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ACKNOWLEDGMENT

I would like to thank my advisor Dr. Phil Wandschneider for his guidance and encouragement for the past four years; Dr. Mark Gibson for working with me for the paper titled “trade, non-homothetic preferences, and the impact of country size on wages” which has inspired the first chapter of this dissertation; and Dr. Ray Batina and Dr. Marketa Halova Wolfe for reading my dissertation and giving very useful comments.

I also thank Tongzhe Li for helping me with my research and having interesting and meaningful conversations with me; Sansi Yang for helping me with my coursework during the first two years; and Ben Weller, Rich Hoeft, Danielle Engelhardt, Tom Dahl, and Jamie Dahl for their help during my Ph.D. studies.
ESSAYS ON IMPERFECT COMPETITION AND INTERNATIONAL TRADE

Abstract

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August 2015

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This dissertation consists of three chapters based on product differentiation and increasing returns to scale. Chapter 1 studies a general equilibrium monopolistic competition model under which a group of worker-consumers are sharing a common utility function which represents their social indifference curves. Hence, product diversity occurs within this group instead of a single individual or the whole economy. In so doing, our model is able to show that group size is important to market demands, markups, the number of firms, and social welfare. We further show that group size also affects relative wage in an open economy, and social expenditure on public good in an economy with a privately provided public good.

Chapter 2 revisits the Alesina et al. (2000) model by taking domestic trade costs into consideration. In so doing, it is able to shed new insights on the relationship between trade costs, country size, and per-capita income. When international trade
costs are exogenous, per-capita income is increasing in country size and decreasing in domestic trade costs and international trade costs. When international trade costs are endogenous, per-capita income does not monotonically change with country size and domestic trade costs.

Chapter 3 examines the impact of manufacturing import growth on agricultural productivity for a small open economy using a $2 \times 2$ sector endogenous growth model. We show that the increase of import variety share in manufacturing, when there is no trade induced labor reallocation, drives the emergence of a sectoral productivity growth gap: productivity growth in agriculture is lower than in manufacturing. This explains, from the viewpoint of international trade, the empirical evidence that cross-country productivity differences in agriculture are larger than in non-agriculture. Our argument is that the recent growth of manufacturing trade in poor countries benefits its own industry but brings costs to agriculture in terms of variety expansion. An effective policy in terms of increasing agricultural productivity in poor countries should be able to alleviate the negative impact of import growth in manufacturing. Cross-country evidence supports our results.
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CHAPTER 1. GENERAL EQUILIBRIUM MODELS OF MONOPOLISTIC COMPETITION: QUANTITY, DIVERSITY, AND WELFARE

1.1 Introduction

The seminal monopolistic competition model by Dixit and Stiglitz (1977) (henceforth DS) has been extensively used in many fields, such as trade and growth. In these models, two approaches have been used to capture consumers’ preferences on differentiated products. One assumes that all worker-consumers share a common utility function (for example, Grossman and Helpman, 1991; Fujita et al., 1999; Melitz, 2003). We call this Case A. The other assumes that all worker-consumers have the same utility function (for example, Krugman, 1980; Blanchard and Kiyotaki, 1987; Ottaviano et al., 2002). We call this Case B. These two different considerations lead to a natural question: do Case A and Case B yield different predictions on product

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1The Dixit-Stiglitz has been used by many fields. For international trade, see Krugman (1979); for growth theory, see Romer (1986); for macroeconomics, see Blanchard and Kiyotaki (1987); for economic geography, see Krugman (1991); and for public goods, see Pecorino (2009).
quantity, product diversity, and welfare?\textsuperscript{2} It is the purpose of this paper to answer this question.

Before starting to describe how we are going to answer this question, note that the question points towards the aggregation problem from individual demands to market demands as well as individual preferences to social preferences. When preferences are homothetic and income distribution is fixed, according to Eisenberg (1961), Chipman (1974), and Shafer (1977), Case A and Case B should yield similar results on product quantity and product diversity because aggregate demand behavior is the same with a single representative consumer. However, we do not know if welfare is the same. When preferences are non-homothetic, we would expect that all product quantity, product diversity, and welfare could be different. If so, how and why are they different?

In order to answer the above questions, this paper analyzes a one-sector general equilibrium monopolistic competition model in which a group of worker-consumers are sharing a common utility function. Group size (the number of individuals in this group) is from one to the total number of individuals on the market, including boundaries. And the total number of groups in the market is the total number of

\textsuperscript{2}In this paper, we assume that each firm is producing one differentiated product (variety). Hence, product quantity is the market demand as well as the output for each firm and product diversity is the number of differentiated products which equals the number of firms on the market. As for welfare, it is represented by the total utility for consumers.
individuals on the market divided by group size. The production side is the same with the literature featuring single product firms, constant marginal costs, and fixed costs. There are two reasons to assume that a group of worker-consumers are a common utility function. First, this includes both Case A and Case B. When group size is one, the model is the same as Case B; and when group size equals the total number of individuals on the market, the model is the same as Case A. Thus, we can compare Case A and Case B by studying whether quantity, diversity, and welfare change with group size. Second, by studying the impact of group size on quantity, diversity, and welfare, we can see why Case A and Case B yield similar or different predictions.

We believe that it is useful to analyze both homothetic and non-homothetic preferences in one integrated framework in order to understand the distinction between those two. In order to do this, we consider a more general utility function which includes both the additively separable preferences as in Zhelobokdo et al. (2012) and the constant elasticity of substitution (CES) preferences as in Melitz (2003).\(^3\) Our

\(^3\)Note that even though the preferences in Zhelobokdo et al. (2012) can be CES, they cannot capture the preferences used in Melitz (2003). More importantly, in so doing, our results apply to some important homothetic and non-homothetic preferences such as constant-elasticity-of-substitution preferences as in Krugman (1980) and Melitz (2003), the hierarchic demand as in Simonovska (2015) and “CRARA” preferences as in Behrens and Murata (2007). However, we do not consider some other equally important homothetic and non-homothetic preferences which are not additively separable, such as the translog expenditure function as in Feenstra (2003) and the quasi-linear quadratic utility function as in Melitz and Ottaviano (2008).
results can be summarized as follows. When preferences are homothetic, product quantity and product diversity do not change with group size, but welfare can change with group size except when utility functions are homogeneous of degree one. When preferences are non-homothetic, product quantity, product diversity, and welfare all change with group size. Hence, except for product quantity and product diversity when preferences are homothetic and for welfare when utility functions are homogeneous of degree one, Case A and Case B yield different predictions.

The result that welfare is not always constant with respect to group size under homothetic preferences implies that the welfare of Case A and Case B could be different. This seems surprising. However, DS has already provided an answer. To see this, we first note that DS belong to Case A. In the introduction part, DS argues that the utility function in their paper can be “regarded as representing Samuelsonian social indifference curves, or (assuming the appropriate aggregation conditions to be fulfilled) as a multiple of a representative consumer’s utility.” Due to this, D-S then argues that “product diversity can be interpreted either as different consumers using different varieties or as diversification on the part of each consumer”. In this regard, Case B is different from Case A by excluding the case that different consumers are using different varieties. In other words, Case B is assuming that each consumer is

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4For discussions on social indifference curves, see the papers by Scitovszky (1942), Samuelson (1956), Chipman and Moore (1979), Varian (1984), and Dow and da Costa Werlang (1988).
consuming all varieties available in the market. This leads to a difference between the two cases that, since the number of differentiated products is the same, the average number of differentiated products for each consumer in Case B is weakly larger than Case A. Hence, it is possible that Case A and Case B yield different predictions on welfare even under homothetic preferences.

In contrast to the results under homothetic preferences, the results under non-homothetic preferences seem more standard. When the elasticity of substitution is decreasing with consumption, larger group size decreases the competitiveness in the market. Due to the decrease of competition, more firms are entering the market and each firm provides a decreased quantity of differentiated products. Thus, compared to Case B, Case A yields a less competitive market, decreased quantity, and more diversity. When the elasticity of substitution is increasing with consumption, larger group size increases the competition in the market. The more competitive market leads to fewer firms entering and each firm in the market will produce more quantity. Hence, compared to Case B, Case A yields more competitive market, more quantity, and less diversity. Since welfare is affected by product quantity, product diversity, and preferences parameters, Case A and Case B can have different welfare predictions.

These results imply that homothetic preferences are important to some issues, but one may need more restrictions on preferences other than homotheticity. In order to see this, we then analyze two additional settings. The first one is a two-country
trade model following Krugman (1980). In this setting, countries are asymmetric only in terms of group size. Under homothetic preferences, since group size does not affect product quantity or product diversity, both countries have the same wage. However, under non-homothetic preferences, the country with larger group size is having lower wage. This shows that homothetic preferences are a dividing line for predictions on trade patterns between Case A and Case B. The second one is a model with a privately provided public good following Pecorino (2009). In this setting, in contrast to the first one, homothetic preferences cannot guarantee that social expenditure does not change with group size. A further restriction that elasticity of sub-utility is constant is needed for Case A and Case B to predict the same result on the equilibrium provision of the public good.

This paper is technically related to the literature analyzing the role of additively separable preferences under general equilibrium monopolistic competition setting. Our results are derived from the restrictions that profit functions are concave, and consumers love variety. These are standard assumptions as in Zhelobodko et al. (2012), Mrázová and Neary (2014a,b), Bertoletti and Epifani (2014), Dhirngra and Morrow (2014), and Behrens et al. (2014, 2015).\(^5\) Especially, we follow Mrázová and Neary (2014a) using price elasticities and the convexity of demand to characterize

\(^5\)Note that these papers belong to Case B.
market equilibrium and using the elasticity of sub-utility to discuss welfare. In an earlier paper, Wang and Gibson (2015) discuss the impact of country size on wages and find that Case A and Case B yield different results on the impact of country size on wages. The current paper uses more general utility functions, considers both market equilibrium and welfare, and provides more general results.

The remainder of this paper is organized as follows. Section 1.2 describes the model and analyzes the impact of group size on product quantity and product diversity. Section 1.3 studies the impact of group size on welfare. Section 1.4 talks about two applications in trade and public economics. Section 1.5 concludes.

1.2 Model

1.2.1 Consumption

We begin with standard assumptions that there are $L$ worker-consumers consuming differentiated products which are indexed by $i$, and each consumer is endowed with one unit of labor which is used to earn money from working for firms. The wage rate is normalized to one. Now we assume that $k$ worker-consumers share a common utility function. Hence, there are $L/k$ groups in the market. Groups are homogeneous, and each group has the following utility function

\[ U = \left( \int_0^N u(c_i) di \right)^{1/\theta} ; 0 < \theta \leq 1, \]  

(1.1)
where $c_i$, $i = 1, 2, 3, ..., n$, represents the consumption of differentiated product $i$ for $k$ individuals, $N$ is the endogenous number of differentiated products, $u$ is the sub-utility from consuming each differentiated product, and $u$ has the following properties, $u' > 0$, $u'' < 0$, and $u(0) \geq 0$.

There are three important features of the above utility function. First, it has the capability of including both additively separable preferences as in Krugman (1979) and Zhelobodko et al. (2012) and the classical constant elasticity of substitution preferences as in Melitz (2003). When $\theta = 1$, the preferences are additively separable; and when $0 < \theta < 1$ and $u(c_i) = c_i^\theta$, the preferences are the same with the ones used by Melitz (2003). Second, since we do not specify which products each consumer consumes, it can be interpreted as, similar to Dixit and Stiglitz (1977), either different individuals consume different products or each individual consume some units of each product. In any cases, $c_i$ is the aggregate consumption of product $i$ and $N$ is the total number of products for $k$ individuals. Third, Case A and Case B as discussed in the introduction are special cases. When $k = 1$, all individuals are using identical utility functions as in Case B; and when $k = L$, all individuals are sharing a common utility function as in Case A.

**Definition 1.** Preferences are homothetic if the marginal rate of substitution is dependent on the ratio of consumption.

**Definition 2.** Preferences are strongly homothetic if utility functions are homoge-
neous of degree one.

The two definitions can help us distinguish some homothetic preferences. For example, the preferences used by Melitz (2003) are strongly homothetic, but the preferences used by Krugman (1980) are not strongly homothetic. Later in this paper, we will show that even though both preferences are homothetic, the difference on whether they are or are not strongly homothetic can imply different predictions for Case A and Case B.

