.

Moorthy (1988) reexamines the channel structure decision when the products are now strategic substitutes or complements. He argues that the channel structure decision does not depend on the level of demand substitution (as suggested in McGuire and Staelin (1983)), but instead depends on whether the products are strategic substitutes or complements. Choi (1991) adds a common retailer to see how the new channel organization affects the equilibrium channel structures. Trivedi (1998) takes Choi (1991) one step further by including retailer substitutability in addition to product substitutability. This introduces a full channel option where both manufacturers sell to both retailers. The retailers then sell competing products in their stores, and they also compete against the rival retailer who sells the same two products. Ha and Tong (2008) continues this research stream on two exclusive supply chains, but they examine Cournot competition and analyze the impact of contracting and information sharing. Xiao and Yang (2008) as well as Xiao and Choi (2009) introduce risk-averse players into the channel structure decision to see how it impacts the equilibrium solutions. Cao et al. (2010) also explore the channel structure decision, but introduce demand uncertainty and use retail price maintenance.

The manufacturer must then balance the incentive of gaining more accurate demand information with the inefficiency of selling directly, instead of selling through an independent retail store.

Lus and Muriel (2009) revisit the demand functions derived in McGuire and Staelin (1983). The latter found that as the two products become more substitutable, the total market size, manufacturer (and supply chain) profits, retail prices, and wholesale prices (in the decentralized channel structure) all increase. These outcomes are a result of the formulated demand function for each channel, and are counterintuitive. As products become more substitutable, the competition level increases, and therefore the prices (and profits) are typically driven down. Lus and Muriel (2009) argue that a new form of demand function derived from the consumer utility function is more realistic and intuitive in terms of how prices and profits change as products become more substitutable. The utility function Lus and Muriel (2009) use is adopted from Singh and Vives (1984). In this chapter, I adopt these demand functions, but generalize them to incorporate the asymmetric parameters.

Despite several academic articles (e.g., see De Jaegher 2009, Kim and Bell 2011, Wu et al. 2012) discussing the potential and usefulness of a model addressing asymmetric substitution, to the best of my knowledge, there are no papers that model channel structure decisions with asymmetric substitution, and I aim to fill that research gap. Shah and Avittathur (2007) discuss one-way substitution, which is an extreme form of asymmetric substitution, when examining the relationship between a customized product and a standard product. Their goal is to determine the correct mixture of customized and standard products. Cao et al. (2010) use asymmetric market share with symmetric substitution to model the channel structure decision with demand uncertainty. Choi and Fredj (2013) recommend the introduction of asymmetric channels,

specifically when the channels are asymmetric in terms of brand equity (i.e., a national brand and a local brand).

Historically, marketing literature has been the primary location for research regarding brand equity or brand recognition, but more and more researchers in operations management are exploring its effects on operations topics (e.g., see Kurata et al. 2007, Groznik and Heese 2010). A national brand typically develops brand equity through national advertising. This brand equity allows the national brand to charge a premium for their products as consumers perceive a relationship between the quality of the product and the brand name (Cunningham et al. 1982). Sethuraman (2003) found that, “consumers state that they would be willing to pay a premium of about 37% for national brands over store brands.” Given the asymmetric pricing between the national brand and local brand, a natural question is, “How do changes in price by the national (local) brand affect the competitor?” Blattberg and Wisniewski (1989) as well as Sethuraman and Srinivasan (2002) examine how a price cut by the high-price (national) brand compares against a price cut by the low-price (local) brand in terms of the effects on sales or market share of the competitor. They argue that the effects are not symmetric. Pauwels and Srinivasan (2004) extend off this point by examining the effect of a store brand entering at a retailer that currently offers a national brand. They find that the entry of a store brand increases the national brand price at the retail and wholesale levels. They contend that the national brand focuses on the quality conscious market segment, and thus does not compete directly with the store brand on price. I apply many of these concepts and ideas to the channel structure decision.

In a duopoly with deterministic demand, traditionally economics literature has two primary forms of competition: Bertrand and Cournot. In Bertrand (Cournot) competition the firms set the retail prices (sales quantities), and the market then subsequently sets the sales

quantities (retail prices) (e.g., see Singh and Vives 1984). Literature typically focuses on Bertrand competition, as many firms set prices, but Cournot competition is still very common (e.g., see Yang and Zhou 2006, Arya et al. 2008a, Wu et al. 2008). A third form of competition, Bertrand-Cournot, is a mixed form of competition, in which one channel competes on price while the other competes on quantity (e.g., see Tremblay and Tremblay 2011). In this chapter, I incorporate all three forms of competition to see how they affect equilibrium decisions on channel structure.

4.3 Model

4.3.1 Notation

w_i	wholesale price offered by manufacturer i to retailer i , $i = 1,2$
c_i	production cost for manufacturer i , $i = 1,2$
p_i	retail price offered by retailer (manufacturer) i to consumer with indirect (direct) sales, $i = 1,2$
q_i	sales quantity by retailer (manufacturer) i with indirect (direct) sales, $i = 1,2$
$\Pi_{h,i}$	profit of h in supply chain i , $i = 1,2$, $h = M$ (manufacturer), R (retailer)
a_i	base price of product sold by supply chain i , $i = 1,2$
b_i	the degree to which product i can substitute product j , where $i, j = 1,2$, $i \neq j$
k	percent price premium of Manufacturer 2 over Manufacturer 1 resulting from the difference in brand equity

4.3.2 Model Development

In the model I consider two manufacturers selling differentiated and competing products through exclusive channels. Unlike most existing studies, I study the more realistic setting where the two products are asymmetric in terms of both brand equity and substitutability. I focus on the strategic channel structure decision, that is, whether each manufacturer should sell directly (e.g., through an online channel or a manufacturer-owned retail store) or indirectly (e.g., through an independent retail store). If a manufacturer sells through an intermediary, then they set a

wholesale price for the retailer, who then sets a retail price for the consumers. Meanwhile, if a manufacturer chooses to sell directly, they set a retail price for the consumers.

Consistent with prior literature, there are a few key assumptions needed to allow for tractable solutions. First off, demand is assumed to be deterministic and a linear function of price. This assumption coupled with a single-period model results in the order quantity of the retailer being equivalent to the sales quantity. Therefore order quantity and sales quantity can be used interchangeably. Secondly, each manufacturer utilizes a linear wholesale contract to sell their single product to their exclusive retailer (when selling indirectly). Finally, manufacturers and retailers behave rationally, and there is no collusion between the two manufacturers and/or the two retailers.

Given that each manufacturer can sell directly to consumers or indirectly through an intermediary, there are four possible channel structure scenarios: (i) both manufacturers sell directly to consumers, (ii) both manufacturers sell indirectly through an intermediary, (iii) Manufacturer 1 sells directly and Manufacturer 2 sells indirectly through intermediary, and (iv) Manufacturer 1 sells indirectly through an intermediary and Manufacturer 2 sells directly. With symmetric substitution and brand equity scenarios (iii) and (iv) are equivalent, but with the introduction of asymmetric characteristics it is crucial to distinguish between the two scenarios.

For each of the four scenarios, each supply chain member maximizes their profit. When selling indirectly, each manufacturer sets its optimal wholesale price to maximize its profit function:

$$\Pi_{M,i} = (w_i - c_i) \cdot q_i(w_i, w_j), \quad i, j = 1, 2 \quad i \neq j. \quad (4.1)$$

In response to the wholesale prices offered by the manufacturers, each retailer sets its optimal retail price (order quantity) in Bertrand (Cournot) competition to maximize its profit function:

$$\Pi_{R,i} = (p_i - w_i) \cdot q_i(w_i, w_j), \quad i, j = 1, 2 \quad i \neq j. \quad (4.2)$$

Demands and profits are then realized. When the manufacturer sells indirectly the solution is found using backward induction, first solving for the retailers' retail prices (order quantities) based off (4.2), and then using (4.1) to find the manufacturers' wholesale prices. Similarly, if a manufacturer sells directly, the independent retailer is removed, and the manufacturer sets a retail price (order quantity) to maximize its profit function:

$$\Pi_{M,i} = (p_i - c_i) \cdot q_i(p_i, p_j), \quad i, j = 1, 2 \quad i \neq j. \quad (4.3)$$

Upon finding the optimal values, I find the corresponding manufacturers' profits (can also be done for supply chains' profits). Finally, the Nash equilibrium is found for each of the four scenarios using the manufacturers' profits.

I implement the solution technique described above for three types of competition: Bertrand, Cournot, and Bertrand-Cournot. The form of competition determines the functional form of the demand function used to determine the optimal wholesale prices, order quantities (retail prices), and ultimately the Nash equilibrium for the channel structure. The following subsections show the demand functions for each form of competition.

4.3.3 Cournot Competition

As mentioned previously, the general demand framework comes from Lus and Muriel (2009) and the utility function they advocate. After introducing modifications to allow for asymmetric parameters, the (inverse) demand functions for quantity competition are as follows:

$$p_1 = a_1 - q_1 - b_2 q_2, \quad (4.4)$$

$$p_2 = a_2 - q_2 - b_1 q_1. \quad (4.5)$$

To allow for asymmetric brand equity ($a_1 \neq a_2$), consider introducing a simplification by using k which is defined as the percent price premium charged by Manufacturer 2 over Manufacturer 1 as a result of the difference in brand equity. Therefore, when $k > 0$ Manufacturer 2 is the national brand with accrued brand equity. By allowing k to be positive or negative, each manufacturer can be the national brand in a single model. The new (inverse) demand functions are as follows:

$$p_1 = a - q_1 - b_2 q_2, \quad (4.6)$$

$$p_2 = (1+k)a - q_2 - b_1 q_1. \quad (4.7)$$

This new formulation does not change anything structurally, but it allows me to vary a single parameter (k) in the comparative study in Section 4.4.

To demonstrate the solution procedure, consider the following steps to find the optimal solutions under Cournot competition when both manufacturers sell indirectly with asymmetric product substitutability and brand equity. First, insert the (inverse) demand equations ((4.6) and (4.7)) into the retailers' profit functions (4.2). That gives me the following updated retailers' profit functions:

$$\Pi_{R,1} = (a - q_1 - b_2 q_2 - w_1)q_1, \quad (4.8)$$

$$\Pi_{R,2} = ((1+k)a - q_2 - b_1 q_1 - w_2)q_2. \quad (4.9)$$

Then use the first order conditions (FOCs) of the profit functions ((4.8) and (4.9)) to find the optimal values of q_1 and q_2 . It can easily be proven that the FOCs guarantee optimality. After solving the system of equations ((4.8) and (4.9)), the optimal order quantities are:

$$q_1^*(w_1, w_2) = \frac{(2 - (1+k)b_2)a - 2w_1 + b_2 w_2}{4 - b_1 b_2}, \quad (4.10)$$

$$q_2^*(w_1, w_2) = \frac{(2(1+k) - b_1)a - 2w_2 + b_1w_1}{4 - b_1b_2}. \quad (4.11)$$

Next, use the results of (4.10) and (4.11) to update the manufacturers' profit functions (4.1).

Once again, use the FOCs, this time of the manufacturers' profit functions, to find the optimal wholesale prices:

$$w_1^* = \frac{(8 - 2(1+k)b_2 - b_1b_2)a + 8c_1 + 2b_2c_2}{16 - b_1b_2}, \quad (4.12)$$

$$w_2^* = \frac{(8(1+k) - 2b_1 - (1+k)b_1b_2)a + 2b_1c_1 + 8c_2}{16 - b_1b_2}. \quad (4.13)$$

The final step is updating the retailers' order quantities to be a function of only the product substitution and the brand equity parameters. To achieve this, insert (4.12) and (4.13) into (4.10) and (4.11). This provides me with the following updated optimal order quantities:

$$q_1^* = \frac{2[(8 - 2(1+k)b_2 - b_1b_2)a - (8 + b_1b_2)c_1 + 2b_2c_2]}{(16 - b_1b_2)(4 - b_1b_2)}, \quad (4.14)$$

$$q_2^* = \frac{2[(8(1+k) - 2b_1 - (1+k)b_1b_2)a + 2b_1c_1 - (8 - b_1b_2)c_2]}{(16 - b_1b_2)(4 - b_1b_2)}. \quad (4.15)$$

With all of this information, the simplified profit functions for Cournot competition when both manufacturers sell to an intermediary and all parameters are asymmetric are:

$$\Pi_{M,i} = \frac{2[(8 - 2(1+k)b_j - b_1b_j)a - (8 + b_1b_j)c_i + 2b_jc_j]^2}{(16 - b_1b_j)^2(4 - b_1b_j)}, \quad i, j = 1, 2 \quad i \neq j, \quad (4.16)$$

$$\Pi_{R,i} = \frac{4[(8 - 2(1+k)b_j - b_1b_j)a - (8 - b_1b_j) + 2b_jc_j]^2}{(16 - b_1b_j)^2(4 - b_1b_j)}, \quad i, j = 1, 2 \quad i \neq j. \quad (4.17)$$

Similar formulations have been done for every combination of channel structure, form of competition, and form of parameters (symmetric or asymmetric). For brevity those have been excluded, but details are available upon request.

4.3.4 Bertrand Competition

Bertrand competition is the most common form of competition in channel structure research. By taking the consumer utility function derived (inverse) demand functions from Cournot competition and solving for q_1 and q_2 the demand functions are as follows:

$$q_1 = \frac{a - (1+k)ab_2}{1 - b_1b_2} - \frac{1}{1 - b_1b_2} p_1 + \frac{b_2}{1 - b_1b_2} p_2, \quad (4.18)$$

$$q_2 = \frac{(1+k)a - ab_1}{1 - b_1b_2} + \frac{b_1}{1 - b_1b_2} p_1 - \frac{1}{1 - b_1b_2} p_2. \quad (4.19)$$

Each retailer simultaneously determines its retail price to offer their customers, and the market in turn sets the demand (sales quantities).

4.3.5 Bertrand-Cournot Competition

Recall that Bertrand-Cournot competition has one manufacturer (without loss of generality I assume it is Manufacturer 1) that competes on price while the rival manufacturer (assume it is Manufacturer 2) competes on quantity. Notice that by allowing k to be positive or negative, I can allow the manufacturer with higher brand equity to compete on price or quantity in a single model. Following similar steps to Section 4.3.4, the derived demand function for Manufacturer 1 and the (inverse) demand function for Manufacturer 2 are:

$$q_1 = a - p_1 - b_2q_2, \quad (4.20)$$

$$p_2 = (1+k)a - ab_1 + b_1p_1 - (1 - b_1b_2)q_2. \quad (4.21)$$

4.4 Results and Discussion

4.4.1 Symmetric Parameters

The analysis begins with the base case, when product substitution and brand equity are symmetric between the two products. In other words, I now have: $b = b_1 = b_2$ and $k = 0$. Table 4.1 displays the equilibrium channel structures, as well as the dominant structures, for each form of competition as the (symmetric) product substitutability varies. The notation (I,D) implies that Manufacturer 1 opts to sell indirectly (I), while Manufacturer 2 opts to sell directly (D). The equilibrium structure is the Nash equilibrium of the duopoly which is well known from the economics literature. A Pareto improvement occurs when a collaborative deviation from the Nash equilibrium benefits (or at least does not harm) both parties. Meanwhile, a dominant strategy can only occur when there are multiple Nash equilibriums, and the dominant strategy is the strategy that benefits both parties (higher profits). It is possible that a dominant strategy does not exist. Appendix C provides an example of how to determine the Nash equilibrium under Cournot competition with symmetric parameters.

Under Bertrand competition, as the product substitutability increases (or competition stiffens) the two manufacturers find it beneficial to put an intermediary between themselves and the fierce competition at the retail level. When the substitutability parameter is $0.7078 \leq b \leq 0.9309$ both manufacturers are more profitable selling indirectly. However, they find themselves in the famous prisoners' dilemma from economics. If the two prisoners (manufacturers) cooperate they can both reduce their prison sentences (improve profitability). Although, if one prisoner, without loss of generality say Prisoner 1 (Manufacturer 1), chooses to cooperate and choose the non-Nash equilibrium solution, and the other prisoner (manufacturer) does not cooperate, then the cooperating Prisoner 1 (Manufacturer 1) ends up with the longest

sentence (lowest profit). In other words, the prisoner (manufacturer) who tries to better both parties ends up with the worst-case scenario if the competitor acts selfishly to minimize their sentence (maximize their profits). This possible worst-case scenario drives both parties stay to the Nash equilibrium and consequently, they do not achieve their highest potential profits, but instead go with the “safe” equilibrium choice factoring in the response of their competitor. Finally, at extremely high substitutability ($b \geq 0.9309$) there is a dual Nash equilibrium, but the dominant strategy is to have both manufacturers sell indirectly.

Table 4.1: Symmetric products under different forms of competition.			
Competition	Nash Equilibrium		Notes
Bertrand	$0 \leq b \leq 0.7078^*$	(D,D)	
	$0.7078^* \leq b \leq 0.9309$	(D,D)	Pareto improvement to (I,I)
	$0.9309 \leq b \leq 1$	(I,I), (D,D)	Dominant strategy is (I,I)
Cournot	$0 \leq b \leq 1$	(D,D)	
Bertrand-Cournot	$0 \leq b \leq 0.8433^\&$	(D,D)	
	$0.8433^\& \leq b \leq 0.8944$	(D,D)	Pareto improvement to (I,I)
	$0.8944 \leq b \leq 1$	(D,I)	

*Assumes $c_1 = c_2$. As $|c_2 - c_1|$ increases, the critical value also increases.
&Assumes $c_1 = c_2$. As $c_1 - c_2$ increases, the critical value also increases.

The less studied, but equally interesting form of competition is Cournot competition. If the two manufacturers are competing on quantities, then the Nash equilibrium is always to sell directly for both manufacturers regardless of the strength of the competition (level of substitutability). A likely reason for this result is the basic reaction of the market to a duopoly in Bertrand competition compared to Cournot competition. For homogeneous products, a duopoly in Bertrand competition creates “perfect competition,” and each firm is forced to set their retail price to marginal cost. Meanwhile, in a duopoly, Cournot competition does not reach “perfect

competition,” but instead the marketplace becomes more competitive as the number of firms grows in the market (Nicholson and Snyder 2008). The stronger level of competition in the Bertrand model leads the manufacturers to desire a buffer between themselves and the competition, and thus introduces the opportunity for selling indirectly. The “weaker” Cournot competition never applies enough pressure on the manufacturers for them to want to remove themselves from selling directly to the consumers.

In Table 4.1, it is interesting to note that the behavior of the Bertrand-Cournot equilibrium channel structure is very similar to the Bertrand equilibrium channel structure. The primary difference in the equilibrium channel structure is when $b \geq 0.8944$. Unlike the dual Nash equilibrium seen in Bertrand competition, Bertrand-Cournot competition has a mixed equilibrium where Manufacturer 1 (M1) sells directly to consumers and Manufacturer 2 (M2) sells indirectly through an intermediary. This result is interesting, as intuitively M1, who is competing on price, would follow the Bertrand result, and M2, who is competing on quantity, would follow the Cournot result. Instead it appears that the driving factor for this mixed result is the *competitor's* form of competition, not your own form of competition. In summary, at high levels of product substitutability, regardless of your own form of competition, if your competitor is competing on price, you choose to sell indirectly; whereas if your competitor is competing on quantity, you choose to sell directly. This rule holds for all the results discussed regardless of the form of competition or the form of the parameters (asymmetric or symmetric).

McGuire and Staelin (1983) came up with identical equilibrium channel structures in Bertrand competition, but later Lus and Muriel (2009) challenged their demand functions on the intuitive results that were not present in McGuire and Staelin (1983). Lus and Muriel (2009) claimed that as product substitutability (strength of competition) increases: prices should

decrease and profits should decrease. Proposition 4.1 confirms these results for all three forms of competition under the symmetric setting.

Proposition 4.1. *Under all three possible types of competitions, namely, Bertrand, Cournot, and Bertrand-Cournot, as product substitutability increases, the optimal selling prices, the maximal channel profits, and the maximal manufacturer profits all decrease.*

Proof. See Appendix D.

These results can be proven under asymmetric settings, but have been excluded for brevity.

Details are available from the authors upon request.

4.4.2 *Asymmetric Substitution*

The first relaxation of the base case is to re-introduce asymmetric substitution into the demand functions. Due to the increased length and complexity of the analytical solutions, the focus of the analysis is through discussion and figures. Figure 4.1 provides a graphical representation of the equilibrium channel structures under Bertrand competition. The regions shown in all subsequent figures (i.e., Region A, B, and C in Figure 4.1) are separated by solid lines. Notice the 45° dashed line that represents the symmetric substitution case analyzed in Section 4.4.1. Also, notice that in equilibrium both manufacturers choose to sell indirectly only when the two products are highly, and nearly symmetrically substitutable to each other. This is an instinctive result as the primary driver for selling indirectly is strong competition. If one firm's product is not substitutable to the competitor's product, then the competitor no longer feels the pressure of the competition, and

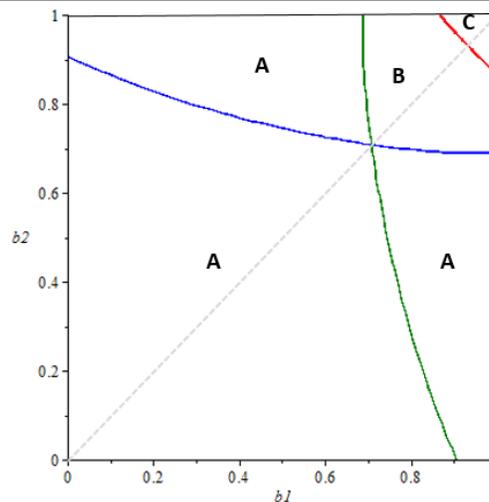
thus the incentive to sell indirectly through an independent retailer decreases. Even a single firm no longer desiring to sell indirectly causes the Nash equilibrium to revert back to both manufacturers selling directly as it was with low levels of substitution.

Figure 4.1: Asymmetric substitution under Bertrand Competition.

A: NE of (D,D)

B: NE of (D,D) with Pareto improvement to (I,I)

C: Dual NE of (I,I) and (D,D) with (I,I) dominant



The equilibrium channel structure for Cournot competition is shown in Figure 4.2. As found in the symmetric case, the Nash equilibrium always has both manufacturers selling directly to consumers. It is interesting to note Regions E and F. The equilibrium channel structure in Regions E and F matches Region A, but these regions show the levels of substitutability for which a single player would unilaterally prefer to switch to selling indirectly through an intermediary. In Region E, Manufacturer 1 (M1) prefers to sell indirectly due to the strong competition felt from Manufacturer 2 (M2). M2 has no incentive to deviate from selling directly due to the low level of substitutability of the product sold by M1. In Region F, the roles of M1 and M2 are reversed.

Figure 4.2: Asymmetric substitution under Cournot Competition.