**Lemma 1.** Preferences are strongly homothetic if and only if \( u(c_i) = ac_i^\theta \).

**Proof.** Homogeneous of degree one implies that

\[
U(\alpha c_1, \alpha c_2, \alpha c_3, \ldots, \alpha c_n) = \alpha U(c_1, c_2, c_3, \ldots, c_n).
\]

(1.2)

In our model, it implies the following

\[
\left( \int_0^N u(\alpha c_i) \, di \right)^{1/\theta} = \alpha \left( \int_0^N u(c_i) \, di \right)^{1/\theta}.
\]

(1.3)

Hence, preferences are strongly homothetic if and only if \( u(\alpha c_i) = \alpha^\theta u(c_i) \). By setting \( \alpha = 1/c_i \), this implies that \( u(1) = c_i^\theta u(c_i) \). Hence \( u(c_i) = ac_i^\theta \). □

In this paper, we focus on symmetric equilibrium, so we drop index \( i \) when convenient.

**Definition 3.** The elasticity of sub-utility is \( \eta(c) = u'(c)c/u(c) \), and \( 0 < \eta(c) < 1 \).
This ensures that preferences exhibit a taste for variety (Vives, 1999). Later in this paper, we will use this to discuss welfare.

The budget constraint for a group of \( k \) consumers is

\[
\int_0^N p(c_i) c_i \, di = k, \tag{1.4}
\]

where \( p(c_i) \) is the price for product \( i \).

Utility maximization yields inverse demands\(^7\) as

\[
p(c_i) = u'(c_i) \left( \frac{\int_0^N u(c_i) \, di}{\theta \lambda} \right)^{(1-\theta)/\theta}, \tag{1.5}
\]

where \( \lambda \) is the marginal utility of income. Plugging the inverse demands back into budget constraint, we get the following expression for \( \lambda \),

\[
\lambda = \frac{\left( \int_0^N u(c_i) \, di \right)^{(1-\theta)/\theta} \int_0^N u'(c_i) c_i \, di}{k \theta}. \tag{1.6}
\]

Hence, in the expression for inverse demands, both \( \lambda \) and \( \left( \int_0^N u(c_i) \, di \right)^{(1-\theta)/\theta} \) are aggregate variables. Following the spirit of Chamberlin (1933), each firm takes them as constants.\(^8\) This enable us to define two more variables to characterize the equilibrium.

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\(^6\)See page 170.

\(^7\)Strictly speaking, these are inverse demands perceived by firms.

\(^8\)The number of firms is very large that each firm is negligible. For this point, see Hart (1985) and Matsuyama (1995).
**Definition 4.** The elasticity of substitution is \( \varepsilon(c) = -u'(c) / (u''(c)c) \), and \( \varepsilon(c) > 1 \).

When \( \varepsilon'(c) = 0 \), preferences feature constant elasticity of substitution (CES); when \( \varepsilon'(c) < 0 \), preferences feature have decreasing elasticity of substitution (DES); and when \( \varepsilon'(c) > 0 \), preferences feature increasing elasticity of substitution (IES).

According to the above two definitions, we have the following relationship between the elasticity of substitution and the elasticity of sub-utility,

\[
\eta'(c) = \frac{\eta(c)}{c} \left( \frac{\varepsilon(c) - 1}{\varepsilon(c)} - \eta(c) \right). \tag{1.7}
\]

**Lemma 2.** Preferences are homothetic if and only if the elasticity of substitution is constant.

*Proof.* \( U \) represents homothetic preferences if and only if \( \partial U / \partial c_i \) are functions of \( c_i / c_j \) for all \( j \neq i \). Since marginal utility is

\[
\frac{\partial U}{\partial c_i} = \left( \int_0^N u(c_i) \, di \right)^{(1-\theta)/\theta} u'(c_i) / \theta \tag{1.8}
\]

the marginal rate of substitution depends only on sub-utility functions. Hence, \( u'(c_i) / u'(c_j) \) should be dependent on \( c_i / c_j \) for all \( i \neq j \). Assume that \( c_i = \alpha c_j \), it is equivalent to that \( u'(\alpha c_j) / u'(c_j) \) is independent on \( c_j \). By log-differentiating it with respect to \( c_j \), we have the following

\[
\frac{\alpha u''(\alpha c_j)}{u'(\alpha c_j)} = \frac{u''(c_j)}{u'(c_j)}. \tag{1.9}
\]
By setting $\alpha = 1/c_j$, we get
\[
\frac{\alpha u''(c_j)}{u'(c_j)} = \frac{u''(1)}{u'(1)},
\] (1.10)
hence, the elasticity of substitution is constant.

**Definition 5.** The convexity of demand is $\rho(c) = -u'''(c)c/u''(c) < 2$, and $\rho(c) < 2$.

The above two definitions ensure that marginal revenue functions are increasing and concave. The reason is that we have $\rho(c) = -\frac{c\varphi'(c)}{\rho'(c)} \leq 0$. For more discussions about this point, see Mrázová and Neary (2014a) and Tirole (1988)\(^9\). Moreover, according to price elasticities and the convexity of demand, the change of price elasticities with consumption is
\[
\varepsilon'(c) = \frac{\varepsilon(c)}{c} \left( \rho(c) - \frac{\varepsilon(c) + 1}{\varepsilon(c)} \right) < \frac{\varepsilon(c) - 1}{c}.
\] (1.11)

### 1.2.2 Production

Each firm produces one differentiated product. Other than that, firms are identical. Market are integrated so that there is no price discrimination across groups. Let $f$ and $\varphi$ denote the common fixed cost and marginal cost in terms of labor. The profit function for each firm is
\[
\pi = p(c)cL/k - \varphi cL/k - f.
\] (1.12)

\(^9\)See page 69.
The markup pricing rule is

\[ p(c) = m(c)\varphi, \tag{1.13} \]

where \( m(c) = \varepsilon(c)/(\varepsilon(c) - 1) \) is the markup factor and it has the following relationship with price elasticity \( \varepsilon(c) \),

\[ m'(c) = -\frac{\varepsilon'(c)}{(\varepsilon(c) - 1)^2}. \tag{1.14} \]

Hence, under CES, markups are constant; under DES, markups are increasing with consumption; and under IES, markups are decreasing with consumption.

Plugging the pricing rule back into the profit function, we get the following expression for profit

\[ \pi = (m(c) - 1)\varphi cL/k - f. \tag{1.15} \]

1.2.3 Equilibrium

There are many potential firms waiting to enter the market. Each firm has the ability to produce one differentiated product. So the profit of each firm in the market is zero. The free entry condition is

\[ (m(c) - 1)\varphi cL = kf. \tag{1.16} \]

Plug \( m(c) = \varepsilon(c)/(\varepsilon(c) - 1) \) into the above equation, we can rewrite the free entry condition as

\[ \varphi cL - kf(\varepsilon(c) - 1) = 0. \tag{1.17} \]
The above equation is used to pin down group demand \( c \) and market demand \( C \equiv cL/k \) of each firm.

**Lemma 3.** The upper bound of \( \varepsilon'(c) \) is \( \frac{L\varphi}{kf} \).

**Proof.** Free entry condition implies \( c = \frac{kf(\varepsilon(c)-1)}{L\varphi} \). Plugging this into \( \varepsilon'(c) \), we get \( \varepsilon'(c) < \frac{L\varphi}{kf} \). \( \square \)

The above result shows that the range of price elasticities is from \(-\infty\) to \( \frac{L\varphi}{kf} \). This is guaranteed by the definition that \( \varepsilon(c) > 1 \) and \( \rho(c) < 2 \). In the following discussions, we will restrict our analysis to this range of price elasticities. The propositions later in this paper automatically satisfy Lemma 1.

Finally, the following labor-market-clearing condition is used to pin down the number of firms in the market,

\[
N(f + \varphi cL/k) = L. \tag{1.18}
\]

Now we are ready to discuss the impacts of group size on group demands, market demands, the number of firms, and market competitiveness.

Group demand can be solved from the free entry condition, \( \varphi cL - kf(\varepsilon(c)-1) = 0 \).

**Proposition 1.** Group demands \( c \) are increasing with group size \( k \) under both homothetic preferences and non-homothetic preferences.
Proof. According to the implicit function theorem and the free entry condition, we have
\[ \frac{\partial c}{\partial k} = \frac{f(\varepsilon(c) - 1)}{L \varphi - k f \varepsilon'(c)} = \frac{f(\varepsilon(c) - 1)}{L \varphi - k f \varepsilon(c) \rho(c) - \varepsilon(c) - 1/c}. \tag{1.19} \]

Due to free entry, we have \( L \varphi = k f (\varepsilon(c) - 1)/c \). Substituting this back into the above equation, we have
\[ \frac{\partial c}{\partial k} = \frac{c \varepsilon(c) - 1}{k \varepsilon(c)(2 - \rho(c))}. \tag{1.20} \]

Since \( C = cL/k \), market demands and group demands have the following relationship
\[ \frac{\partial C}{\partial k} = \frac{c}{k \varepsilon(c)(2 - \rho(c))} - 1. \tag{1.21} \]

**Proposition 2.** When preferences are homothetic, market demands, \( C \), do not change with group size \( k \); and when preferences are non-homothetic, market demands are decreasing with group size under DES and increasing with group size under IES.

Proof.
\[ \frac{\partial C}{\partial k} = C \frac{\varepsilon'(c)c}{k \varepsilon(c)(2 - \rho(c))}. \tag{1.22} \]
Since \( \rho(c) < 2 \), the sign of \( \partial C/\partial k \) is the same with the sign of \( \varepsilon'(c) \) and the results are proved. \( \square \)

In general we have market demands changing with group size. One special case is when preferences are homothetic. In this case, market demands do not change with
group size.

Markups charged by each firm represent the competitiveness of the market.

**Proposition 3.** *When preferences are homothetic, markups do not change with group size; and when preferences are non-homothetic, markups are increasing with group size under DES and decreasing with group size under IES.*

*Proof.* Since we have

\[
\frac{\partial m(c)}{\partial k} = m'(c)\frac{\partial c}{\partial k}.
\]

(1.23)

According to the definition of markups and Proposition 1, the above results are proved.

Similar to market demands, markups does not change with group size when preferences are homothetic. However, when preferences are non-homothetic, markups change differently from market demands. Under DES, larger group size lowers the competitiveness of the market, and the markups charged by each firm increases. Under IES, larger group size increases the competitiveness of the market, and the markups charged by each firm decreases.

According to the labor-market-clearing condition, the number of firms in the market is

\[
N = \frac{L}{f + \varphi C}.
\]

(1.24)

**Proposition 4.** *When preferences are homothetic, the number of firms \( N \) does not change with group size \( k \); and when preferences are non-homothetic, the number of
firms $N$ is increasing with group size $k$ under DES and decreasing with group size $k$ under IES.

Proof. Standard calculation yields

$$\frac{\partial N}{\partial k} = \frac{\partial N}{\partial C} \frac{\partial C}{\partial k} = -\frac{L\varphi}{(f + \varphi C)^2} \frac{\partial C}{\partial k} = -\frac{N\varepsilon(c) - 1}{C} \varepsilon(c) \frac{\partial C}{\partial k}. \quad (1.25)$$

Hence, we know that the sign of $\partial N/\partial k$ is negatively related to the sign of $\partial C/\partial k$. Then according to Proposition 1, the results are proved.

Thus, similar to market demands and markups, the number of firms does not change with group size under homothetic preferences. When preferences are non-homothetic, the number of firms is increasing with group size under DES and decreasing with group size under IES.

The change of markups provides an explanation on the negative relationship between the impacts of group size on market demands and the number of firms. Under DES, larger group size decreases the competition in the market. Due to the decrease of competition, more firms are entering the market and each firm provides less quantity of differentiated product. Under IES, larger group size increases the competition in the market. The more competitive market leads to fewer firms entering and each firm in the market will produce more quantity. The intuition is that group size affects group demand, $c$. Under homothetic preferences, the change of group demand $c$ does not change firms’ markups because $m'(c) = 0$. However, under non-homothetic preferences, the change of group demand $c$ will affect firms’ markups $m(c)$. 

1.3 Welfare

In this model, since consumers are also workers, it is enough to examine social welfare from the consumption side.\(^{10}\) We can define social welfare as

\[
W = UL/k.
\]

It is easy to show the following,

\[
\frac{dW}{dk} = \frac{W}{k} \left( \frac{N}{U} \frac{\partial U}{\partial N} \frac{k}{dk} + \frac{c}{U} \frac{\partial U}{\partial c} \frac{k}{dc} - 1 \right)
\]

Proposition 5. When preferences are strongly homothetic, welfare \(W\) does not change with group size \(k\); otherwise, welfare \(W\) can be increasing or decreasing with group size \(k\).

Proof. Under symmetric equilibrium, the utility function can be written as \(U = (Nu(c))^{1/\theta}\). Hence, we have \(Nu(c) = U^\theta\) and the following

\[
\frac{\partial U}{\partial N} = \frac{U^{1-\theta} u(c)}{\theta}; \quad \frac{\partial U}{\partial c} = \frac{U^{1-\theta} Nu'(c)}{\theta}.
\]

According to the above equations, the change of welfare \(W\) with respect to \(k\) can be written as

\[
\frac{dW}{dk} = \frac{W}{k} \left( \frac{1}{\theta} \frac{k}{dk} + \frac{\eta(c)}{\theta} \frac{k}{dc} - 1 \right).
\]

\(^{10}\)Another reason for this is that, since there are free entry conditions, each firm makes zero profit. Dhingra and Morrow (2014) use this approach to discuss welfare under firm heterogeneity. Using the change of real income will yield similar results.
According to the relationship between group demands and market demands, the above expression can be written as

\[
\frac{dW}{dk} = W \left( \frac{1}{\theta N} \frac{dN}{dk} + \frac{\eta(c)}{C} \frac{dC}{dk} + \frac{\eta(c)}{\theta} - 1 \right). \tag{1.30}
\]

As for IES and DES, note that according to the proof of Proposition 4, we have the following

\[
\frac{d \ln W}{d \ln k} = \frac{1}{\theta} \left( \frac{\eta(c)}{c} - \frac{\varepsilon(c) - 1}{\varepsilon(c)} \right) \frac{k}{\theta} \frac{dC}{dk} + \frac{\eta(c)}{\theta} - 1. \tag{1.31}
\]

According to the proof of Proposition 2, we can rewrite the above equation as

\[
\frac{d \ln W}{d \ln k} = \frac{\eta(c)}{\theta} - 1 - \frac{1}{\theta} \frac{\eta'(c)c}{\varepsilon(c)} \frac{\varepsilon'(c)c}{2 - \rho(c)} \frac{1}{\varepsilon(c)\theta} - 1. \tag{1.32}
\]

Under homothetic preferences, we have \( \varepsilon'(c) = 0 \), and the change of welfare with respect to group size as

\[
\frac{dW}{dk} = \frac{W}{k} \left( \frac{\eta(c)}{\theta} - 1 \right). \tag{1.33}
\]

Hence, welfare still change with group size. The only exception is that preferences are strongly homothetic. Hence, the results are proved. \( \square \)

The above results show that even under homothetic preferences, welfare could change with group size. The only exception is that preferences are strongly homothetic. This implies that Case A and Case B predicts different welfare except for the case that \( u(c_i) = ac_i^\theta \). This is surprising. This is due to the “love-of-variety” property for preferences which restricts \( \eta(c) \) to be less than one. Under homothetic
preferences, since group size does not affect market demands and the number of firms, \( \eta(c) \) and \( \theta \) becomes very important. When \( \eta(c) > \theta \), welfare is increasing with group size. Hence, *Case A* predicts more welfare than *Case B*. When \( \eta(c) < \theta \), welfare is decreasing with group size. Hence, *Case A* predicts less welfare than *Case B*.

These results actually are not counterintuitive. If all consumers are sharing a common utility function, as argued by Dixit and Stiglitz (1977), the market can be considered as either that different consumers are consuming different products or that each consumer consumes a part of each product. However, the case that all consumers have identical utility functions only capture the latter case. Hence, these two cases should have different welfare implications.

Under non-homothetic preferences, the results become more complicated. Under different utility functions and different values of \( \theta \), welfare can be increasing or decreasing with group size. Thus, it is hard to say which of *Case A* and *Case B* predicts higher level of welfare.

Since the change of welfare with respect to group size depend on the assumption of preferences, we will discuss two examples. One for homothetic preferences, and the other for non-homothetic preferences.
1.3.1 An Example of Homothetic Preferences

Consider a simple CES preferences as in Krugman (1980) with $\theta = 1$ and $u(c) = x^\beta$, where $0 < \beta < 1$. In this case, we have $\eta(c) = \beta$. Hence, the change of welfare with respect to group size is

$$\frac{d \ln W}{d \ln k} = \beta - 1 < 0.$$  \hfill (1.34)

Hence, Case A predicts smaller level of welfare than Case B. Note that if we change the assumption of $\theta$, welfare predictions of Case A could be lower than Case B.

1.3.2 An Example of Non-Homothetic Preferences

Now we examine social welfare under a commonly used non-homothetic preferences as in Jackson (1984), Young (1991), Murata (2009), Sauré (2012), Pieters (2014), and Simonovska (2015). In this case, $\theta = 1$ and the sub-utility function is

$$u(c) = \log(c + \theta),$$  \hfill (1.35)

where $\theta$ is a constant and positive. Under this, we have the following

$$\eta(c) = \frac{c}{(c + \theta) \log(c + \theta)}; \xi(c) = \frac{c + \theta}{c}; \varrho(c) = \frac{2c}{c + \theta}.$$  \hfill (1.36)

The above expressions add an additional restriction on the parameter $\theta$, which is

$$\log(c + \theta) > \frac{c}{c + \theta}.$$  \hfill (1.37)
Furthermore, plugging price elasticities into free entry conditions, we get

\[ c = \sqrt{\frac{k_f \theta}{L \varphi}}. \]  \hspace{1cm} (1.38)

With the above expressions, it is easy to show that

\[ \frac{\partial W}{\partial k} W = \frac{3c - (c + \theta + c\theta) \log(c + \theta)}{2(c + \theta) \log(c + \theta)} < 1 - \frac{c\theta}{2(c + \theta)}. \]  \hspace{1cm} (1.39)

The above equation could be positive or negative under different consumption levels. This shows that, under non-homothetic preferences, social welfare can either increase or decrease with group size.

1.4 Applications

1.4.1 The Pattern of Trade

In this subsection, we analyze an open economy with two countries, Home and Foreign. Each country has the same setup as in the last section. The only difference between these two is that they are allowed to have different group sizes. This could be due to that the two countries having different cultures or government policies which affect consumers’ decisions. We first describe the setup of the open economy, and then analyze the role of group size in determining the relative wage between Home and Foreign.

With differentiated products supplying from two countries, the utility function
for \( k \) individuals in Home becomes

\[
U = \left( \int_0^N u(c_i)di + \int_0^{N^*} u(c_{ix})di \right)^{1/\theta},
\]

(1.40)

where \( N^* \) is the endogenous number of firms in foreign country, and \( c_{ix} \) is the consumption of foreign product.

The budget constraint becomes

\[
\int_0^N p(c_i)c_i di + \int_0^{N^*} p(c_{ix})c_{ix} di = wk,
\]

(1.41)

where \( w \) is the wage for worker-consumers in Home, and \( p(c_{ix}) \) is the price of foreign product.

Foreign is identical to Home except that the group size is \( k^* \). We normalize the wage in Foreign to one, hence \( w \) is the relative wage between Home and Foreign.

Similar to the last section, we have inverse demands and price elasticities for \( c, c_x, c^*, \) and \( c^*_x \).

Home and Foreign markets are segmented\(^\text{11}\) and there are no trade costs. The profit function for each firm in Home becomes

\[
\pi = p(c)cL/k + p(c^*_x)c^*_xL/k^* - w\varphi cL/k - w\varphi c^*_xL/k^* - wf.
\]

(1.42)

\(^{11}\)Note that the market within each country is still integrated that there is no price discrimination across groups as in the closed economy.
Firms in Foreign have similar profit functions. Hence, the markup pricing rules are
\[ p(c) = m(c) w \varphi; p(c^*_x) = m(c^*_x) w \varphi; p(c^*) = m(c^*) \varphi; p(c_x) = m(c_x) \varphi. \] (1.43)

The labor market clearing conditions are
\[ N(f + c \varphi L/k + c^*_x \varphi L/k^*) = L; N^*(f + c^* \varphi L/k^* + c_x \varphi L/k) = L, \] (1.44)
and the free entry conditions are
\[ (m(c) - 1)c \varphi L/k + (m(c^*_x) - 1)c^*_x \varphi L/k^* = f; \] (1.45)
and
\[ (m(c^*) - 1)c^* \varphi L/k^* + (m(c_x) - 1)c_x \varphi L/k = f. \] (1.46)
The above conditions imply that the numbers of firms in Home and Foreign follow
\[ N = N^* \frac{m(c^*)c^*/k^* + m(c_x)c_x/k}{m(c)c/k + m(c^*_x)c^*_x/k^*} \] (1.47)

With the above expression, we can use the trade balance condition to pin down the relative wage of Home \( w \). The trade balance condition is
\[ N p(c^*_x) c^*_x L/k^* = N^* p(c_x) c_x L/k. \] (1.48)
Substituting the number of firms and prices into the above equation, we get the following expression for relative wage in Home
\[ w = \frac{k^* \alpha k^* + k}{k \alpha^* k + k^*}. \] (1.49)
where $\alpha = \frac{m(c)c}{m(c)c_x}$, and $\alpha^* = \frac{m(c^*)c^*}{m(c^*)c_x}$.

Standard comparative statics yield the following results.

**Proposition 6.** When preferences are homothetic, the relative wage is one ($w = 1$); and when preferences are non-homothetic, the relative wage is the inverted ratio of group sizes ($w = k^*/k$).

**Proof.** We first prove the case of $\varepsilon'(c) = 0$. Applying Proposition 1 to both Home and Foreign, the market demands for each firm in both countries do not change with $k$. This leads to

$$c/c_x = k^*/k; \ c^*/c_x = k/k^*. \quad (1.50)$$

At the same time, markups are also constant, so we have $\alpha = k^*/k$ and $\alpha^* = k/k^*$. Therefore $w = 1$ as long as $\varepsilon'(c) = 0$.

We now turn to the case of $\varepsilon'(c) \neq 0$. Since there are no trade costs, and markets are segmented, marginal revenue equals marginal cost in both markets. And because firms in two countries have the same marginal cost $\varphi$, we have the following

$$\frac{\partial (m(c)c)}{\partial c} = \frac{\partial (m(c_x)c_x^*)}{\partial c_x^*}; \ \frac{\partial (m(c^*)c^*)}{\partial c^*} = \frac{\partial (m(c_x)c_x)}{\partial c_x}. \quad (1.51)$$

Since $\rho(x) < 2$ guarantees that the second order condition is negative, we have $m(c)c = m(c_x)c_x^*$ and $m(c^*)c^* = m(c_x)c_x$, and $\alpha = \alpha^*$. Based on these results, the relative wage becomes

$$w = \frac{k^*k + k}{k + k^*} = \frac{k^*}{k}. \quad (1.52)$$
Hence, the results are proved.

The above results show that, with preferences non-homotheticity and income redistribution among a group of individuals, wages inequality does not need to come from country size differences and iceberg trade costs as in Krugman (1980) and Chen and Zeng (2014). Our model has neither country size differences nor iceberg trade costs. In our model, when preferences are non-homothetic, wages inequality emerges as long as group sizes are different. Hence, it highlights the impact of consumption patterns on international inequality. In our case, globalization favors the country with smaller group size.

1.4.2 Public Goods Provision

In this subsection, we consider the public goods provision problem in a large economy following Pecorino (2009). We first describe the setup, and then discuss the impact of group size\(^\text{12}\) on public good expenditure.