A: NE of (D,D)

E: NE of (D,D), but M1 prefers I

F: NE of (D,D), but M2 prefers I

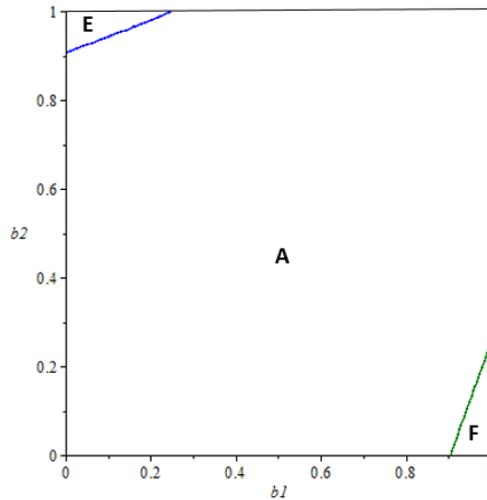
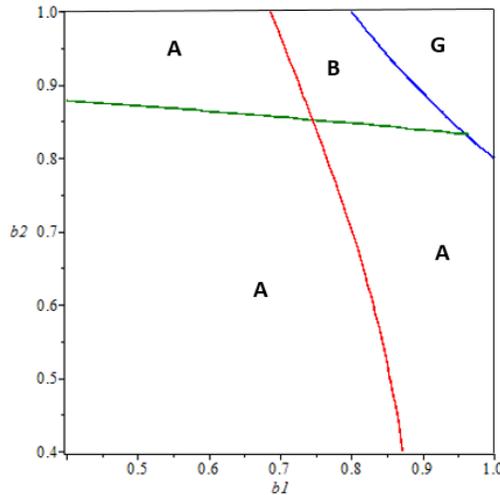


Figure 4.3 shows the equilibrium channel structure under Bertrand-Cournot competition with asymmetric product substitution. The reader may recognize similarities between Figure 4.1 (Bertrand) and Figure 4.3, but there are two key differences. First, the asymmetry in the figure. The mixed form of competition causes the asymmetry seen in Figure 4.3, and hence the symmetric substitutability case is not as obvious in Figure 4.3 compared to Figure 4.1. The second primary difference between Figure 4.1 (Bertrand) and Figure 4.3 (Bertrand-Cournot) is the upper-right-hand region (Region G in Figure 4.3 and Region C in Figure 4.1). Recall that under Bertrand competition, at high (and similar) levels of product substitution there were dual Nash equilibriums with both manufacturers selling indirectly as the dominant strategy. In Bertrand-Cournot competition I found a mixed equilibrium where Manufacturer 1, competing on price, sells directly, and Manufacturer 2, competing on quantity, sells indirectly; similar to what I found in Section 4.4.1 with Bertrand-Cournot competition and symmetric substitution. To

reiterate Section 4.4.1, the primary driver for channel structure appears to be your *rival's* form of competition, instead of your own form of competition.

Figure 4.3: Asymmetric substitution under Bertrand-Cournot Competition.

- A: NE of (D,D)
 - B: NE of (D,D) with Pareto improvement to (I,I)
 - G: NE of (D,I)
-



Asymmetry in the degree of product substitutability impacts not only the equilibrium channel structures, but also the equilibrium retail prices and profits for each manufacturer. Bertrand and Bertrand-Cournot competitions result in optimal retail prices that decrease as a function of both self-substitutability and rival substitutability. As the degree of substitutability grows, the difference in retail prices increases. The manufacturer with higher self-substitutability relative to their rival's substitutability experiences a decrease in retail price. This is likely due to an increased market share, resulting in a lower price per unit. Cournot competition results in a slightly different result. Following Proposition 4.1, there is a net or average decrease in retail price for the duopoly as the net product substitutability increases, but the result for each manufacturer differs. The manufacturer achieving higher self-substitutability compared to their

rival's substitutability experiences an increase in retail price. Again, following Proposition 4.1, the net or average manufacturer profits in the duopoly decrease as product substitutability increases, but there are opposite results for each manufacturer. For all three forms of competition, the manufacturer with higher self-substitutability relative to their rival's substitutability experiences an increase in manufacturer profits. For brevity, the proofs have been omitted, but they are available upon request.

4.4.3 Asymmetric Brand Equity

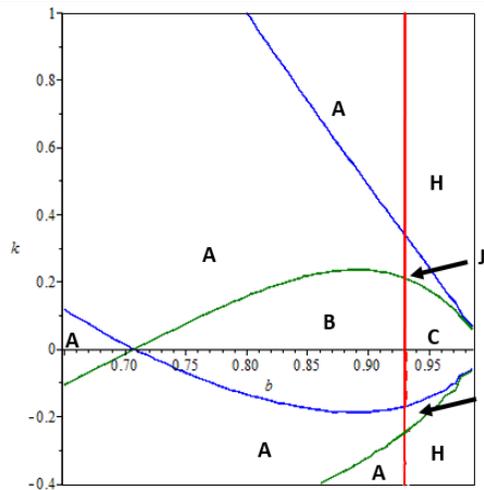
The second relaxation off the base case (Section 4.4.1) is to re-introduce asymmetric brand equity or recognition. Note that I use symmetric product substitution to isolate the effects of asymmetric brand equity. Recall that k is the percent price premium of Manufacturer 2 over Manufacturer 1 resulting from the difference in brand equity. Figure 4.4 graphically shows the equilibrium channel structures as the product substitution and brand equity parameters vary under Bertrand competition. The introduction of asymmetric brand equity provides some unique characteristics not seen when analyzing only product substitution. As mentioned previously, consumers in 2002 were willing to pay 37% more on average for a national brand (i.e., $k = 0.37$). The analysis extends past this threshold to examine the results under extreme scenarios.

Note that Figure 4.4 does not provide the full range of product substitutability. This is done to focus on the interesting aspects of the results. The Nash equilibrium for any $b < 0.65$ has both manufacturers selling directly (same as Region A). One interesting characteristic seen with asymmetric brand equity occurs when there is a significant price premium. Specifically, the elimination of the indirect channel structure as the dominant strategy at high levels of product substitutability for both manufacturers. Regions C, J, and H (regions with product substitutability

higher than 0.9309) all have dual Nash equilibriums. In Region C, selling indirectly is the dominant strategy. This occurs when the two products have similar (or symmetric) brand equity. As the difference in brand equity grows, selling indirectly becomes less attractive, and ultimately selling directly becomes the dominant strategy for both manufacturers in Region H. Intuitively, this makes sense, if the two manufacturers have significant differences in brand equity, then they likely are not competing for the same customers. The national brand is targeting the price insensitive customers with a penchant for name brands, where the lower priced generic brand is targeting the price sensitive customers. This difference in target market segments results in weaker competition, and thus the benefit of selling indirectly decreases.

Figure 4.4: Asymmetric brand equity under Bertrand Competition.

- A: NE of (D,D)**
 - B: NE of (D,D) with Pareto improvement to (I,I)**
 - C: Dual NE of (I,I) and (D,D) with (I,I) dominant**
 - H: Dual NE of (I,I) and (D,D) with (D,D) dominant**
 - J: Dual NE of (I,I) and (D,D) with no dominant structure**
-



The introduction of brand equity has no impact on the equilibrium channel structures for Cournot competition. As seen in Bertrand competition, a difference in brand equity or

recognition lowers the competition strength, which is already lower for Cournot competition. A primary driver for a manufacturer to engage in selling indirectly is the strength of competition at the retail level, therefore a reduction in the level of competition decreases the incentives to sell indirectly through an intermediary, and thus manufacturers competing on quantity continue to sell directly to consumers at all levels of product substitutability and brand equity.

Finally, in Bertrand-Cournot competition, similar to Bertrand competition, there are some interesting results when introducing asymmetric brand equity. Figure 4.5 shows the equilibrium channel structures with asymmetric brand equity for Bertrand-Cournot competition. There are a two interesting characteristics of the results shown in Figure 4.5. First, there is significant asymmetry in Region B, where there is a Pareto improvement for the manufacturers to switch from selling directly to selling indirectly. Recall that a positive value of k implies that Manufacturer 2 (competing on quantity) is the national brand with higher brand equity. If Manufacturer 1 (competing on price) is the product with lower brand equity, there is no incentive for the manufacturers to both switch to indirect sales. The second interesting aspect of the results occurs in Region G, where there are high levels of product substitutability ($b > 0.8944$). At these high levels of product substitutability, the effect of brand equity is eliminated in terms of its impact on the equilibrium channel structure. Regardless of the level of asymmetric brand equity, the equilibrium structure always has Manufacturer 1 (competing on price) selling directly and Manufacturer 2 (competing on quantity) selling through an intermediary. In summary, I found that for a fixed level of product substitutability, under Bertrand-Cournot competition, the position of the manufacturer as the national brand or local brand has no impact on the Nash equilibrium (recall Region B has the same Nash equilibrium as Region A).

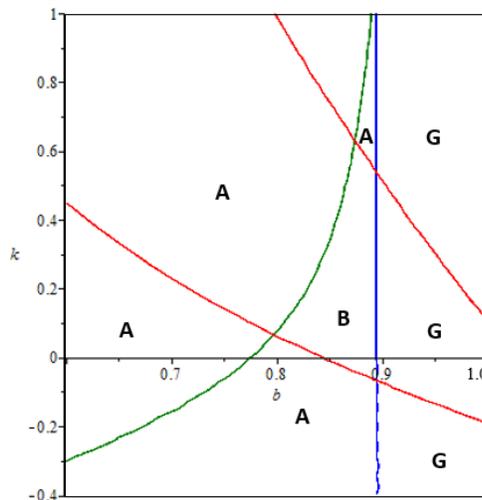
Asymmetry in the brand equity of the two manufacturers impacts not only the equilibrium channel structures, but also the equilibrium retail prices and profits of each manufacturer. The manufacturer with higher brand equity achieves high retail prices and profits as the relative difference in brand equity increases. The benefits for the well-known manufacturer significantly outweigh the negative impact for the lesser-known manufacturer, resulting in a system-wide increase in profit and retail price as the brand equity difference grows. This helps explain why the Nash Equilibrium recommends the manufacturers to sell directly when there is a large difference in brand equity. The well-known manufacturer is not pressured by the other manufacturer, thus lessening the strength of competition and incentive to sell indirectly. Once again, for brevity, the proofs have been omitted, but they are available upon request.

Figure 4.5: Asymmetric brand equity under Cournot Competition.

A: NE of (D,D)

B: NE of (D,D) with Pareto improvement to (I,I)

G: NE of (D,I)



4.5 Extensions

4.5.1 Two-Part Tariff Contracts

In the Section 4.4 analysis, there is an underlying assumption that the manufacturer is able to sell directly. With the increasing presence of internet-based commerce, this is not a restrictive assumption, but there may be situations where the manufacturer cannot or chooses not to sell their product directly. Independent retailers specialize in, and likely are more efficient at, the intricacies of selling to consumers (e.g., maintaining website or retail store, shipping in small quantities, advertising, etc.) in comparison with manufacturers. Thus, many manufacturers may not want to undertake these challenges as part of their operation. In these situations, the manufacturer will ideally design a contract with the retailer to achieve profit levels identical to the profits seen when the manufacturer sold their product directly. The most common form of contract is a linear wholesale contract (see Section 4.4). The second most commonly used contract form is a two-part tariff (Anand et al. 2008). Oi (1971) was one of the first to study the two-part tariff contract utilizing a Disneyland case study. Under a two-part tariff contract, each buyer (retailer) must pay a fixed fee F for the right to enter the amusement park (purchase units of inventory), and then pay an additional price per ride (per unit ordered). The proposed solution sets the price per unit to the marginal cost of each ride (unit) and then extracts the remaining consumer surplus (retailer profit) through the fixed fee.

Transitioning to a supply chain setting, a two-part tariff contract with the solution proposed by Oi (1971) can achieve centralized profits in a decentralized system (Anand et al. 2008). In this case, the centralized system is when the manufacturer sells directly, and the decentralized system is when the manufacturer sells indirectly through an independent retailer. In most scenarios considered in Section 4.4, the Nash equilibrium is to have each manufacturer sell

directly. In such scenarios, if the manufacturer cannot sell directly, they can achieve identical profits through the utilization of a two-part tariff contract with a per unit wholesale price equal to marginal cost ($w_i = c_i$) and a fixed fee equal to the retailer profits ($F = \Pi_{R,i}$) found in Section 4.4 with a linear wholesale contract.

On the other hand, how can the manufacturer utilize a two-part tariff contract when the Nash equilibrium channel structure recommends selling indirectly through an independent retailer? Similar logic still holds, but now I utilize a wholesale price greater than marginal cost. The price per unit will equal the wholesale price from the linear wholesale price contract solution, and the fixed fee will extract the remaining retailer profit. It is important to note, the ability to extract all the retailer profit is heavily dependent on the power of the manufacturer in the supply chain. Arya et al. (2008b) explore the relationship of power in supply chain contract negotiations, and similar logic can be applied to this situation.