Individuals care about a public good in the economy. The utility function for \(k\)

\(^{12}\text{Note that the group size in this paper is different from the definition in earlier discussions such as Olson (1965). The group size in this paper is the number of individuals } k \text{ sharing a common utility; whereas the "group size" in the traditional public goods provision literature corresponds to the total number of individuals in the market } L \text{ in this paper.}
individuals is

\[ V = \left( \int_0^N u(c_i) \, di \right)^{1/\theta} S^\gamma \]  \hspace{1cm} (1.53)

where \( c_i \) is the consumption of private good \( i \) by \( k \) individuals and \( S \) is the consumption of a public good for \( k \) individuals. We assume that all groups are consuming all units of the public good available. Hence \( S \) is also the output of public good for this economy. In order to focus on the role of homothetic preferences, we assume the following production for producing public good

\[ S = lL/k. \]  \hspace{1cm} (1.54)

The production of private goods is the same as the one-sector economy.

The budget constraint for \( k \) individuals is

\[ \int_0^N p(c_i) c_i \, di = k - l, \]  \hspace{1cm} (1.55)

where \( l \) is the total labor expenditure of \( k \) individuals for producing the public good.

To solve the consumers’ problem, we maximize \( \ln U \) subject to the budget constraint. It yields the following inverse demand

\[ p(c_i) = \frac{u'(c_i)}{\theta \lambda \int_0^N u(c_i) \, di}, \]  \hspace{1cm} (1.56)

and the following condition for labor expenditure

\[ Sk\lambda = \gamma L, \]  \hspace{1cm} (1.57)
where $\lambda$ is an aggregate variable defined by the following expression

$$
\lambda = \frac{\int_0^N u'(c_i)c_i \, di}{(k - l)\theta \int_0^N u(c_i) \, di},
$$

(1.58)

and, similar to the one-sector economy, each firms take it as a constant.

The free entry condition for firms producing private goods remains the same with the closed economy. Hence, the impact of group size $k$ on market demands of private goods $C$ satisfy Proposition 1. With this in mind, we have two conditions for solving for the number of firms $N$ and social expenditure on the public good $S$. The first one is the labor market clearing condition which is the following (symmetric equilibrium),

$$
N(f + \varphi cL/k) = L - S.
$$

(1.59)

The second one is the relationship between private goods and the public good. According to the first order conditions of the utility maximization problem and the markup pricing rules (the same with the one-sector economy), we have the following,

$$
Sku'(c) = Nu(c)\gamma Lm(c)\varphi \theta.
$$

(1.60)

Therefore, social expenditure on the public good is

$$
S = \frac{\gamma \theta}{\eta(c) + \gamma \theta} L.
$$

(1.61)

**Proposition 7.** When $\eta'(c) = 0$, social expenditure $S$ does not change with group size $k$; when $\eta'(c) > 0$, social expenditure $S$ is decreasing with group size $k$; and when $\eta'(c) < 0$, social expenditure $S$ is increasing with group size $k$. 
Proof. The change of social expenditure on the public good \( S \) with respect to \( k \) is

\[
\frac{dS}{dk} = -\frac{\gamma \theta \eta'(c)L}{(\eta(c) + \gamma \theta)^2} \frac{dc}{dk}.
\]

According to Proposition 1, \( dc/dk > 0 \), the sign of the above equation is negatively related to the sign of \( \eta(c) \). Hence, the above results are proved.

The above example shows that the public good provision problem does not depend on the elasticity of substitution, but depends on the elasticity of sub-utility. When \( \eta(c) \) is constant, Case A and Case B yield the same prediction on social expenditure on the public good; when \( \eta(c) \) is decreasing with consumption, Case A predicts more social expenditure on the public good than Case B; and when \( \eta(c) \) is increasing with consumption, Case A predicts less social expenditure on the public good than Case B.

1.5 Conclusion

It is standard that general equilibrium models of monopolistic competition in the spirit of Dixit and Stiglitz (1977) assume either that all individuals share a common utility function or that all individuals have identical utility functions. This paper has discussed whether these two approaches yield different prediction on product quantity, product diversity, and welfare. It has been shown that homothetic preferences can guarantee that the two approaches yield similar predictions on product quan-
tity and product diversity. However, in order to have similar prediction on welfare, homothetic preferences are not enough. In fact, we need stronger assumption that utility functions are homogeneous of degree one. One implication derived from this paper is that, since group size affects the predictions of general equilibrium models of monopolistic competition, empirical work on testing this group size in different industries or different countries is needed. Since our framework does not include quasi-linear quadratic preferences as in Melitz and Ottaviano (2008), we will discuss these preferences in future drafts.
CHAPTER 2. TRADE COSTS, COUNTRY SIZE, AND PER-CAPITA INCOME

2.1 Introduction

Early studies by Ades and Glaeser (1999), Frankel and Romer (1999), Alesina et al. (2000), and Alcalá and Ciccone (2004) show that larger country size and lower international trade costs lead to higher income levels. These studies do not consider domestic trade costs. A recent study by Ramondo et al. (2014) shows that domestic trade costs are also important to income inequality across countries. According to this, this paper revisits the Alesina et al. (2000) model with the consideration of both domestic trade costs and international trade costs, and answers the questions on how domestic trade costs and international trade costs affect per-capita income and scale effects differently.

Alesina et al. (2000) consider a dynamic model with a continuum of individuals endowed with differentiated intermediate inputs. These individuals exchange with others to improve outputs according to a production function featuring increasing
returns to scale. Exchange with foreign individuals incurs trade costs, but exchange with domestic individuals does not incur trade costs. Under this framework, they show that the growth rate and the steady state level of income are increasing with country size and decreasing with international trade costs.

In this paper, we simplify their model into a static one and add domestic trade costs into our analysis. Similar to Alesina et al. (2000), after normalizing technology and individual labor endowment to one, final product is produced by intermediate varieties under constant elasticity of substitution (CES) production function. Each individual is endowed with one unit of intermediate input variety and is seeking to exchange varieties with others to obtain economies of scale. Exchanging with other individuals from the same country incurs domestic trade costs, and exchanging with other individuals from other countries incurs international trade costs. Both types of costs are in iceberg form: more than one unit of that variety is needed in order to exchange one unit of each variety.

The model yields a simple per-capita income formula. Lowering domestic trade costs and international trade costs both increase per-capita income. At the same time, the impact of one type of trade costs is also affected by the other type of trade costs. The larger the one type of trade costs, the larger the impact of the other type.

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13Increasing returns to scale come from the constant elasticity of substitution aggregation of intermediate inputs. See Romer (1987) and Jones (2005) for discussions on this.
of trade costs on per-capita income. The impacts of trade costs on per-capita income change with country size as well. The impact of domestic trade costs decreases with country size, but the impact of international trade costs increases with country size.

These results provide a variety of trade policy\textsuperscript{14} implications. First, domestic trade policies and international trade policies are complements. Imposing one policy will eventually affect the adoption of the other policy in the future. Second, countries with different country sizes should focus on different trade policies. Domestic trade policy is important for larger countries, and international trade policy is important for smaller countries.

In addition to the impacts of trade costs on per-capita income, we also consider the impacts of trade costs on scale effects\textsuperscript{15}. Due to increasing returns to scale, countries with larger country size have higher per-capita income. Trade costs affect the scale effects differently. Lowering domestic trade costs increases the scale effects; in contrast, lowering international trade costs decreases the scale effects. We also consider a case where international trade costs are endogenous and depend on domestic trade costs and country size. In this case, interestingly, the scale effects are no

\textsuperscript{14}In this paper, trade policy is a government policy which is used to lower trade costs, either domestic trade costs or international trade costs.

\textsuperscript{15}See Jones (1999) for more discussions on this problem in endogenous growth literature. The scale effects are not consistent with empirical evidence, for example Backus et al. (1992), Jones (1995a,b), and Rose (2006).
longer a necessary feature of the model. Whether larger countries are richer depends on domestic trade costs. This is different from the result of Spolaore and Wacziarg (2005) that larger countries are richer when there are no domestic trade costs.

Ramondo et al. (2014) show that the scale effects property of the “idea-based” trade and growth models are due to the missing of domestic trade costs in these models. This paper complements their by including both domestic trade costs and international trade costs into a simple model. We show that the endogenous international trade costs case is consistent with their claim.

The remainder of this paper is organized as follows. Section 2.2 describe the setup of the model and analyze relationship between trade costs, country size, and per-capita income. Section 2.3 consider a case where international trade costs are endogenous. Section 2.4 concludes.

2.2 Model

There are $S$ countries in the world. In each country $k$, there are $N_k$ individuals; and each individual $i$ is endowed with one unit of labor and one unit of intermediate variety for producing final good $y_{ik}$. To produce final good, each individual can use its own intermediate variety $x_{iik}$, intermediate variety $x_{jik}$ from another individual $j$ in the same country, or/and intermediate variety $x_{jil}$ from another individual $j$ in another country $l$. 
The production function for each individual $i$ in country $k$ is

$$y_{ik} = A \left( \sum_i \sum_j x_{jil}^\theta \right) L_{ik}^{1-\theta},$$

(2.1)

where $0 < \theta < 1$ and it represents two important concepts: the elasticity of substitution between labor and intermediate varieties, and the elasticity of substitution between intermediate varieties themselves, $\sigma = 1/(1 - \theta)^{16}$. The first item is worldwide technology level, the second term is an aggregate of non-labor input (physical capital or human capital), and the third term is the input of labor. In this paper, we focus on the second term, so we assume each individual in the world has the same first term and third term.

The second term shows that the elasticity of substitution between varieties are constant.\textsuperscript{17} The production functions capture the insight of increasing returns to scale\textsuperscript{18}.

Each individual has three production choices. If individual $i$ in country $k$ uses

\textsuperscript{16}As we can see, the value of elasticity of substitution is increasing with the the value of $\theta$.

\textsuperscript{17}There are two forms representing CES functions. The one used in this paper follows Krugman (1980), and the other one is used by Melitz (2003). In terms of increasing returns to scale, these two forms capture the same idea, and only differ in terms of quantity.

\textsuperscript{18}Increasing returns to scale come from the externalities of differentiated intermediate varieties. For the details of this, see the discussions by Romer (1987) and Jones (2005).
its own intermediate variety only, the final good production function is

\[ y_{ik} = x_{iik}^\theta. \]

If this individual uses its own intermediate variety and intermediate varieties from others in the same country, the production function is

\[ y_{ik} = x_{iik}^\theta + \sum_{j=1}^{N_k} x_{jik}^\theta; j \neq i. \]

If this individual chooses to use intermediate varieties from both its own country and other countries, the production function is

\[ y_{ik} = x_{iik}^\theta + \sum_{j=1}^{N_k} x_{jik}^\theta + \sum_{l=1}^{S} \sum_{j=1}^{N_l} x_{jil}^\theta; j \neq i; l \neq k. \] (2.2)

Due to increasing returns to scale, each individual will choose production choice 3.

As shown by the production function above, intermediate varieties are imperfect substitutes. Each individual is willing to exchange with others to obtain more intermediate varieties. To get intermediate varieties from others, each individual needs to pay the same units of intermediate variety from its own. In the meantime, exchanging with others incurs trade costs. There are iceberg trade costs both domestically and internationally. In country \( k \), if one unit of intermediate variety is exchanged between individual \( i \) and \( j \), only \( 1 - \rho_k \) units of intermediate variety will be received by each individual; and if two individuals are from different countries \( k \) and \( l \) are exchanging
one unit intermediate variety, only $1 - \tau_k - \tau_l$ units of intermediate variety will be received by each individual.

2.2.1 Equilibrium and Per-Capita Income

To simplify the model, we assume that final good is for own consumption only. Hence, consumption $c_{ik}$ is equal to the output $y_{ik}$ of each individual. Besides, we consider the symmetric case in which the choice of individuals are identical, so $y_{ik}$ is equal to per-capita income $Y_k$.

Since each individual $i$ in country $k$ chooses to exchange with others both domestically and internationally, the resource constraint is

$$x_{iik} + \sum_{j=1}^{N_k} [x_{jk}/(1 - \rho_k)] + \sum_{l=1}^{N_l} [x_{jl}/(1 - \tau_k - \tau_l)] = 1; j \neq i; l \neq k. \quad (2.3)$$

Each individual maximizes (2.2) subject to (2.3). The first order conditions yield the following relationship among intermediate varieties used by this individual

$$x_{iik} = (1 - \rho_k)^{1/(\theta - 1)}x_{ijk} = (1 - \tau_k - \tau_l)^{1/(\theta - 1)}x_{jil}; j \neq i; l \neq k. \quad (2.4)$$

Plugging this back into resource constraint, we get the intermediate input from its own as

$$x_{iik} = \frac{1}{1 + (N_k - 1)(1 - \rho_k)^{\theta/(1-\theta)} + \sum_{l=1}^{S}[N_l(1 - \tau_k - \tau_l)^{\theta/(1-\theta)}], l \neq k;} \quad (2.5)$$
the intermediate input from others in the same country as

\[ x_{jik} = \frac{(1 - \rho_k)^{1/(1-\theta)}}{1 + (N_k - 1)(1 - \rho_k)^{\theta/(1-\theta)} + \sum_{l=1}^{S} [N_l(1 - \tau_k - \tau_l)^{\theta/(1-\theta)}]}; l \neq k, \quad (2.6) \]

and the intermediate input from others in other countries as

\[ x_{iik} = \frac{(1 - \tau_k - \tau_l)^{1/(1-\theta)}}{1 + (N_k - 1)(1 - \rho_k)^{\theta/(1-\theta)} + \sum_{l=1}^{S} [N_l(1 - \tau_k - \tau_l)^{\theta/(1-\theta)}]}; l \neq k. \quad (2.7) \]

Plugging (2.5)-(2.7) back into (2.2), we can get the following per-capita income in country \( k \),

\[ Y_k = \left\{ 1 + (N_k - 1)(1 - \rho_k)^{\theta/(1-\theta)} + \sum_{l=1}^{S} [N_l(1 - \tau_k - \tau_l)^{\theta/(1-\theta)}] \right\}^{1-\theta}; l \neq k. \quad (2.8) \]

The above expression is the basis of our analysis. Each country takes the elasticity of substitution as given, and is different in terms of country size and trade costs. Hence, per-capita income depends on country specific parameters given other countries’ parameters and world’s technology level, \( Y_k = f(N_k, \rho_k, \tau_k; \theta, N_l, \tau_l) \).

**Proposition 8.** Per-capita income is increasing with country size, and decreasing with domestic trade costs and international trade costs.

**Proof.** The change of per-capita income with respect to country size is

\[ \frac{\partial Y_k}{\partial N_k} = (1 - \theta)Y_k^{\theta/(\theta-1)}(1 - \rho_k)^{\theta/(1-\theta)} > 0; \]

with respect to domestic trade costs is

\[ \frac{\partial Y_k}{\partial \rho_k} = -\theta Y_k^{\theta/(\theta-1)}(N_k - 1)(1 - \rho_k)^{(2\theta-1)/(1-\theta)} < 0; \]
with respect to international trade costs is

$$\frac{\partial Y_k}{\partial \tau_k} = -\theta Y_k^{\theta/(\theta-1)} \sum_{l=1}^S \left[ N_l (1 - \tau_k - \tau_l)^{(2\theta-1)/(\theta-1)} \right] < 0; l \neq k.$$

Hence, Proposition 8 is proved.

Proposition 8 shows that the model captures some basic implications of the effects of country size and trade costs on per-capita income. The first result is the scale effect, larger countries are richer, and the last two results are trade costs effects, lowering trade costs is beneficial to the economy.

For the change of the impact of domestic (international) trade policy on per-capita income with respect to international (domestic) trade costs, we have the following result.

**Proposition 9.** Per-capita income is decreasing with domestic trade costs multiplied by international trade costs.

**Proof.** According to Young’s theorem and Proposition 8, we have

$$\frac{\partial^2 Y_k}{\partial \rho_k \partial \tau_k} = \frac{\partial^2 Y_k}{\partial \tau_k \partial \rho_k} = \frac{\theta^2}{1 - \theta} Y_k^{1/(\theta-1)} (N_k - 1) (1 - \rho_k)^{(2\theta-1)/(1-\theta)} \frac{\partial Y_k}{\partial \tau_k} < 0.$$

Hence, the result is proved.

The above proposition shows that when international trade costs are larger, the domestic trade costs effects increase, and it becomes more important to reduce domestic trade costs to improve per-capita income; the economy follows the same
pattern when domestic trade costs are larger. Combining these two cases together, the model shows that under increasing returns to scale and iceberg trade costs, domestic trade costs and international trade costs are complements and they have mutual effects.

For the change of the impact of domestic (international) trade costs on per-capita income with respect to country size, we have the following result.

**Proposition 10.** Per-capita income is decreasing with domestic trade costs multiplied by country size, and increasing with international trade costs multiplied by country size.

*Proof.* The change of domestic trade costs effects with respect to country size is

$$\frac{\partial^2 Y_k}{\partial \rho_k \partial N_k} = -\theta Y_k^{\theta/(\theta-1)}(1-\rho_k)^{(2\theta-1)/(1-\theta)}$$

$$+ \frac{\theta^2}{1-\theta} Y_k^{1/(\theta-1)}(N_k-1)(1-\rho_k)^{(2\theta-1)/(1-\theta)} \frac{\partial Y_k}{\partial N_k}.$$  

Plugging $\frac{\partial Y_k}{\partial N_k}$ and (2.5) into above expression, we get

$$\frac{\partial^2 Y_k}{\partial \rho_k \partial N_k} = -\theta Y_k^{\theta/(\theta-1)}(1-\rho_k)^{(2\theta-1)/(1-\theta)}$$

$$\times \left[ 1 - \frac{(N_k-1)(1-\rho_k)^{\theta/(1-\theta)}}{1 + (N_k-1)(1-\rho_k)^{\theta/(1-\theta)} + \sum_{l=1}^{S} [N_l(1-\tau_k - \tau_l)^{\theta/(1-\theta)}]} \right] < 0.$$  

The change of international trade costs effects with respect to country size is

$$\frac{\partial^2 Y_k}{\partial \tau_k \partial N_k} = \theta^2 Y_k^{(1+\theta)/(\theta-1)} \sum_{l=2}^{S} [N_l(1-\tau_k - \tau_l)^{(2\theta-1)/(1-\theta)}] (1-\rho_k)^{\theta/(1-\theta)} > 0; l \neq k.$$  

Hence, the result is proved. \qed
For the impact of country size on domestic trade costs effects, we have several interesting points. Firstly, when there is only one individual in country $k$, there is no trade costs effects. This might seem a little obvious and unrealistic. However, it does show the case that when country is small enough, we can observe low domestic trade costs effects. Secondly, domestic trade costs effects are larger in absolute value when countries are larger. If two countries have similar level of domestic trade costs, the larger country is worse. Hence, domestic trade policy is more important to larger countries.

Different from domestic trade costs effects, larger countries have smaller international trade effects. The intuition is that since larger countries have more domestic resources, larger international trade costs will not bring too much harm. With high international trade costs, individuals can simply choose to exchange with others from the same country to avoid international trade costs.

The above results also show that domestic trade costs and international trade costs affect the scale effects differently. Lowering domestic trade costs will increase the scale effects; however, lowering international trade costs will decrease the scale effects.
2.3 Endogenous International Trade Costs

The above result and the results in Section 2.2 are derived from the assumption that trade costs are exogenous, and when government lowers one trade costs, the other trade costs remain constant. This is of course not always true. As documented by Fajgelbaum and Redding (2014) and Donaldson (2014), domestic trade costs play a role in shaping international trade costs. In this section, we consider an endogenous international trade costs case to test the robustness of the scale effects property.

Following the technique used by Spolaore and Wacziarg (2005), we extend the current model into the case where international trade costs are endogenous and are dependent on domestic trade costs and country size. International trade costs are assumed as the following function

$$\tau_k = \frac{\tau}{2} - \lambda_k,$$  \hspace{1cm} (2.9)

where $\lambda_k$ is the choice variable by the government in country $k$. The government of country $k$ chooses $\lambda_k$ to maximize the following

$$\eta_k \tau_k + (1 - \eta_k) Y_k - \frac{\phi_k}{2} \lambda_k^2,$$  \hspace{1cm} (2.10)

where $\eta$ is the weight on the benefit from trade barriers, $1 - \eta$ is the weight on per-capita income, and $\phi$ captures the costs from trade barriers. We follow Spolaore and Wacziarg (2005) to use this specific objective function, which, as they argued, shows
the tradeoff between the benefits and costs of the government from imposing trade policy. Besides, we also assume that government has measure zero in each country, which will ensure that per-capita income is not affected by the benefits and costs of the government. Plugging (2.9) into (2.10), the objective function of the government becomes

$$\eta_k \left( \frac{\tau}{2} - \lambda_k \right) + (1 - \eta_k)Y_k - \frac{\phi_k}{2} \lambda_k^2; l \neq k.$$  

where $Y_k$ is given by equation (2.8). The first order condition is given by the following

$$\eta_k + (1 - \eta_k) \frac{\partial Y_k}{\partial \tau_k} + \phi_k \lambda_k = 0. \quad (2.11)$$

**Proposition 11.** When $\phi_k > (1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k^2}$, international trade costs are increasing with country size and decreasing with domestic trade costs; and when $\phi_k < (1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k^2}$, international trade costs are decreasing with country size and increasing with domestic trade costs.

**Proof.** We first prove that international trade costs are increasing with country size. According to (2.11) and the implicit function theorem, we have the following

$$\frac{\partial \lambda_k}{\partial N_k} = -\frac{(1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k \partial N_k}}{-(1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k^2} + \phi_k}. \quad (2.12)$$

According to Proposition 10, we have $\frac{\partial^2 Y_k}{\partial \tau_k \partial N_k} > 0$. For $\frac{\partial^2 Y_k}{\partial \tau_k^2}$, we have

$$\frac{\partial^2 Y_k}{\partial \tau_k^2} = \frac{\theta^2}{1 - \theta} Y_k^{1/(\theta - 1)} \frac{\partial Y_k}{\partial \tau_k} \sum_{l=1}^{S} \left[ N_l (1 - \tau_k - \tau_l)^{(2\theta - 1)/(1 - \theta)} \right] + \theta \frac{2\theta - 1}{1 - \theta} Y_k^{\theta/(\theta - 1)} \sum_{l=1}^{S} \left[ N_l (1 - \tau_k - \tau_l)^{(3\theta - 2)/(1 - \theta)} \right]$$
The above equation is negative as long as $\theta \leq 1/2$. In this case, we have $\partial \lambda_k / \partial N_k < 0$. Hence, we have $\partial \tau_k / \partial N_k > 0$.

Similarly, the change of $\lambda_k$ with respect to domestic trade costs is given by the following

$$\frac{\partial \lambda_k}{\partial \rho_k} = -\frac{(1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k \partial \rho_k}}{-(1 - \eta_k) \frac{\partial^2 Y_k}{\partial \tau_k^2} + \phi_k}. \tag{2.13}$$

According to Proposition 9, we have $\frac{\partial^2 Y_k}{\partial \tau_k \partial \rho_k} < 0$. Hence, we have $\partial \lambda_k / \partial \rho_k > 0$, and $\partial \tau_k / \partial \rho_k < 0$. \hfill \Box

The intuition for the first case is that when country size is very large or domestic trade costs are very small, the costs of lowering international trade costs become lower. Hence, governments tend to choose higher international trade costs. The intuition for the second case follows the same logic, but the signs are different. Thus, we can conclude that the choice of international trade costs by the government is dependent on values of parameters. However, in any cases, the consideration of endogenous international trade costs changes the scale effects and domestic trade costs.

**Proposition 12.** When international trade costs are endogenous, there are no scale effects or domestic trade costs effects.