4.5.2 *Dual Channels*

In all analysis to this point I assume that each manufacturer sells through an exclusive channel, namely, either directly or indirectly through an independent retailer. With the growing presence of internet-based commerce, more and more manufacturers have the opportunity to open dual channels. Specifically, simultaneously sell directly *and* indirectly through an independent retailer. The analysis in Section 4.4 shows that under Bertrand competition and with high product substitutability the Nash equilibrium switches from each manufacturer selling directly to each selling indirectly. What happens if I allow each manufacturer to split their sales between their direct and indirect channels?

To allow for a true comparison between exclusive channels and dual channels, I assume the market size is fixed (Anderson and Bao 2010, Cai et al. 2012). Given the fixed market size, each manufacturer has an additional decision variable, namely, the proportion of units sold directly, $\gamma_i \in [0,1]$. If $\gamma_i = 0$, then manufacturer i sells in an exclusive indirect channel, whereas if $\gamma_i = 1$, then manufacturer i sells in an exclusive direct channel. I assume that the retail price for each manufacturer's product is equal in the direct and indirect channels, and the retail price decision is made by the independent retailer (except in the exclusive direct channel situation when the independent retailer is eliminated) (Federal Trade Commission 2015).

The subsequent analysis focuses on Bertrand competition with symmetric substitutability and brand equity. The addition of a dual channel for both manufacturers impacts the profit equations for each manufacturer and retailer. The updated profit equations are:

$$\Pi_{M,i} = (w_i - c_i) \cdot (1 - \gamma_i) \cdot q_i + (p_i - c_i) \cdot \gamma_i \cdot q_i, \quad i = 1, 2, \quad (4.22)$$

$$\Pi_{R,i} = (w_i - p_i) \cdot (1 - \gamma_i) \cdot q_i, \quad i = 1, 2. \quad (4.23)$$

Notice at the extreme points, $\gamma_i = 0$ and $\gamma_i = 1$, the profit equations match the cases of an exclusive indirect channel and an exclusive direct channel, respectively. Using the new profit equations, equilibrium retail prices, order quantities, and wholesale prices are created as a function of γ_1 and γ_2 . To help facilitate the Nash equilibrium solution when facing dual channels, each manufacturer is limited to five options for their proportion of units sold directly: an exclusive indirect channel ($\gamma_i = 0$), an indirect-focused channel ($\gamma_i = 1/4$), a split channel between indirect and direct ($\gamma_i = 1/2$), a direct-focused channel ($\gamma_i = 3/4$), and an exclusive direct channel ($\gamma_i = 1$).

Table 4.2 shows the Nash equilibrium and the first-best channel structure for each manufacturer for all levels of symmetric product substitutability under Bertrand competition. As expected, at low levels of product substitutability, each manufacturer continues to use an exclusive direct channel ($\gamma_i = 1$). At higher levels of product substitutability, unlike in Section 4.4., there is no longer a dominant Nash equilibrium solution for an exclusive indirect channel ($\gamma_i = 0$). Instead each manufacturer finds a direct focused channel ($\gamma_i = 3/4$) is sufficient enough to remove themselves from the stiff retail competition. Despite the change in the Nash equilibrium, it is interesting to note that the first-best channel structure is still to have each manufacturer utilize an exclusive indirect channel ($\gamma_i = 0$) when $b \geq 0.85$. Additionally, all five levels of the proportion of units sold directly is the first-best channel structure for some level of product substitutability.

Table 4.2: Dual Channels – Nash equilibrium and first-best solution.		
Level of Substitutability (b)	Nash Equilibrium (γ_1, γ_2)	First-Best Solution (γ_1, γ_2)
0 – 0.58	(1, 1)	(1, 1)
0.59 – 0.80	(1, 1)	(3/4, 3/4)
0.81 – 0.82	(1, 1)	(1/2, 1/2)
0.83 – 0.84	(1, 1)	(1/4, 1/4)
0.85 – 0.86	(1, 1)	(0, 0)
0.87 – 1	(3/4, 3/4) and (1, 1) (3/4, 3/4) dominant	(0, 0)

4.6 Conclusions

Since McGuire and Staelin (1983), there has been extensive research studying equilibrium channel structures for two symmetric competing products under Bertrand (price) competition. A classic finding is that indirect channel structures are optimal when the products are sufficiently substitutable to each other. This is because under intense competition, the manufacturers are better off selling to consumers indirectly and hence minimizing bitter price wars.

In this chapter, I extend the previous research on equilibrium channel structures in three directions. First, I allow for asymmetry in both brand equity and substitutability between products, which is more likely in business practice. Second, I allow for Cournot (quantity) competition for one or both manufacturers. As a result, there are three possible types of competitions, Bertrand, Cournot, and Bertrand-Cournot. Third, I also depart from most previous studies (e.g., see McGuire and Staelin 1983) by adopting a refined demand model, which is in turn derived from the representative consumer utility theory (e.g., see Singh and Vives 1984, Lus and Muriel 2009).

This study thoroughly investigates the equilibrium channel structures when the two competing products differ in terms of brand equity and substitutability under three possible types of competition: pure Cournot competition, pure Bertrand competition, and mixed Bertrand-Cournot competition. The results show that whenever a manufacturer's competitor engages in Cournot competition, its equilibrium channel structure is always direct. This implies that whether a manufacturer should sell directly or not depends more on how its competitor competes (either via price or quantity) than anything else (e.g., self-form of competition, product characteristics). Second, product substitutability and brand equity do influence the equilibrium channel structures under Bertrand and Bertrand-Cournot. Specifically, as the level of asymmetry grows, the

competition level decreases, and consequently the benefit for selling indirectly for the manufacturer decreases. In other words, the classic finding that selling indirectly can improve profitability of the manufacturers depends greatly on the assumption that the two products are symmetric. When the two products are asymmetric, as is common in business practice, selling directly and the elimination of double marginalization still benefits the manufacturers (and supply chains).

As commonly done in the literature, the study is limited to two products to preserve analytic tractability. Further research can extend this work to the general case of three or more products or manufacturers. Moreover, I assume away demand uncertainty. Hence future research can study the case of price-dependent uncertain demand. Additionally, further exploration of dual channels with asymmetry appears to be a fruitful future research stream. Finally, empirical studies can be done to see if these theoretical results hold in practice. For example, is a supply chain more likely to sell directly when it engage in Cournot (quantity) competition? Is a supply chain more likely to sell directly when its product differ significantly from its competitors? The answers to those questions can help managers to better design their supply chains.

CHAPTER FIVE
THE RHYMES AND REASONS OF QUANTITY DISCOUNTS:
A PRACTICAL PERSPECTIVE

Abstract

This study utilizes two managerial surveys to provide an up-to-date overview of common practices associated with quantity discounts from the both the buyers' and sellers' perspectives. Additionally, the surveys provide information regarding the usage of profit and cost analysis in inventory decisions in the presence of quantity discounts as well as the common factors in these analyses. The inquiries to the usage of quantitative analysis led to an interesting finding: many buyers order sub-optimally when facing all-units and incremental quantity discounts. This indicates the need for better training for managers on the intricacies of quantity discounts. I develop two fundamental scenarios to help participants better understand the differences between all-units and incremental quantity discounts. Finally, from the results of the survey, I identify opportunities for future research.

5.1 Introduction

Since the 1960s, researchers have been developing quantity discount solution techniques. For buyers, researchers develop mathematical models to determine order quantities that minimize total cost. For sellers, to maximize profits and entice the buyer to deviate from their traditional order quantity, researchers derive the correct discount size and order quantity requirement. Fast forward to the 21st century, and there are hundreds of academic articles exploring best practices associated with quantity discounts from the buyers', sellers', and joint perspectives. The interest in quantity discounts remains strong due to the omnipresence of quantity discounts in both business-to-consumer and business-to-business transactions. As consumers, we see examples of quantity discounts on a daily basis: at fast food restaurants, "upsized for an extra fifty cents," at grocery stores, "buy in bulk for per unit savings," or even the generic "BOGO" or "buy-one-get-one" offers.

There are two primary objectives of this study. Munson and Rosenblatt (1998) performed open-ended interviews with 39 firms to better understand the key drivers of quantity discounts. Utilizing that information, they describe key areas in academic literature that remain understudied. The first objective of this study is to update the uses and real-world practices of quantity discounts. I explore the similarities and differences between these new survey results and the survey results found in Munson and Rosenblatt (1998). I aim to identify any new trends in practice with regard to quantity discounts. Additionally, I seek to better understand the needs of practicing managers who offer or receive quantity discounts.

The survey results indicate that many purchasing and sales managers are unaware of the key tradeoffs in quantity discounts, or the general guidelines associated with better business practices when facing or offering different forms of quantity discounts. This leads to a major

component of the second objective of this study: to identify potentially fruitful areas for future research. One very promising area, is the development of quantity discount training scenarios. The goal of these scenarios is to help train managers and business students on the appropriate techniques to determine efficient (if not optimal) discount pricing and ordering policies under different forms of quantity discounts from both the buyers' and sellers' perspectives. Furthermore, researchers can utilize these potential scenarios to better understand the rationale behind managers' inventory decisions.

The remainder of the paper is organized as follows. Section 5.2 briefly reviews the relevant quantity discount literature. Section 5.3 provides background information on the buyers and sellers who participated in the surveys. Analysis on the survey results is provided in Section 5.4. Section 5.5 describes potential areas for future research. Finally, Section 5.6 concludes the study.

5.2 Literature Review

There are a vast number of quantity discount publications in the marketing, economics, and operations management literature. Each year, at least ten quality publications continue to appear in the operations management field alone. Several literature review papers address much of the work in this area from various viewpoints. Dolan (1987) analyzes the quantity discount problem from the seller's perspective utilizing four industry examples. Goyal and Gupta (1989), and later Sarmah et al. (2006), explore the academic literature on joint perspective or coordinated quantity discount models. Benton and Park (1996) provide an overview of quantity discount academic models for buyers and buyer-seller coordinated systems.

This study is not explicitly expanding upon the current quantity discount literature, but instead it attempts to bridge the gap between the wide array of academic literature and the real-world applications of quantity discounts. Fifteen-plus years ago, Munson and Rosenblatt (1998) performed similar analysis, and this study updates the uses and practices of quantity discounts in industry. The subsequent literature review discusses articles that analyze different forms of quantity discounts from both the buyers' and sellers' perspectives, specifically: (1) all-units, (2) incremental, (3) time aggregated, and (4) item aggregated quantity discounts.

Most introductory operations management textbooks cover all-units quantity discounts (e.g., see Hadley and Whitin 1963, Heizer and Render 2014). Over the years, numerous authors have expanded upon the applications and intricacies of all-units quantity discounts (e.g., Das 1990, Khouja 1996, Abad 2003, and Li et al. 2012). Hu and Munson (2010) and Krajewski et al. (2013) are two articles of particular interest for this study due to their applicability to practitioners. Hu and Munson (2010) modify the standard solution procedure to facilitate easy spreadsheet implementation. Furthermore, Krajewski et al. (2013) provide a streamlined analytical approach to solving the all-units quantity discount problem. Both approaches are easily implemented in spreadsheet software such as Microsoft's Excel®.

The incremental quantity discount solution procedure is not as readily available, but it is still common in intermediate operations management or supply chain management textbooks (e.g., see Chopra and Meindl 2010). Similar to the all-units case, researchers have taken the base incremental quantity discount and utilized it in many different applications and scenarios (e.g., Abad 1998, Haksever and Moussourakis 2008, Chen and Ho 2011). Hu and Munson (2010) also modify the traditional incremental quantity discount solution procedure to facilitate easy

implementation in spreadsheets. This potential tool for managers is relatively unknown, but can be invaluable to anyone with a working knowledge of spreadsheet software.

There are two main tracks of research for item aggregated quantity discounts, both of which are significantly more rigorous than the all-units or incremental cases. The first is a constrained, multi-item system where each item has a unique quantity discount schedule, but all items are subject to a common resource constraint (e.g., warehouse space) that limits the order quantities and creates interdependencies between the decision variables. Solution techniques are developed for both all-units quantity discounts (e.g., Pirkul and Aras 1985, Mehrez and Ben-Arieh 1991, Rubin and Benton 1993, Moussourakis and Haksever 2008) and incremental discounts (e.g., Güder et al. 1994, Rubin and Benton 2003, Haksever and Moussourakis 2008). The second stream of item aggregated research handles business volume discounts. Business volume discounts apply to all items that are ordered simultaneously and typically come in all-units or incremental quantity discounts based on the number of cumulative units ordered across all items or the total dollar amount. The determination of optimal order quantities becomes quite difficult due to the numerous order quantity combinations capable of achieving the discount, and the interdependence of the decision variables. Nevertheless, solution techniques are available for both all-units (e.g., Chakravarty 1984, van der Duyn Schouten et al. 1994) and incremental business volume discounts (e.g., Chakravarty 1985, van der Duyn Schouten et al. 1994). More recently, van de Klundert et al. (2005) analyze business volume discounts for mobile phone operations who need to efficiently select telecommunication carriers (sellers) for international calls. Each carrier offers a unique business volume discount. Dahel (2003) explores this supplier selection decision in the presence of business volume discounts from a more general framework.

Literature on time aggregated quantity discounts is far sparser in comparison with the other three forms of quantity discounts. Sadrian and Yoon (1994) present a mixed integer program to deal with business volume discounts aggregated over a year-long period. Qin et al. (2007) utilize time aggregated quantity discounts to achieve channel coordination. Meanwhile, Hammami et al. (2014) use stochastic programming to solve a multi-period, multi-product, multi-site production planning problem with quantity discounts aggregated over both quantity and time. Similar to item aggregation, time aggregated quantity discounts present difficulties when trying to determine the optimal procurement policy due to the often lax restrictions on *when* a buyer must order.

The articles discussed previously focus on the buyers' order quantity decisions, there is also a large stream of literature related to the sellers' decisions on the pricing structure to offer to their buyers. The seminal sellers' perspective article on quantity discounts is Monahan (1984), which promotes price discounts as a marketing tool to alter the *timing* of a buyer's orders. Lee and Rosenblatt (1986) extend off the seminal piece by removing the lot-for-lot assumption of Monahan (1984) to allow for *lumpy demand*. Munson and Rosenblatt (2001) extend the sellers' perspective research to three levels in the supply chain, where the middle firm must handle incoming and outgoing quantity discounts. Finally, Ke and Bookbinder (2012) introduce a solution technique for sellers offering business volume discounts.

There are a handful of academic articles on quantity discounts which were built around a specific application in industry. Often, large firms forge partnerships with external consultant (often academics) to solve the firm's quantity discount procurement problem. By far, the most common problem is supplier selection, where each supplier offers a unique quantity discount schedule, and the firm must decide how to allocate their purchases (e.g., Gaballa 1974, Bender et

al. 1985, Katz et al. 1994, Hohner et al. 2003, and Sandholm et al. 2006). In these articles, the most common form of quantity discount is a business volume discount, and the most common solution technique is mixed integer programming. Many firms do not have the necessary institutional knowledge in these operations research techniques to solve these problems. Additionally, as the survey results indicate, some managers also lack a solid foundation in the quantity discount fundamental to fully understand and be able to utilize such models.

5.3 Data

Two managerial surveys were created using Qualtrics (www.qualtrics.com). Each survey assesses a firm's quantity discount practices. The first survey is for sellers who offer quantity discounts to their buyers, and the second survey is for buyers who received quantity discounts from their suppliers. I allow for a single firm to participate in both the seller and buyer surveys if they offer and receive quantity discounts. The buyer survey has three versions based on the purchasing power of the participant within their firm. The differences revolve primarily around the number of facilities or locations a firm has. The first version is for firms with a single facility, where the second and third versions are for firms with multiple facilities. These versions are split based upon the participant's purchasing responsibilities. Namely, do they purchase for a single facility or multiple facilities.

Survey participants were found using a variety of sources: local chapters of the Institute of Supply Management (ISM), the Washington State University (WSU) Carson College of Business National Board of Advisors, WSU MBA students (online, face-to-face, and executive), the Cougar Business Network (made up of WSU alumni), and personal contacts. Within each of these sources, only a subset of the potential participants have the necessary knowledge and

position to participate in the surveys. For this reason, reporting an accurate response rate is very difficult. Seventy-three participants began the surveys, 17 from the seller survey and 56 from the buyer survey. Of the 73 participants who began the survey, 49 completed the survey. I removed all incomplete surveys from the analysis, leaving a sample size of 49 participants, divided into 13 sellers and 36 buyers.

Table 5.1: Survey respondents by industry classification.

SIC Industry	Number of Sellers	Number of Buyers
Agriculture, Forestry, and Fishing	0	5
Construction	1	1
Food Manufacturing	2	2
Metal Fabrication	1	1
Paper Manufacturing	1	0
Electronics Manufacturing	1	4
Transportation, Communications, Electric, Gas, and Sanitary Services	0	3
Wholesale Trade	3	3
Retail Trade	1	9
Finance, Insurance, and Real Estate	0	1
Health Care	0	3
Other	Printing: 1 Mail Advertising: 1 Unspecified: 1	Aerospace: 4

Table 5.1 provides the Standard Industrial Classification (SIC) industry for each firm represented in the surveys. The firms surveyed varied significantly in terms of workforce size and annual sales. The workforce size ranged from less than 100 employees up to over 10,000 employees. In terms of sales, the lowest annual sales totals were less than \$1 million, and on the higher end, annual sales exceeded \$1 billion. Nearly all the survey participants held positions as executives or upper-level managers (sales managers for sellers and purchasing managers for

buyers). Of the buyers, 12 control the purchasing for their firm's only facility, 7 control the purchasing for one of many facilities, and 17 control multiple facilities. Note that of the 17 participants who control multiple facilities, there is no differentiation between the participant controlling all facilities and the participant controlling a subset (≥ 2) of the facilities.

5.4 Research Questions

This study addresses four primary research questions. Both the seller and buyer surveys have questions aimed to better understand the quantity discount practices and perceptions associated with the research questions.

The first research question (RQ1) explores the reasons for sellers to offer quantity discounts. Munson and Rosenblatt (1998) identify *marketing reasons* and *operations reasons* as the two major categories for reasons to offer quantity discounts. *Marketing reasons* include: sales stimulation, requirements from large buyers, and customer acquisition and retention. *Operations reasons* primarily revolve around better production planning, economies of scale in production, savings in shipping cost, and inventory reduction. RQ1 aims to provide an up-to-date reference for the primary reasons to offer quantity discounts. Potentially confirming the reasons found in Munson and Rosenblatt (1998) or uncovering new trends in industry unseen in academic literature.

RQ1. What are the primary reasons for offering quantity discounts?

The second research question (RQ2) looks to determine the most prevalent forms of quantity discounts. As mentioned previously, there are four common forms of quantity discounts,

namely: all-units, incremental, time aggregated, and item aggregated. Munson and Rosenblatt (1998) found that all-units quantity discounts were the most prevalent, but all forms were common. Similar to RQ1, RQ2 provides an update on the usage of each form of quantity discount. In addition, this study analyzes the common characteristics associated with each form of quantity discount, and determines if managers are performing efficiently (or optimally) in their inventory management. More specifically, I analyze characteristics such as the number of price breakpoints in the discount schedule, the length and form of contract, and the flexibility in order and delivery timing. The easiest determination of efficient performance in inventory management with quantity discounts is to check if managers are ordering at price breakpoints when facing all-units and incremental quantity discounts. Operations Management textbooks (e.g., see Hadley and Whitin 1963) prove that it is *never* optimal to order at a price breakpoint when facing incremental quantity discounts, but it is *often* optimal to order at a price breakpoint when facing all-units quantity discounts.

RQ2a. What are the primary forms and characteristics of quantity discounts?

RQ2b. When facing all-units and incremental quantity discounts, are managers ordering at a price breakpoint?

The third research question (RQ3) explores the effects of quantity discounts in business-to-business transactions on consumers and each level of the supply chain. Does each level of the supply chain (i.e., supplier, retailer, and consumer) see the same effect? Two common viewpoints on the effect of quantity discounts are: that quantity discounts cause an increase in annual demand, or that they just change the timing of the orders (i.e., increases order quantities

and decreases order frequency) without impacting annual demand. In the sellers' survey, I examine if the introduction of quantity discounts increases their annual demand. Then, in the buyers' survey, I analyze the impact of quantity discounts on the number of suppliers for a particular item as well as their annual demand.

RQ3. Are annual demand and the number of suppliers used effected by quantity discounts?

The fourth research question (RQ4) is a major driver for this study. Historically, there is a lack of continuity between academic research and real-world practice. There is very little known about the use of profit or cost analysis, or more formal mathematical models, in inventory management decisions when facing or offering quantity discounts. I seek to better understand the usage of profit and cost analysis in inventory management decisions as well as identify the reasons for the lack of synergy between academic literature and industry practices associated with quantity discounts. In addition, to the best of my knowledge, no study has explored both the buyers' and sellers' abilities to estimate holding and setup costs of the buyer. The benefits for researchers is two-fold. First, it will provide real-world estimates of inventory-related costs to justify numerical examples utilized in research. And secondly, it will help validate the assumption that sellers can estimate or know their buyers' inventory-rated costs.

RQ4. Do firms use profit or cost analysis to make better inventory management decisions?

5.5 Results

5.5.1 RQ1: What are the primary reasons for offering quantity discounts?

Table 5.2 provides the reasons for offering quantity discounts from this study as well as Munson and Rosenblatt (1998). Following Munson and Rosenblatt (1988), I separate the results into operational reasons and marketing reasons. The study results find many of the same primary justifications for quantity discounts seen fifteen-plus years ago, but there are two attention-grabbing findings in Table 5.2. First, a common research justification for utilizing quantity discounts to modify the order size and timing is to coordinate the supply chain without affecting annual demand. This study shows 31% of sellers using quantity discounts to modify the order size and timing of their buyers' orders, but less than 10% of sellers use quantity discounts to coordinate or synchronize their supply chain. Moreover, no firms in the study modify order timing to synchronize orders across multiple buyers to potentially save on setup and transportation costs.

The second attention-grabbing result from Table 5.2 is the frequency for which sellers offer quantity discounts as a potential "loss leader." This phenomenon occurs when the seller offers a quantity discount such that the per-unit price is below their own cost. This often occurs when a seller attempts to "lock in customers for the long term" or "win large orders." The seller knows that the long-term business with the large buyer will easily recoup their losses and more through service and post-sales extras (e.g., upgrades, service, and peripherals).

Table 5.2: Survey comparison on reasons for quantity discounts.

Issue	Munson and Rosenblatt (1998)	This Study
<i>Operational Reasons</i>		
Cost Savings	56%	54%
Shipping: Truckload/Transportation Discounts	Common	38%
Shipping: To Ship in Standard Package/Container Sizes	N/A	23%
Increase Order Size and Decrease Order Frequency	27%	31%
Reduce Inventory	Some	8-15%
Coordinate the Supply Chain	N/A	8%
Synchronize Customers' Order Timing	N/A	0%
<i>Marketing Reasons</i>		
Lock in Customers for the Long Term	Some	62%
Increase Annual Demand	56%	54%
Win Large Orders	N/A	46%

5.5.2 RQ2a: What are the common forms and characteristics of quantity discounts?

Table 5.3 compares the results on the prevalence of each form of quantity discount between this study and Munson and Rosenblatt (1998). Overall, despite slightly lower percentages in this study, the prevalence of all four forms of quantity discounts remains strikingly similar. All-units and aggregated discounts remain the most popular forms. A potential feature of any form of quantity discount is an up-front fixed fee. Often times a fixed fee is included in a “two-part tariff” type of quantity discount, where there is a fixed fee and a per-unit cost. Both studies found that 29% of firms have fixed fees in their transactions.