**Proof.** To see that larger country size does not lead to higher income level, we differ-
entiate (2.8) with respect to country size,

\[
\frac{\partial Y_k}{\partial N_k} = (1 - \theta) \frac{\theta}{(1 - \theta)} (1 - \rho_k) Y_k^{\theta/(1-\theta)} - \theta Y_k^{\theta/(1-\theta)} \sum_{i=1}^{S} \left[ N_i (1 - \tau_k - \tau_i)^{(2\theta - 1)/(1-\theta)} \frac{\partial \tau_i}{\partial N_k} \right].
\]

(2.14)

Since we do not know the sign of the second term, so the sign of the above expression is ambiguous.

To see that larger domestic trade costs does not lead to lower income level, we differentiated (2.8) with respect to domestic trade costs,

\[
\frac{\partial Y_k}{\partial \rho_k} = \frac{\partial Y_k}{\partial \rho_k} (-) + \frac{\partial Y_k}{\partial \tau_k} (-) \frac{\partial \tau_k}{\partial \rho_k} (?) \]

(2.15)

According to previous propositions, the sign of the above equation is indecisive. □

The story behind the above results is that country size and domestic trade costs affect the choice of international trade costs. Hence, country size and domestic trade costs can no longer guarantee the advantages or disadvantages of a country. These results are very different from what we have known from Spolaore and Wacziarg (2005). In their paper, without domestic trade costs, even though international trade costs are endogenous and are dependent on country size, larger country size is always an advantage. By including domestic trade costs, we show that this result is not always true and it is dependent on domestic trade costs. This echoes the argument of Ramondo et al. (2014) that missing domestic trade costs is responsible for the scale effects in idea-based trade and growth theories.
2.4 Conclusion

By considering domestic trade costs and international trade costs into a simple model, this paper has studies the relationship between trade costs, country size, and per-capita income. It highlights the differences and similarities between domestic trade costs and international trade costs. On one hand, per-capita income is decreasing with both domestic trade costs and international trade costs; on the other hand, per-capita income is decreasing with domestic trade costs multiplied by country size but increasing with international trade costs multiplied by country size.

We also consider a case where international trade costs are endogenously chosen by the government. In this case, international trade costs can increase or decrease with country size and domestic trade costs under different values of parameters. In any cases, a surprising result emerges that pre-capita income does not monotonically change with domestic trade costs and country size. This echoes the argument by Ramondo et al. (2014) that, when domestic trade costs are taken into consideration, larger country size does not lead to larger income level.
CHAPTER 3. IMPORT GROWTH AND THE EMERGENCE OF AN AGRICULTURAL PRODUCTIVITY GAP

with Philip R. Wandschneider

3.1 Introduction

Agriculture is a large and unproductive sector in poor countries; it is a small, but highly productive sector in rich countries. It is well established that low labor and total factor productivity in agriculture are central factors in explaining cross-sectional and inter-temporal differences in income level across nations (Caselli, 2005; Gollin et al., 2014). Hence the agricultural productivity gap in poor countries presents both an important social problem and a compelling theoretical puzzle.

When and how does the productivity gap between manufacturing and agricultural sectors emerge? Interestingly, in richer economies, agriculture labor and factor productivity is again growing at the same rate as the general economy (Martin and Mitra, 2001). As economies grow from a Malthusian subsistence trap economy, there must be a period in which this productivity gap emerges, though it disappears later.
A classic explanation is Schultz (1953) “food problem”. Food is a necessity. Workers stay in agriculture to avoid starvation, even when productivity and wages become higher in industrial sectors. However, the income elasticity of food is low: while it dominates the budget of the poor, it dwindles to 20% or less of the rich consumer’s budget. The agricultural sector falls to 5% or less by employment.

In this paper, we propose a different and complimentary mechanism for the emergence of the productivity gap using a $2 \times 2$ sector endogenous growth model based on Romer (1990) and Grossman and Helpman (1991). International trade is modeled in the spirit of Krugman (1980). We begin with a baseline state in which there is no difference in productivity growth and we analyze the impact of international trade change on this economy. We impose two key assumptions: increased openness to trade does not lead to labor reallocation, and import variety share in manufacturing increases faster than in agriculture. Given these two assumption, international competition in the manufacturing sector drives higher productivity and growth in manufacturing, while productivity grows slowly or is stagnant in agriculture.

There are two primary reasons for these results. Firstly, under an R&D based

\footnote{Rural to urban migration issues are beyond the scope of the present paper, but we note that issues include barriers to mobility and the high unemployment which reduces expected returns from higher wages.}
endogenous growth framework, existing varieties have a spillover effect on new intermediate variety creation. More imports from other countries in one industry will generate expansion in the domestic industry’s own new variety creation, and hence generate an expansion in productivity. Secondly, more imports in one industry will leave more room for the relative price of the other industry to grow. The former captures the benefit of imports for its own industry, and the latter captures the cost for the other industry.

The mechanism presented in this paper provides an explanation for economies with the following features: (i) the economy has two major sectors, a rural-agriculture sector and an emerging urban industrial sector; (ii) there are initially identical levels of growth rates in the two sectors; (iii) and there is a restricted flow of labor between sectors when trade policies change.

Cross-country trade and agricultural productivity data provides empirical evidence on the above analytical results. By giving different variety weights for each product according to the export of each country, we compared agricultural productivity and weighted import variety of each country in 1985. We find that poor countries import more variety from other countries as well as have lower labor productivity in agriculture.

This paper is related to the growing literature analyzing which factors are responsible for the large cross-country productivity differences in agricultural sectors,
but relatively small productivity differences in the non-agricultural sector\textsuperscript{20}. In the paper by Lagakos and Waugh (2013), self-selection of heterogeneous workers determines sector productivity. In poor countries, subsistence requirements induce low productive workers into agriculture where they may be “stuck” (non-mobile) for institutional and economic reasons. Using a two-sector general equilibrium model, Restuccia et al. (2008) show that differences in economy-wide productivity, barriers to modern intermediate inputs in agriculture, and barriers in the labor market are the key elements generating productivity difference. Other factors investigated include: transportation (Adamopoulos, 2011), farm size (Adamopoulos and Restuccia, 2014), intermediate inputs risks (Donovan, 2014), and financial frictions (Liao and Wang, 2014).

This paper enriches this literature in two ways. First, we focus on the impact of international trade on sectoral productivity growth difference. While Uy et al. (2013) study an open multi-sector economy similar to ours; they only focus on the Korean case. Moreover, their results are mostly based on simulations. Fadinger and Fleiss (2011) estimate how trade affects sectoral productivity, but they only consider the manufacturing industries. Tombe (Forthcoming) analyzes the impact of international trade on changes in agriculture and manufacturing sectors - including different

\textsuperscript{20}See Caselli (2005), Chanda and Dalgaard (2008), and Vollrath (2009b).
growth rates and a resulting productivity gap. Swięcki (2014) shows that domestic in-
tersectoral distortion affects gains from trade. This paper follows this trade-oriented
literature, but it adds consideration of how intersectoral market imperfections, in-
cluding labor market immobility, might affect sectoral growth.

Second, we model an emerging two-sector economy in which the subsistence
level is already met in both agriculture and manufacturing and the final product is
produced by differentiated intermediate varieties in each industry. Hence the theory
describes an economy that may have escaped Malthusian and Schultzian traps, but
still is relatively poor and only partially linked to world markets. This is different from
a static approach with calibration or simulation, or either a neoclassical or a unified
growth approach (Vollrath, 2009a). Furthermore, we add realism and richness by
allowing for monopolistic competition in agriculture instead of the usual assumption
of agriculture as a homogeneous and perfectly competitive sector (Grossmann, 2013).
People eat a variety of foods, processed in a variety of ways; they don’t eat raw grain,
vegetables, and livestock.

The paper is organized as follows. Section 3.2 describes the basic setup of
dual economy endogenous growth model with international trade. Section 3.3 shows
analytical results on balanced growth path. Section 3.4 discusses the impact of trade
on productivity growth. Section 3.5 shows empirical evidence. Section 3.6 discusses
the model and concludes.
3.2 Model

We now introduce an endogenous growth framework in which there are two countries: home and foreign, \( i \in \{H, F\} \). In each country, there are two industries, agriculture and manufacturing, \( j \in \{a, m\} \). There is international trade in both industries.

3.2.1 Consumption and the Inter-Temporal Choice

The representative consumer in each country consumes final products from both agriculture and manufacturing. The consumer maximizes its inter-temporal utility from aggregate consumption of agriculture \((C^i_a)\) and manufacturing \((C^i_m)\) commodities,

\[
U^i(t) = \int_t^\infty e^{-\rho(t-\tau)} \left[ \psi \log C^i_a(\tau) + (1 - \psi) \log C^i_m(\tau) \right] d\tau,
\]

where \( C \) is the consumption of final product, \( \rho \) is the subjective discount rate, and \( \psi \) is the relative taste for agricultural versus manufacturing goods, and the inter-temporal elasticity of substitution is one. The budget constraint is

\[
\int_t^\infty e^{-[R^i(\tau)-R^i(t)]} [P^i_a(\tau)C^i_a(\tau) + P^i_m(\tau)C^i_m(\tau)] \leq \int_t^\infty e^{-[R^i(\tau)-R^i(t)]} w^i(\tau)L^i d\tau + W^i(t),
\]

where \( R^i(\tau) \equiv \int_0^\tau r^i(s)ds \) represents the discount factor from time \( \tau \) to time 0, \( P \) is the price of the final product, \( w \) is the countrywide wage rate, and \( W \) is the value of
the household’s asset holding. By maximizing the representative consumer’s utility subject to budget constraint, we obtain the following Euler equation,

\[
\frac{\dot{C}_i^j(t)}{C_i^j(t)} = r^i(t) - \rho - \frac{\dot{P}_i^j(t)}{P_i^j(t)},
\]

(3.1)

where \(r^i\) is the countrywide interest rate for both agriculture and manufacturing, and \(\dot{C}\) and \(\dot{P}\) represent the change of consumption and price, respectively.

### 3.2.2 Final Good Production and the Intermediate Goods Sets

At the aggregate level, in each industry of each country, the final good is produced from intermediate goods with a constant elasticity of substitution (CES) production function,

\[
Y_j^i(t) = \left( \int_0^{N_j^i(t)} y_j^i(k, t)^{\sigma_j/(\sigma_j-1)} \, dk \right)^{\sigma_j/(\sigma_j-1)},
\]

(3.2)

where \(Y\) is the output of final product, \(y\) is the input of intermediate product, \(\sigma\) is the elasticity of substitution between varieties, \(N\) is the number of intermediate varieties. We use \(k\) to denote each intermediate variety. We will assume trade in intermediate products but no trade in final products (since the final products in both countries are identical). Hence, final domestic consumption will equal final domestic production. We assume that, in each country, elasticity of substitution for agriculture and manufacturing are different, and the elasticity of substitution in agriculture is
larger\textsuperscript{21}; also we assume that both countries have the same elasticity of substitution parameters. It is worth noting that the agricultural sector can be either homogeneous or heterogeneous, depending on the value of elasticity of substitution. Both cases capture important characteristics of agricultural sectors in the real world.

Since the final good is produced from a set of intermediate products, based on Dixit and Stiglitz (1977), the price index of the final good can be written as a function of prices of intermediate products as

$$P^i_j(t) = \left( \int_0^{N^j(t)} p^j_i(k, t)^{(1-\sigma_j)} dk \right)^{1/(1-\sigma_j)},$$

where $p$ is the price of the intermediate variety.

Each country produces a different set of intermediate products, and the final product can be produced in both countries. Hence, the intermediate product set for producing the final good can divided into domestic set and importing set as the following,

$$N^i_j(t) = N^i_{j,i}(t) + N^i_{j,-i}(t),$$

where $N^i_{j,i}$ is the number of intermediary varieties produced domestically, and $N^i_{j,-i}$ is the number of intermediate varieties imported from the other country.

\textsuperscript{21}This assumption ensures that manufacturing products are more heterogeneous, and agricultural products are more homogeneous.
3.2.3 Sectoral Productivity

As shown in Chapter 10 of Feenstra (2003), the above endogenous growth setup has a traditional (neoclassical) productivity implication. In the symmetric case, each intermediate variety will have the same property and the final good production function (3.2) can be rewritten as

\[ Y^i_j(t) = N^i_j(t) \sigma_j / (\sigma_j - 1) \phi^i_j(t) = N^i_j(t)^{1/(\sigma_j - 1)} N^i_j(t) y^i_j(t). \]

The above expression can be understood as \( Y^i_j(t) \equiv A^i_j(t) \Phi^i_j(t) \), where \( A^i_j(t) \equiv N^i_j(t)^{1/(\sigma_j - 1)} \) is technology in the neoclassical growth framework, and \( \Phi^i_j(t) \equiv N^i_j(t) y^i_j(t) \) is the input to final good production. Hence, this “endogenous growth” approach can be easily related to the neoclassical growth literature and the development accounting literature in which productivity is the key to growth in the economy. Moreover, in this framework, productivity is entirely related to the number of intermediate varieties. The relationship between the productivity growth rate and the number of intermediate varieties growth rate is given in the following lemma.

**Lemma 4.** The growth rate of productivity is \( g = g_N/(\sigma - 1) \).

**Proof.** From the definition of productivity. □
3.2.4 Intermediate Goods Production and Firms’ Decisions

Labor is the only input for producing intermediate products. To simplify the analysis, we assume a relationship of one labor unit to one variety product type as

\[ x^i_j(k, t) = l^i_j(k, t), \]

where \( l \) is the labor input, and \( x \) is the output of intermediate variety.

The above setup is similar to standard monopolistic competition models, so we have the standard monopolistic competition solution in final good production. The markup pricing rule is

\[ p^i_j(k, t) = w^i(t)\sigma_j/\left(\sigma_j - 1\right), \]

and the profit for each variety is

\[ \pi^i_j(k, t) = w^i(t)l^i_j(k, t)/\left(\sigma_j - 1\right). \quad (3.3) \]

In this basic setup, labor is perfectly mobile. This ensures a natural (baseline) state which is useful for comparing the subsequent state with international trade policy changes. In Section 3.4, when trade policy changes, we have the labor market shares in agriculture and manufacturing remain the same based on empirical evidence.

Following Grossman and Helpman (1991), we find the no-arbitrage condition for each variety as the following

\[ \dot{v}_j^i(k, t) = r^i(t)v_j^i(k, t) - \pi_j^i(k, t), \quad (3.4) \]
where $v$ is the stock market value of a particular intermediate variety.

### 3.2.5 Intermediate Variety Expansion and Free Entry

Firms with potential new varieties are always trying to enter the market in each industry. In order to enter the market, each variety (firm) needs to hire labor. The entrance of each new variety is affected by existing varieties and labor input. We assume variety growth functions as

$$
\dot{N}^{i,i}_j(t) = \frac{1}{\eta_j} [N^{i,i}_j(t) + N^{i,-i}_j(t)] \gamma^i [N^{i,i}_j(t) + N^{i,-i}_j(t)]^{1-\gamma} l^{i}_{j,n}(t),
$$

where $l_{j,n}$ is the labor input, $\eta$ is the productivity parameter, $\dot{N}$ is the variety set of new varieties, $-i$ denotes the other country, $-j$ denotes the other industry, $\gamma$ is an exogenous parameter for existing varieties’ externality, and the Cobb-Douglas term captures the spillover effect from existing varieties. We consider three sources of the externality from existing varieties: domestic variety, foreign variety, and variety from the other sector\textsuperscript{22}. Moreover, as discussed above, based on the assumption of intermediate goods production (one unit of labor to one variety), the variety growth functions can capture two kinds of innovation: process and production innovation\textsuperscript{23}.

\textsuperscript{22}A good example of inter-industry spillover effect is biofuel. Biofuel is a combination of industrialized technology and traditional agriculture.

\textsuperscript{23}See a discussion by Atkeson and Burstein (2010) and Dhingra (2013).
Hence, variety expansion can be seen as either the creation of a new product (product innovation) or the improvement of an existing product (process innovation).

As shown in Section 3.2.3, each variety has a value function $v$, so the free entry condition for new varieties can be written as

$$v_j^i(t) N_j^{i,i}(t) = w_j^i(t) r_{j,n}(t).$$

(3.6)

### 3.2.6 Equilibrium

In the intermediate goods market, the output of an intermediate variety can be used for either domestic final production or exporting or both. So we have:

$$y_j^{i,i}(k,t) + y_{-j}^{i,i} = x_j^i(k,t),$$

(3.7)

where $y_j^{i,i}$ and $y_{-j}^{i,i}$ represent the intermediate products used for domestic final good production and exporting, respectively. The above setup implies that trade occurs in intermediate products only. Therefore final good equals consumption as $C_j^i(t) = Y_j^i(t)$. Hence, condition (3.1) holds for final goods output as well.

In the labor market, labor endowment $L$ in each country is constant. Labor is used to produce both existing varieties and new varieties. So the labor market clearing condition is

$$\sum_j \left( \int_0^{N_j^{i,i}(t)} l_j^i(k, t) dk + \int_{j,n}^i l_j(t) \right) = L^i.$$

(3.8)
3.3 Baseline State of the Home Country

Now we consider the case where the home country is only a tiny part of the globe. We add a labor endowment constraint $L^F \gg L^H$ to the setup of the last section. This is the case for most developing countries. Now the foreign country is just a much larger version of the home country, and we can take it as the rest of world.

In this case, we analyze the balanced growth path for the home country. Specifically, we consider a state in which the government of the home country imposes a trade policy to each industry such that, on the balanced growth path, import variety set is proportional to domestic variety set. This is assured by two characteristics of the home country: one is that it is a small open economy; the other is that agriculture and manufacturing are symmetric. Therefore, in symmetric equilibrium, for the home country, we have

$$\alpha_j^H N_j^{H,F}(t) = \alpha_j^H N_j^{H,H}(t), \quad (3.9)$$

where $\alpha_j^H$ denotes the proportion of the import intermediate variety set to the domestic intermediate variety set, and we call it the import variety share of sector $j$. Given the implied balanced trade condition, exports will also be proportional to domestic consumption for each variety as the following

$$y_j^{F,H}(t) = \beta_j^H y_j^{H,H}(t), \quad (3.10)$$
where $\beta_j^H$ denotes the percentage of export to domestic use for a given intermediate variety.

For the home country, we normalize the price index of the manufacturing final product to one. The remaining price index $P$ is now simply the relative agricultural price for the home country. According to the above considerations and dropping the superscript of country index $i$ for each variable, the conditions for solving the balanced growth path of home country are:

$$g_Y = r(t) - \rho - g_P, \quad (3.11)$$
$$g_Y = r(t) - \rho, \quad (3.12)$$
$$Y_j(t) = (1 + \alpha_j)^{\sigma_j/(\sigma_j - 1)}N_j(t)^{\sigma_j/(\sigma_j - 1)}(1 + \beta_j)^{-1}l_j(t), \quad (3.13)$$
$$\pi_j(t) = w(t)l_j(t)/(\sigma_j - 1), \quad (3.14)$$
$$r(t)v_j(t) - \pi_j(t) = 0, \quad (3.15)$$
$$\dot{N}_j(t) = \eta_j^{-1}(1 + \alpha_j)^{\gamma_j}(1 + \alpha_{-j})^{1-\gamma_j}N_j(t)^{\gamma_j}N_{-j}(t)^{1-\gamma_j}l_{j,n}(t), \quad (3.16)$$
$$v_j(t)\dot{N}_j(t) = w(t)l_{j,n}(t), \quad (3.17)$$
$$(1 + \beta_j)y_j(t) = x_j(t), \quad (3.18)$$
$$\sum_j (N_j(t)l_j(t) + l_{j,n}(t)) = L, \quad (3.19)$$

where $g$ is the growth rate, and $\dot{v} = 0$.

Using the above equilibrium conditions, we can solve for the productivity growth
rate for each sector. The following proposition shows the result.

**Proposition 13.** The productivity growth rate in each industry is affected by import variety and relative price as the following

\[
g_a = \frac{1}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}L - \frac{1}{2}\rho - \frac{b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}g_P; \quad (3.20)
\]

\[
g_m = \frac{1}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}L - \frac{1}{2}\rho + \frac{b/(1 + \alpha_a)}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}g_P, \quad (3.21)
\]

where \( b = \eta_a(\sigma_a - 1)(\eta_a/\eta_m)^{(1-\gamma_a)/(\gamma_a - \gamma_m)} > 0 \) and \( d = \eta_m(\sigma_m - 1)(\eta_a/\eta_m)^{-\gamma_m/(\gamma_a - \gamma_m)} > 0 \).

**Proof.** See Appendix A.

In the above proposition, \( b \) and \( d \) are exogenous variables, as well as \( L \) and \( \rho \). These variables represent the preference and technology for home economy. Three variables are critical to labor productivity growth rate: \( \alpha_a, \alpha_m, \) and \( g_P \). Relative price has opposite effects on agriculture and manufacturing, and the values of \( \alpha_a \) and \( \alpha_m \) determines how much relative price affects labor productivity.

Since agriculture and manufacturing are symmetric, there is no relative price growth \( (g_P = 0) \) in our baseline state. Intuitively, if there is a change in the relative price in the initial state, labor will migrate until the growth of relative price is zero (since labor is perfectly mobile). So the growth rates for agriculture and manufacturing are equal. However, we still keep \( g_P \) in Proposition 14 as we want to compare this with the case when trade policy changes and labor market distortion exists.
We emphasize that this baseline state is a convenient starting point for analyzing the effect of international trade. In fact, it represents a case of a country in which there is a fixed import variety share and no growth difference between agriculture and manufacturing. Now we can examine the impact of altering import variety share. This is our major task in next section.

3.4 The Impact of Import Growth on Agricultural Productivity

Based on the above results, we can now examine how sectoral productivity changes when the home country becomes more open to trade. We start by considering trade induced labor reallocation.

3.4.1 Labor Reallocation

There are two possible channels of trade induced labor reallocation: intra-industry reallocation, in which labor move from unproductive firms to productive firms; and inter-industry reallocation, in which labor move from comparatively disadvantaged industries to comparatively advantaged industries. Since we do not incorporate firm productivity heterogeneity (Melitz, 2003), inter-industry reallocation is the only channel available in our model, and it is the one we are concerned with here.
First, we review some empirical results to help justify the labor market assumption in our model.

Papageorgiou et al. (1990) analyze 19 episodes of liberalization in developing countries, and find very little relationship between trade liberalization and employment shift. Similarly, using a larger set of 25 liberalization episodes, Wacziarg and Wallack (2004) show that trade liberalization has weakly negative effects on intersectoral reallocation between agriculture and manufacturing. In addition Menezes-Filho and Muendler (2011) examine inter-industry labor reallocation for Brazil's trade liberalization in the 1990s. Individual workers jobs show that lowering tariff induces worker displacements, but the displaced labor is absorbed neither by exporters nor sectors with comparative advantage.

In summary, the literature shows that there is little or no trade-induced labor reallocation between aggregate industries (agriculture and manufacturing). We adopt this as a stylized fact and impose the following restriction:

**Assumption 1.** *Labor share for agriculture and manufacturing remains the same when trade share changes.*

The strategy in this paper is to let the model be consistent with this stylized labor reallocation restriction. A more complicated way to do this is to add labor market frictions in Section 3.2. However, explicit modeling of labor market friction would not change the results we need and would add to mathematical complexity.
Hence we simply impose the labor market restriction in order to focus on our main topic.

3.4.2 Relative Price

As shown in Proposition 13, in our $2 \times 2$ sector model, the relative price is very important to labor productivity growth. We document the effect of trade on the relative price in the following proposition.

Proposition 14. When manufacturing import variety share increases, relative agricultural price growth rate is positive; and the larger the import share of manufacturing, the higher the growth rate of relative agricultural price.

Proof. See Appendix B.

The importance of relative price is well supported by cross-country agricultural price data in Prasada Rao (1993). In the current literature, most empirical evidence about agricultural productivity is calculated from the data of year 1985\textsuperscript{24}. Figure 3.1 is borrowed from Page 68 of Prasada Rao (1993). It shows that in 1985, the agricultural price level is generally higher than the overall price level across all countries;

\textsuperscript{24}See the work by Caselli (2005), Restuccia et al. (2008), and Lagakos and Waugh (2013).
and poor countries have a higher price level difference between the general price level and the agricultural price level. That is food is relatively costly in poor countries.