Table 5.3: Survey comparison on forms of quantity discounts.

Form	Munson and Rosenblatt (1998)	This Study
All-Units	95%	71%
Incremental	37%	33%
Time Aggregation	76%	43%
Item Aggregation	63% units or BVD	27% units, 54% BVD

Time aggregated quantity discounts have two key contract characteristics: length and discount timing. By far, the most common contract length is one year. In terms of discount timing, 67% of firms utilize a quantity commitment contract where the discount is given up-front with the buyer promising to order a minimum quantity. This is opposed to a retro-active contract, such as a year-end rebate associated with the final annual order quantity. For item aggregated quantity discounts, the key characteristic is the way in which the discount is determined: by units or total dollar value of the order. These types of discounts are often called business volume discounts (BVD).

Within aggregated quantity discounts (item or time), there is the potential for flexibility with respect to the order and delivery timing. Only 34% of sellers require that items aggregated in a single discount are ordered simultaneously, and only 14% of sellers require those products to be delivered simultaneously. This flexibility in order timing and delivery indicates that aggregated discounts often have time and item aspects working in unison. The combination of item and time aggregation is an understudied area in academic literature, and based off these results represents a very promising avenue for future research. This applies to both buyers, who need to know how to structure their procurement policy, and sellers who need to understand how to restrict this flexibility to maximize their profits.

Table 5.4 reports the results on key quantity discount characteristics for this study and Munson and Rosenblatt (1998). Negotiations were very common around the turn of the century, and remain prevalent now with 67% of managers indicating that they negotiate more than half of their prices. This naturally leads to only 38% of sellers offering the same quantity discount schedule to all of their buyers, despite 92% of them selling their major products to over 50 buyers. There are three primary aspects of quantity discounts that are commonly negotiated

between a buyer and a seller: (1) the quantity required to receive the discount (65%), (2) the size of the discount (61%), and (3) the form of the discount (45%). Other negotiations include payment terms and the sale of peripheral items that run alongside the major product, but do not count towards the primary quantity discount. All negotiations require an instigator, and interestingly, both parties believe they initialize quantity discount negotiations. Sixty-two percent of sellers claim to take the initiative to offer quantity discounts *and* 64% of buyers claim to request or demand the seller to offer some form of quantity discount.

Table 5.4: Survey comparison on *characteristics of quantity discounts*.

Characteristic	Munson and Rosenblatt (1998)	This Study
Negotiate More Than Half of Their Prices	87%	67%
Number of Price Breakpoints	Most < 5	Most 2-6
Use of Continuous Discount Schedules	0%	6%
Maximum Discount of 20% or Less	N/A	74%

The key characteristics of quantity discount schedules remain consistent with Munson and Rosenblatt (1998) in terms of the typical usage of six or less price breakpoints, and the very low usage of continuous discount schedules. Seventy-four percent of sellers and buyers report a maximum discount of 20% or less. Buyers indicate an equal distribution of maximum discounts between 1-5%, 6-10%, and 11-20%, whereas sellers primarily report a maximum discount of 11-20%. The difference in maximum discount distribution between buyers and sellers is intuitive. Sellers likely offer their maximum discount to their biggest or “most influential” buyers, but not every buyer is their suppliers’ biggest or “most influential” customer.

5.5.3 *RQ2b. When facing all-units and incremental quantity discounts, are managers ordering at a price breakpoint?*

Judging the effectiveness or optimality of a particular procurement policy is difficult, if not impossible, as an outsider due to the complex nature of each firm's cost structure. There are only two "guidelines" that apply to a majority of situations. First, when facing all-units quantity discounts, the optimal order quantity *often* falls on a price breakpoint. And second, when facing incremental quantity discounts, the optimal order quantity *never* falls on a price breakpoint. Therefore, I utilize RQ2b as a "quick-and-dirty" way to gain insight into the effectiveness of each purchasing manager's procurement policy. Given the "guidelines" above, I expect a high percentage of buyers facing all-units quantity discounts to order at a price breakpoint, but only 31% of buyers do. Similarly, I expect no buyers facing incremental quantity discounts to order at a price breakpoint, but a surprising 25% do. These results indicate that purchasing managers may lack some institutional knowledge regarding the fundamental guidelines for procurement when facing quantity discounts, further emphasizing the potential benefits of developing quantity discount training scenarios.

5.5.4 *RQ3: Are annual demand and the number of sellers used effected by quantity discounts?*

Sixty-nine percent of managers from selling firms indicate that offering quantity discounts results in an increase in their annual demand from that individual buyer. Meanwhile, only 39% of buyers report an increase in their annual demand for a particular product as a result of quantity discount offerings. Instead, 58% of buyers indicate that quantity discounts reduce the number of suppliers they order from for a particular product. Therefore, the increase in demand for the seller is a result of a reduction in the supplier base for a buyer instead of an increase in end-

consumer demand. Sole sourcing was a major push as part of the just-in-time (JIT) philosophy, but these results seem to indicate that buyers are moving towards the use of multiple sources. In fact, 47% of buyers order from more than five suppliers for their major products, whereas only 15% of buyers utilize sole sourcing. The recent natural disasters and increase in awareness of supply chain risk management are likely major culprits leading to this change in philosophy. For example, Toyota is historically a strong proponent of sole sourcing as part of their JIT practices, but recent experience with fires, earthquakes, and tsunamis has led Toyota “to have at least two suppliers, each in a different geographical region, for each component” (Heizer and Render 2014).

5.5.5 RQ4: Do firms use profit or cost analysis to make better inventory management decisions?

A common assumption in academic quantity discount models is that each party has knowledge of the other party’s cost structure. Of the sellers surveyed, over one-third of them do not know the cost structure of their primary buyers. More surprisingly, nearly 30% of buyers do not know their *own* cost structure. Estimates of the buyers’ cost structure from the buyers and sellers varies widely. Holding cost estimates vary from less than 5% annually of the item’s purchase price, up to over 40%. The two most common responses are less than 5% (21%) and 11-20% (18%), but overall the distribution of responses is relatively uniform. Estimates of the buyers’ setup costs are as low as \$10, and as high as \$10,000. With the largest proportion of responses falling between \$100 and \$1,000.

There is little evidence of firms using published academic inventory management models that are not specifically designed for them, but most firms do appear to incorporate some profit or cost analyses into their decision making. Of the 85% of sellers who implement some form of

profit analysis, all of them utilize their own cost structure, roughly half of them incorporate demand curve information (55%) and production batch sizes (45%), and some of them integrate container or shipping sizes (31%) and estimates of buyers' holding and setup costs (27%). Recall that roughly two-thirds of sellers were able to estimate their primary buyers' cost structure, but less than half of those sellers utilized those costs in developing quantity discount pricing structures. This perhaps indicates a lack of institutional knowledge on how to implement these costs into their decision making.

Meanwhile, only 42% of buyers include some form of cost analysis in their purchasing decisions. Of those, 64% incorporate transportation costs and 57% utilize the size of the quantity discount, but only 50% incorporate their own holding cost and 36% include their own setup cost. It is surprising to see the significantly lower usage of cost analysis from the buyers, as the models and analysis required is historically more available and easier to implement than similar models for sellers. Note the large difference between the usage of holding cost and setup cost in the buyers' cost analysis. Holding cost appears to be a bigger factor. This difference may be a result of the different firm types, manufacturing versus non-manufacturing. A manufacturing firm has a true setup cost, where a non-manufacturing firm has an ordering cost, for which technology (i.e., online ordering and electronic processing) has significantly reduced through quicker and less complex procedures associated with ordering inventory. This potentially reduces the "setup cost" for non-manufacturing firms down to an insignificant amount. Therefore, it may make more sense to balance *transportation* and holding costs for non-manufacturing firms instead of the traditional setup and holding costs.

Of the buyers and sellers not utilizing some form of cost or profit analysis, 31% of sellers and 26% of buyers use *intuition* to make discount pricing and order quantity decisions,

respectively. The two most common reasons cited for not using academic or mathematical models are, “They do not work well for our particular situation,” and, “We are unaware of their existence.” These two responses indicate a need for tailored academic models that bridge the gap between current literature and the needs of practitioners as well as training scenarios that can help both buyers and sellers make more informed decisions when facing and offering quantity discounts.

5.5.6 *Other Insightful Findings*

The surveys led to multiple insightful findings outside of the study’s research questions. First, buying firms with multiple locations often separate purchasing and storage decisions. Fifty-six percent of buyers purchase over half of their products *locally*, opposed to through a regional or corporate office. Meanwhile, 68% of buyers have over half of their products stored at a *central* warehouse. These results are counterintuitive, because a natural assumption to make is that the two decisions go hand-in-hand. In other words, if a firm utilizes centralized purchasing, then they would also employ a centralized warehouse, and vice-versa. Instead the survey results indicate many firms use centralized warehouses with local purchasing. A possible driver for this setup is the existence of centralized pricing negotiations that are aggregated across the firm and apply to all local purchases regardless of the individual order quantity.

Continuing along the line of centralization and coordination, 92% of sellers indicate that they regularly receive quantity discounts from their own suppliers, but only 42% jointly determine best ordering and selling practices. This thought-provoking finding indicates the need for a better understanding of how to contend with simultaneous incoming and outgoing quantity discounts. As well as the struggles of creating effective cross-functional teams within a firm,

resulting from a lack of communication and collaboration between the purchasing and sales departments.

Fifty-four percent of sellers indicate that they sell some products to multi-firm purchasing organizations, but only 17% of the surveyed buyers participate in these group purchasing organizations. The primary reasons why buyers participate in the purchasing groups are: (1) to receive quantity discounts (83%), (2) to increase purchasing power (67%), (3) to outsource price negotiation activities (67%), (4) to share information (33%), and to outsource supplier selection decisions (17%). On the other hand, several buyers indicate concerns and limitations associated with the purchasing groups, including: prescribed order quantities (67%), prescribed timing of orders (50%), and a restricted supplier selection (50%). Of the buyers who participate in multi-company purchasing groups, none of them purchase more than half of their products through the purchasing group.

5.6 Opportunities for Future Research

The design of this study is to initiate future research to help bridge the gap between academic literature on quantity discounts and the needs of practitioners who face or offer quantity discounts. From the initial design, I identify two major areas for future research opportunities. First, survey respondents cited a lack of fit between the quantity discount mathematical models seen in literature and the needs of their particular procurement or sales situation. Therefore, we as academics need to ensure our future quantity discount research has practical ties and applicability. Table 5.5 lists a few issues practitioners are facing which I believe are currently understudied, and simultaneously suggest opportunities to fill those research gaps.

Table 5.5: Future research opportunities.

Practitioner Issue	Research Opportunity
Increasing use of BVD	Buyers: Supplier selection models when multiple suppliers offer unique BVD Sellers: Model to create efficient BVD schedules
Continuing presence of time aggregated discounts with up-front commitment contracts	Buyers: Model to determine order <i>size</i> and <i>timing</i> Sellers: Determination of optimal contract
Sellers utilizing quantity discounts as <i>loss leader</i> to lock-in long-term business	Sellers: Model of trade-off between upfront losses and long-term benefits
Buyers moving away from sole sourcing	Buyers: Model of competitive environment with multiple sellers offering quantity discounts
Increasing presence of combination or hybrid quantity discounts	Buyers: Efficient procurement policies when facing combination/hybrid discounts Sellers: Determination of optimal combinations of quantity discounts
Firms offering and facing quantity discounts simultaneously	Need for models to coordination incoming and outgoing quantity discounts

The second major area for future research comes from the reasons that practitioners do not use mathematical models in their analysis: they did not know how to use or that such models existed. This presents researchers with an excellent opportunity to develop hypothetical dynamic training scenarios that are multifunctional. They provide opportunities: to interact (e.g., seminars or workshops) with purchasing and sales managers to help them better understand the inner-workings of quantity discounts; to connect with undergraduate business students to train the future business leaders of the world; and to better understand the rationale behind managers' decisions on inventory management. Scenarios can be from any perspective and pertain to any form of quantity discount, therefore, the opportunities are endless.

5.6.1 Fundamental Pricing Scenario for All-Units and Incremental Quantity Discounts

In this section, I formulate two fundamental quantity discount scenarios that I hope will be utilized as the foundation for further scenario development. I focus on all-units and incremental quantity discounts from the buyer's perspective as they are the easiest to understand and are the building blocks for more intricate discounts (e.g., time aggregated and item aggregated). A similar approach could be done from an alternative perspective. The primary goal for this fundamental quantity discount scenario is: to ensure the participants can interpret and understand the pricing schemes of all-units and incremental quantity discounts. Once the participant understands the pricing scheme, they should begin to fully understand how to efficiently utilize the quantity discount to minimize their inventory-related costs. Within this goal, I hope to instill the key characteristic that differentiates all-units and incremental quantity discounts. More specifically, when facing all-units quantity discounts it is *often* beneficial to order at a price breakpoint q_j . Meanwhile, when facing incremental quantity discounts it is *never* beneficial to order at a price breakpoint q_j .

Scenario 1 provides the participant with an all-units quantity discount schedule with three price levels. The participant must calculate the average per-unit purchasing cost associated with q_{j-1} and q_j for each price level. This scenario allows them to see the potentially significant purchasing cost savings associated with ordering at a price breakpoint when facing all-units quantity discounts. Similarly, Scenario 2 provides the participant with an incremental quantity discount schedule with three price levels. In these two scenarios, they recognize the key characteristic that differentiates all-units and incremental quantity discounts. More specifically, there are virtually no benefits (savings) associated with ordering at a price breakpoint in the presence of incremental discounts. The discount does not kick in until larger order quantities. I

highly recommend participants to utilize spreadsheet software in their determination of the average per-unit cost in both scenarios. This allows for more efficient calculations, and experience utilizing spreadsheet software.

Scenario 1:

You work for a firm who has a supplier that offers them a quantity discount on Item J. The undiscounted price for Item J is \$5, and the quantity discount schedule is provided in Table 5.6. Assume the price per unit in Table 5.6 applies to all units in the order (this is called an all-units quantity discount). There are two “price breakpoints” in this quantity discount schedule: 200 and 400 units.

Order Quantity	Price Per Unit
1 – 199	\$5
200 – 399	\$3
400 +	\$1

- (a) What is the average per-unit cost associated with ordering 199, 200, 300, 399, 400, and 500 units?
- (b) Assume that without quantity discounts you order 150 units at a time. Could it be beneficial to order 200 or 400 units instead?

Scenario 2:

Now consider another supplier that offers quantity discounts to your firm on Item A. The undiscounted price for Item A is \$5, and the quantity discount schedule is provided in Table 5.7. Assume the price per unit in Table 5.7 only applies to units within the range provided (this is

called an incremental quantity discount). For example, if 300 units are ordered, units 1 – 200 are \$5, and units 201 – 300 are \$3.

Table 5.7: Incremental quantity discount scenario

Order Quantity	Price Per Unit
1 – 200	\$5
201 – 400	\$3
401 +	\$1

- (a) What is the average per-unit cost associated with ordering 199, 200, 300, 399, 400, and 500 units?
- (b) Assume that without quantity discounts you order 150 units at a time. Could it be beneficial to order 200 units instead?

Scenario 1 Answer:

The solution for Scenario 1 is listed in Table 5.8 and graphed in Figures 5.1 and 5.2. The graphs provide illustrations to help the participant see the benefits of ordering at a price breakpoint when facing an all-units quantity discount schedule. Figure 5.1 provides the average per-unit prices and Figure 5.2 provides the total purchasing price associated with ordering Q units. The participant should be able to see the potential purchasing cost benefit of ordering at a price breakpoint through Table 5.8 and Figures 5.1 and 5.2. Further scenario development would then introduce setup and holding costs to aid the participant in understanding the relationship between the three inventory-based costs as the order quantity changes.

Table 5.8: Results for Scenario 1.

Order Quantity (Q)	Average Per-Unit Price	Total Price of Ordering Q Units
199	\$5.00	\$995
200	\$3.00	\$600
300	\$3.00	\$900
399	\$3.00	\$1197
400	\$1.00	\$400
500	\$1.00	\$500

Figure 5.1: Average per-unit price for all-units quantity discounts.

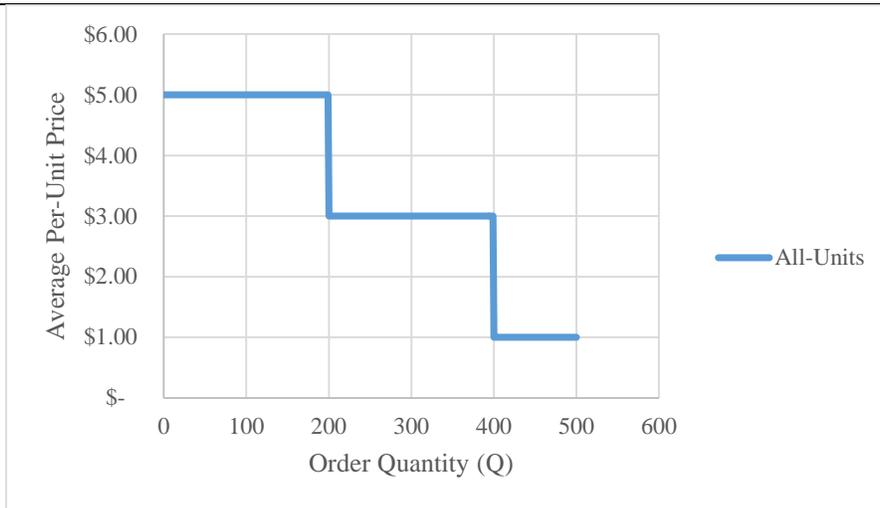


Figure 5.2: Total purchasing price of Q units for all-units quantity discounts.



Scenario 2 Answer:

The solution for Scenario 2 is listed in Table 5.9 and graphed in Figures 5.3 and 5.4. The graphs provide illustrations to help the participant see the difference in purchasing cost between all-units and incremental quantity discounts. Figure 5.3 provides the average per-unit prices and Figure 5.4 provides the total price associated with ordering Q units. The participant should be able to see the lack of purchasing cost benefit of ordering at a price breakpoint through Table 5.9 and Figures 5.3 and 5.4. Further scenario development would then introduce setup and holding costs to aid the participant in understanding the relationship between the three inventory-based costs as the order quantity changes.

Table 5.9: Results for Scenario 2.

Order Quantity (Q)	Average Per-Unit Price	Total Price of Ordering Q Units
199	\$5.00	\$995
200	\$5.00	\$1000
300	\$4.33	\$1300
399	\$4.00	\$1597
400	\$4.00	\$1600
500	\$3.40	\$1700

Figure 5.3: Average per-unit price for incremental quantity discounts.

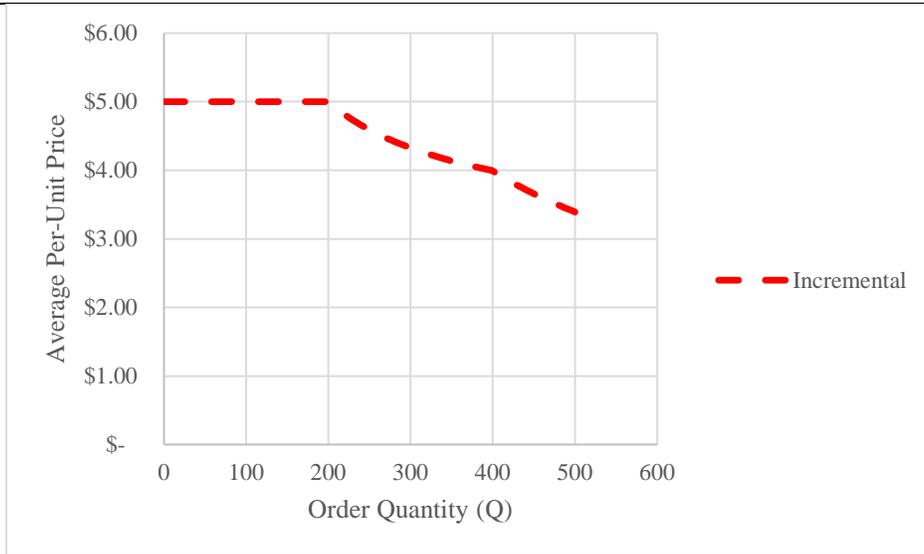


Figure 5.4: Total purchasing price of Q units for incremental quantity discounts.



5.7 Conclusions

This study provides insights into the common practices associated with quantity discounts from both the buyers' and sellers' perspectives through two managerial surveys. To the best of my knowledge, the latest update to real-world quantity discount practices is from over fifteen years

ago (Munson and Rosenblatt 1988). A second key contribution of this study is a better understanding of the usage of profit and cost analysis in inventory decisions in the presence of quantity discounts as well as the factors that managers include in those analyses. As part of this, I performed a “quick-and-dirty” optimality check of the buyers’ ordering policies when facing all-units and incremental quantity discounts. The surprising sub-optimal results lead to the third contribution of the study: the development of two fundamental quantity discount scenarios to help managers and business students understand the differences between, and characteristics of, all-units and incremental quantity discounts.

The survey results led to some interesting insights. First, all-units quantity discounts remain the most prevalent form of quantity discounts, but the use of aggregated (both time and item) quantity discounts continues to increase. Incremental discounts remain present as well, but to a lesser extent in comparison with the other forms. Second, the impact of quantity discounts on annual demand differs between the buyers’ and sellers’ perspectives. Many sellers acknowledge increases in annual demand from a particular buyer upon the introduction of quantity discounts. Meanwhile, many buyers claim no increase in annual demand from their customers with quantity discounts (implying no change in their prices), but instead a reduction in the number of suppliers for that particular product. Despite this decrease in their supplier base, very few buyers utilize the JIT practice of sole sourcing. Third, there are many firms that offer quantity discounts as well as receive quantity discounts from their suppliers. This emphasizes the need for more three-level analysis within a supply chain when quantity discounts are present between each link in the supply chain.

In addition to some interesting insights, the survey results also aided in the identification of areas for future research. For brevity, I point readers to review Section 5.6, instead of re-

listing the opportunities here. There are still numerous topics related to quantity discounts that are currently understudied despite the hundreds of previously published articles. The most obvious issue with much of the quantity discount literature is lack of applicability of the research to industry. Therefore, we need to ensure that our future research has real-world applications.

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APPENDICES

APPENDIX A: ALGORITHMS ADAPTED FROM PIRKUL AND ARAS (1985)

Algorithm A.1: Optimal Order Quantities with a Fixed Capacity Level and All-Units Quantity

Discounts

Step 1 For each item i and each price level j in segment m' determine $x_{ij}^{m'}(\lambda)$ using:

$$x_{ij}^{m'}(\lambda) = \sqrt{(2D_i S_i) / (p_{ij}^A h_i + 2\lambda k_i)}. \quad (\text{A.1})$$

Step 2 Determine $Q_i^{m'}(\lambda)$:

$$Q_i^{m'}(\lambda) = \max(x_{ij^*}^{m'}, q_{ij^*}), \quad (\text{A.2})$$

where $j^* = \text{Argmin}_{\{j=0,1,\dots,J\}} Z_{ij}^{m'}(\lambda)$, and:

$$Z_{ij}^{m'}(\lambda) = \left(\frac{D_i}{\max(x_{ij}^{m'}, q_{ij})} \right) S_i + \left(\frac{\max(x_{ij}^{m'}, q_{ij})}{2} \right) h_i p_{ij}^A + p_{ij}^A D_i + \lambda k_i \max(x_{ij}^{m'}, q_{ij}). \quad (\text{A.3})$$

Step 3 Find the best Lagrangian multiplier λ^* . Start at $\lambda = 0$, and increase λ until:

$$K_{m'} = \sum_i k_i Q_i^{m'} < y_{m'+1}. \quad (\text{A.4})$$

Then the order quantity for item i is:

$$Q_i^{m'} = Q_i^{m'}(\lambda^*). \quad (\text{A.5})$$

Algorithm A.2: Optimal Order Quantities with a Fixed Capacity Level and Incremental Quantity

Discounts

Step 1 For each item i and each price level j in segment m' determine $x_{ij}^{m'}(\lambda)$ using:

$$x_{ij}^{m'}(\lambda) = \sqrt{2D_i(S_i + R_{ij} - p_{ij}^I q_{ij}) / (h_i p_{ij}^I + 2\lambda k_i)}. \quad (\text{A.6})$$

Step 2 Determine $Q_i^{m'}(\lambda)$:

$$Q_i^{m'}(\lambda) = x_{ij^*}^{m'}, \quad (\text{A.7})$$

where $j^* = \text{Argmin}_{\{j=0,1,\dots,J\}} Z_{ij}^{m'}(\lambda)$, and:

$$Z_{ij}^{m'}(\lambda) = \left(\frac{D_i}{x_{ij}^{m'}} \right) (S_i + R_{ij} - p_{ij}^I q_{ij}) + \left(\frac{h_i}{2} \right) (R_{ij} + p_{ij}^I (x_{ij}^{m'} - q_{ij})) + p_{ij}^I D_i + \lambda k_i x_{ij}^{m'}. \quad (\text{A.8})$$

Step 3 Find the best Lagrangian multiplier λ^* . Start at $\lambda = 0$, and increase λ until:

$$K_{m'} = \sum_i k_i Q_i^{m'} < y_{m'+1}. \quad (\text{A.9})$$

Then the order quantity for item i is:

$$Q_i^{m'} = Q_i^{m'}(\lambda^*). \quad (\text{A.10})$$

APPENDIX B: PARAMETERS FOR THE BENTON EXAMPLE AND THE FORTUNE

500 EXAMPLE FROM RUBIN AND BENTON (1993)

For the Benton Example, $S_i = \$5$, and $h_i = 100\%$, $\forall i$.

Table B.1: Parameters for the Benton Example.

Parameter	Item (i)									
	1	2	3	4	5	6	7	8	9	10
Annual Demand (D_i)	600	900	2400	12,000	18,000	3000	2500	180	50	146
Space Required per Unit (k_i)	1.00	1.50	0.50	2.00	1.00	1.00	0.50	0.20	0.25	0.50

Table B.2: All-units quantity discount schedule for the Benton Example.

Item (i)	Order Quantity (units)		
	1 – 99	100 – 199	200 +
1	\$2.10	\$1.70	\$1.35
2	\$10.00	\$10.00	\$10.00
3	\$4.25	\$3.85	\$3.20
4	\$4.60	\$4.25	\$4.10
5	\$1.00	\$1.00	\$1.00
6	\$4.00	\$3.45	\$3.00
7	\$5.00	\$4.75	\$4.10
8	\$3.60	\$3.60	\$3.60
9	\$8.20	\$7.56	\$6.90
10	\$8.00	\$6.90	\$6.00

Table B.3: Incremental quantity discount schedule for the Benton Example.

Item (i)	Order Quantity (units)		
	1 – 49	50 – 199	200 +
1	\$2.70	\$2.30	\$1.90
2	\$9.82	\$9.25	\$8.75
3	\$4.90	\$4.50	\$3.70
4	\$4.60	\$4.30	\$3.90
5	\$0.90	\$0.75	\$0.60
6	\$4.40	\$4.25	\$4.00
7	\$5.85	\$5.36	\$5.00
8	\$3.00	\$2.85	\$2.60
9	\$12.20	\$11.30	\$10.00
10	\$9.20	\$9.00	\$8.88

For the Fortune 500 Example, $S_i = \$125$, and $h_i = 36\%$, $\forall i$.

Table B.4: Parameters for the Fortune 500 Example.

Item (i)	Annual Demand (D_i)	Space Required per Unit (k_i)
1	12,500	2.00
2	5,012	1.50
3	10,100	1.00
4	15,320	2.00
5	22,300	3.00
6	14,500	2.00
7	5,180	1.75
8	3,250	1.25
9	7,330	0.50
10	18,240	0.25
11	8,942	1.35
12	9,640	1.67
13	3,060	2.00
14	11,200	1.10
15	5,640	0.90

Table B.5: All-units and incremental quantity discount schedules for the Fortune 500 Example.

Item (i)	Order Quantities (units)			
	1 – 899	900 – 1499	1,500 – 2499	2500+
1	\$34.00	\$32.70	\$30.00	\$28.80
2	\$45.50	\$43.22	\$40.95	\$40.00
3	\$19.50	\$18.91	\$18.33	\$17.74
4	\$22.00	\$21.67	\$21.34	\$21.01
5	\$11.50	\$11.38	\$11.32	\$11.27
6	\$5.26	\$5.26	\$5.26	\$5.26
7	\$6.00	\$6.00	\$6.00	\$6.00
8	\$12.45	\$12.32	\$12.20	\$12.07
9	\$14.00	\$14.00	\$14.00	\$14.00
10	\$26.20	\$25.67	\$25.41	\$25.20
11	\$4.85	\$4.78	\$4.60	\$4.48
12	\$3.40	\$3.20	\$3.00	\$2.89
13	\$8.69	\$8.47	\$8.25	\$8.17
14	\$10.00	\$9.25	\$9.00	\$9.00
15	\$42.10	\$41.67	\$41.26	\$41.00

APPENDIX C: DETERMINATION OF THE NASH EQUILIBRIUM UNDER COURNOT COMPETITION WITH SYMMETRIC PARAMETERS

Consider a duopoly in Cournot competition (quantity) and assume the products have symmetric substitution and brand equity. Nicholson and Snyder (2010) give the following definition of a Nash equilibrium:

In a two-player game, (s_1^*, s_2^*) is a Nash equilibrium if s_1^* and s_2^* are the mutual best responses against each other (maximizing utilize u_i):

$$u_1(s_1^*, s_2^*) \geq u_1(s_1, s_2^*) \quad \forall s_1 \in S_1, \quad (\text{C.1})$$

$$u_2(s_1^*, s_2^*) \geq u_2(s_1^*, s_2) \quad \forall s_2 \in S_2. \quad (\text{C.2})$$

In the scenario, each player can choose to sell directly (D) or to sell indirectly (I). For simplicity, we have assumed that $c_1 = c_2 = 0$. The manufacturers' profit functions for each scenario are listed below.

$$\Pi_{M,1}^{DD} = \frac{a^2}{(2+b)^2} \quad (\text{C.3})$$

$$\Pi_{M,2}^{DD} = \frac{a^2}{(2+b)^2} \quad (\text{C.4})$$

$$\Pi_{M,1}^{DI} = \frac{(4+b)^2 a^2}{16(2+b)^2} \quad (\text{C.5})$$

$$\Pi_{M,2}^{DI} = \frac{(2-b)a^2}{8(2+b)} \quad (\text{C.6})$$

$$\Pi_{M,1}^{ID} = \frac{(2-b)a^2}{8(2+b)} \quad (\text{C.7})$$

$$\Pi_{M,2}^{ID} = \frac{(4+b)^2 a^2}{16(2+b)^2} \quad (\text{C.8})$$

$$\Pi_{M,1}^{II} = \frac{2(2-b)a^2}{(4-b)^2(2+b)} \quad (\text{C.9})$$

$$\Pi_{M,2}^{II} = \frac{2(2-b)a^2}{(4-b)^2(2+b)} \quad (\text{C.10})$$

If Manufacturer 2 (M2) chooses to sell directly, then Manufacturer 1 (M1) finds it more profitable to also sell directly.

$$\Pi_{M,1}^{DD} - \Pi_{M,1}^{ID} = \frac{(4+b^2)a^2}{8(2+b)^2} > 0 \quad \rightarrow \Pi_{M,1}^{DD} > \Pi_{M,1}^{ID} \quad (\text{C.11})$$

If M2 chooses to sell indirectly, then M1 finds it more profitable to sell directly.

$$\Pi_{M,1}^{DI} - \Pi_{M,1}^{II} = \frac{(128+b^4)a^2}{(4-b)^2(2+b)^2} > 0 \quad \rightarrow \Pi_{M,1}^{DI} > \Pi_{M,1}^{II} \quad (\text{C.12})$$

Knowing that M1 chooses to sell directly regardless of the choice by M2, M2 must decide which setting is most profitable. Given this information, M2 also prefers to sell directly.

$$\Pi_{M,2}^{DD} - \Pi_{M,2}^{ID} = \frac{(4+b^2)a^2}{8(2+b)^2} > 0 \quad \rightarrow \Pi_{M,2}^{DD} > \Pi_{M,2}^{ID} \quad (\text{C.13})$$

Therefore, the Nash equilibrium is (D,D) for all levels of product substitutability under Cournot competition with symmetric substitution and brand equity.

APPENDIX D: PROOF OF PROPOSITION 4.1

For brevity, I will only consider Bertrand competition with symmetric substitution and brand equity. This process can be repeated for all other combinations of type of competition and nature of parameters (symmetric or asymmetric). Details available upon request.

Let me begin by examining the retail prices under Bertrand competition with symmetric parameters.

$$p_1 = \frac{(2-b-b^2)a + 2c_1 + bc_2}{4-b^2} \quad (\text{D.1})$$

$$p_2 = \frac{(2-b-b^2)a + bc_1 + 2c_2}{4-b^2} \quad (\text{D.2})$$

Due to the symmetric nature of the competition and parameters, the result for one manufacturer will be identical to the other, therefore I only show the solutions for Manufacturer 1.

$$\frac{\delta p_1}{\delta b} = -\frac{(4+4b+b^2)a - 4bc_1 - (4+b^2)c_2}{4-b^2} \quad (\text{D.3})$$

Intuitively, as product substitution increases, the retail price should decrease; therefore, I expect the first derivative to be less than zero.

$$- [(4+4b+b^2)a - 4bc_1 - (4+b^2)c_2] < 0 \quad (\text{D.4})$$

Therefore, the following inequality must hold.

$$a > \frac{4bc_1 + (4+b^2)c_2}{4+4b+b^2} \quad (\text{D.5})$$

Notice that if $c_1 = c_2 = c$ then $a > c$. This is an intuitive and realistic constraint. Therefore I can confirm under Bertrand competition, retail prices decrease as product substitution increases.

I follow a similar process when examining the manufacturers' profit functions.

$$\Pi_{M,1}^{DD} = \frac{[(2-b-b^2)a - (2+b)c_1 + bc_2]^2}{(2-b)^2(2+b)^2(1-b^2)} \quad (D.6)$$

$$\frac{\partial \Pi_{M,1}^{DD}}{\partial b} = -\frac{2[(2-b-b^2)a - (2+b)c_1 + bc_2][(4-4b+b^2+2b^3-2b^4-b^5)a + (4b-2b^3+b^5)c_1 - (4+b^2-2b^4)c_2]}{(2-b)^3(2+b)^3(1-b)^2(1+b)^2} \quad (D.7)$$

Naturally, as product substitution increases, manufacturers' profits should decrease; therefore, I expect the first derivative to be less than zero. Therefore, the following inequality must hold.

$$a > \frac{(2-b^2)c_1 - bc_2}{2-b-b^2} \quad \text{and} \quad a > \frac{(4+b^2-2b^4)c_2 - (4b-2b^3+b^5)c_1}{4-4b+b^2+2b^3-2b^4-b^5} \quad (D.8)$$

Again, notice that if $c_1 = c_2 = c$ then $a > c$. This again is an intuitive and realistic constraint.

Therefore, I can confirm under Bertrand competition, the manufacturers' profits decrease as product substitution increases.