Proposition 14 offers an explanation for Figure 3.1. Since poor countries generally produce low quality industrial products as in the work by Hausmann et al. (2007), and Mitchener and Yan (2014), they import more manufacturing varieties. In contrast, trade in agriculture is more restricted (Xu, Forthcoming). The result is that poor countries have a higher relative agricultural price than rich countries which is well documented by Figure 3.1.
3.4.3 The Emergence of Low Agricultural Productivity Growth

Based on the restriction of labor reallocation, and the impact of trade on relative price, growth rate difference between manufacturing and agriculture emerges. The following proposition states this result.

**Proposition 15.** As the import variety share in manufacturing increases, the productivity growth rate of manufacturing is larger than agriculture; and the larger the import varieties share in manufacturing, the greater the difference of the productivity growth rate between agricultural and manufacturing.

*Proof.* See Appendix C.

Proposition 15 shows that, as import variety share in manufacturing increases, the difference in growth rates between agriculture and manufacturing also increases. The model we discussed above provides a mechanism to explain this story. When import variety share in manufacturing is larger, the relative agricultural price is more likely to increase. This will lead to more labor allocated to final good production and less labor to new products creation. Since the creation of new products is the pure engine of economic growth, agricultural productivity growth rate will be lower.

Figure 3.2 shows the transition from the baseline state to the diverging productivity growth state. In the baseline state, relative price is zero, so the productivity growth rate in manufacturing and agriculture are the same. In the new state, agricul-
Figure 3.2: The Emergence of Productivity Growth Difference

natural productivity growth could be lower (dotted line in Figure 3.2) or higher (solid line in Figure 3.2) than before (though more likely higher). However, what matters is its rate relative to manufacturing: where it is always the same in the baseline state, and is dependent on the change of import variety in manufacturing when trade policy changes. In the figure, the distance of productivity growth rates between solid lines and dotted lines are the same in the new state. In summary, the diagram illustrates that the productivity growth rate difference between agriculture and manufacturing emerges from changing import shares.

Proposition 15 has an intuitive cross-country implication. In the very beginning, countries are not at the same income level, both manufacturing and agriculture. All countries are looking for different methods to improve real income. Obviously,
international trade is one of them. According to Proposition 13, introducing more trade in manufacturing will increase the productivity growth rate in manufacturing, but only slightly increase or even decrease the productivity growth rate in agriculture. When there is trade between a rich and a poor country, since they have different initial income levels, poor countries are more likely to have more import variety share (the ratio of import and domestic production) in manufacturing than in agriculture. Hence, we should see that poor countries have larger import variety share in manufacturing, but lower agricultural productivity which is the result of lower productivity growth rate in agriculture. Therefore, we have greater cross-country productivity differences in agriculture than non-agriculture.

3.5 Quantitative Analysis

The main result derived from the model is that countries with large manufacturing import share have lower agricultural productivity. In this section, we use cross-country data to test if this result matches data. Specifically, we calculate the import share parameters for agriculture and manufacturing.
3.5.1 Import Variety Measurement

According to the production type of the model, the first thing we need to do is to give an appropriate variety weight for each product. And by adding the varieties of each import products, we obtain all the gross import varieties. We then need to delete the varieties that this country is producing domestically as well. We do not know what a country is truly producing, but we do know what they are exporting. So we use export varieties to represent the variety set that is producing domestically. This is consistent with the model, in which country exports all varieties one can produce domestically. The measurement procedure is the following steps:

a) In this step, we calculate artificial productivity for each product. Let $I_k$ be the export indicator vector of country $k$, and $x_k$ is the export value vector. Total export from country $k$ can be written as

$$X_k = I_k x'_k.$$ 

For each product, we sum the value percentage from all the exporters weighting by GDP per capita. In this way, we can solve the artificial productivity for product $j$ as

$$\lambda_j = \frac{\sum_k x_{jk} Y_k}{\sum_k x_{jk} X_k},$$

where $\lambda_j$ is artificial productivity of product $j$, $Y_k$ is GDP per capita for country, and $x_{jk}$ is export value of product $j$. 
b) Given the artificial productivity for each product, we attribute a variety weight for each product. We choose the least productive product as unit variety, and give other product variety weight by

\[ n_j = \frac{\lambda_j}{\lambda_{\text{min}}} . \]

c) Let \( IM_{ik} \) be import indicator for industry \( i \), \( EK_{ik} \) be export indicator vector, and \( N_i \) be the variety weight vector for industry \( i \) consists of varieties calculated in the last step. Import variety share parameter can be calculated by

\[ \alpha_i = \frac{IM_{ik}N_i' - EX_{ik}N_i'}{EX_{ik}N_i'} . \]

When import variety is less than export variety, we take \( \alpha_i \) as zero.

3.5.2 Data

To fulfill the measurement of import variety, we use two datasets, Penn World Table 7.1 and Feenstra et al. (2005). We delete countries without GDP per capita data and varieties that are not consistent with Standard International Trade Classification, Rev. 4. Finally, we have 130 countries and 870 varieties left for calibrating artificial productivity for each product.

Eventually, we want to know which countries are importing more varieties. So we focus our exercise on the year of 1985, in which agricultural productivity data is
available from Prasada Rao (1993). We have 94 countries which are covered by all datasets and can be utilized in analysis.

3.5.3 Results

Figure 3.3: Import of Manufacturing and Agricultural Productivity

Figure 3.3 shows the relationship between manufacturing import variety percentage and agriculture productivity. Countries with high manufacturing import variety share tend to have lower labor productivity in agriculture; however, countries with low manufacturing import variety share tend to have higher labor productivity in manufacturing. The result is consistent with our model. In this exercise, we also
find that agriculture import variety share does not have significant relationship with agriculture productivity. One reasonable explanation is agricultural trade between countries are always under regulation due to social security consideration.

3.6 Discussion and Conclusion

This paper analyzes the role of international trade in explaining cross-country agriculture productivity differences and the productivity gap between agriculture and other sectors. It shows that, increased import variety share in manufacturing largely improves productivity growth in manufacturing, but only slightly increases, or even decreases, agricultural productivity growth.

Intuitively, the increase in competition from international trade drives greater innovation in the industrial sector, while the agricultural sector lags. When the poor economy is exposed to increased openness in international trade, it experiences entry of a large variety of foreign goods and services. Sectoral productivity growth is stimulated by competition from multiple competing varieties. The industrial-manufacturing sector is exposed to competition from a wider variety of foreign competitors than the rural-agricultural sector. An intuitive explanation is that industrial technology is relatively portable, whereas the technology in agriculture is relatively place-bound.

As productivity increases, real goods output increases in both sectors; but out-
put grows faster in the manufacturing sector. Wages and prices must adjust to allow real income to rise enough to purchase the increase in real output. This relationship can be viewed as either a rise in wages or a fall in the price of final goods or some equivalent combination. For convenience, assume that the final price of manufacturing is fixed - it was set as the numéraire. In this case, as productivity increases, wages increase. In the baseline case wages increase equally in both sectors and prices of final goods remain the same. When import variety share in manufacturing increases, wages in agriculture grow more slowly than wages in manufacturing. The relative price of the final goods in agriculture also rise more quickly than the price of final goods in manufacturing. (Alternatively, manufacturing goods prices fall if agricultural prices are constant.)

The results of these price movements reinforce the initial productivity differences. Workers in both industries buy final products from both industries. However, since the price of agricultural goods is rising relative to manufacturing goods, consumers shift the shares in their market basket towards the industrial sector. Notice that, since wages are increasing, the agricultural sector may increase, decrease or stay the same in absolute size (physical output), but in value terms the industrial sector grows larger relative to the agricultural sector.

In summary, a logical consequence of the faster increase in productivity in the manufacturing sector is an agricultural sector that has: lower wages, slower growing
productivity, and is relatively smaller than the manufacturing sector. These results reproduce the observed changes in the two sectors described in the introduction of the paper.

While our model superficially resembles traditional two sector economic growth models, there is a critical difference. In employing modern endogenous growth theory, our model features heterogeneity in both agricultural and industrial sectors instead of assuming homogeneous sectors as traditional growth models do. Increasing productivity is associated with the increasing availability of innovation and variety in the intermediate goods markets, in agriculture as in manufacturing. We posit that as economies enter a transitional phase, the rural-agricultural economy becomes more diversified, though not as diversified as manufacturing.

The mechanism of this paper can be applied to other contexts as long as there are multiple sectors. Our result can be interpreted as poor countries are producing low quality products as signified by lower variety. More importantly, due to many reasons, poor countries are more likely to import manufactured products rather than agricultural products. Finally, industrial technology is generally transferable, while much agricultural technology is place specific. The result is that poor countries have a larger import variety share in manufacturing and a lower productivity growth rate in agriculture.

These results imply several policy options. For instance, agricultural and trade
policies in terms of increasing agricultural productivity in poor countries could try to control the trade-induced relative agricultural price increase. Another policy approach would be to address labor mobility gaps between agriculture and other industries. Given that many poor countries have large urban unemployment problems, the actual problem may be one of "mobility" in labor quality rather than in physical location, but that is clearly a topic for another paper.

3.7 Appendices

3.7.1 Appendix A: Proof of Proposition 14

According to (3.14), since there is no labor market friction, the labor share between agriculture and manufacturing for each variety is,

\[ l_a(t)/l_m(t) = (\sigma_a - 1)/(\sigma_m - 1). \]  

(3.22)

Using (3.14)-(3.17), (3.19), and (3.22), we can solve the interest rate as

\[ r(t) = (1 + \alpha_j)^{\gamma_j}(1 + \alpha_{-j})^{1-\gamma_j}(N_j(t))^{\gamma_j}(N_{-j}(t))^{1-\gamma_j}l_j(t)\eta_j^{-1}(\sigma_j - 1)^{-1}. \]  

(3.23)

Since agriculture and manufacturing have the same interest rate, we have the following inter-industry variety set relationship

\[ [(1 + \alpha_a)N_a(t)]/[(1 + \alpha_m)N_m(t)] = (\eta_a/\eta_m)^{1/(\gamma_a-\gamma_m)}. \]  

(3.24)
We can obtain labor share of final good production for each industry from (3.23) and (3.24) as
\[ N_j(t)l_j(t) = br(t)/(1 + \alpha_j). \] (3.25)

Plugging (3.25) back into (3.16), we get labor share for creating new varieties as,
\[ l_{j,n} = b(r(t) - \rho - g_P)/(1 + \alpha_a). \] (3.26)

Interest rate on the balanced growth path is solved by using (3.19), (3.25), and (3.26) as
\[ r = \frac{1}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}L + \frac{1}{2}\rho + \frac{b/(1 + \alpha_a)}{2b/(1 + \alpha_a) + 2d/(1 + \alpha_m)}g_P. \] (3.27)

In each sector, final product is
\[ Y_j = (1 + \alpha_j)^{\sigma_j/(\sigma_j - 1)}N_j(t)^{1/(\sigma_j - 1)}(1 + \beta_j)^{-1}N_j(t)l_j(t). \] (3.28)

It implies that the relationship between final product and variety set is \( g_j = g_{N_j}/(\sigma_j - 1) \). Plug the interest rate back into (3.11) and (3.12), we can get sectoral growth rates.

3.7.2 Appendix B: Proof of Proposition 15

To examine the effect of trade, we change import parameters from initial level \((\alpha_j)\) to a new level \((\alpha'_j)\), and keep the labor share in manufacturing as constant, which
is $L_m = L'_m$. This gives us the relationship between interest rates in two periods as,

$$2r/(1 + \alpha_m) - \rho/(1 + \alpha_m) = 2r'//(1 + \alpha'_m) - \rho//(1 + \alpha'_m). \quad (3.29)$$

The relationship between relative prices in two cases can be calculated as

$$g'_P = \frac{(1 + \alpha'_m)b + (1 + \alpha'_a)d}{(1 + \alpha_m)b + (1 + \alpha_a)d} g_P - \frac{(1 + \alpha'_m)b + (1 + \alpha'_a)d}{b^2(1 + \alpha'_m)/(1 + \alpha'_a) + bd} \left[ \frac{(1 + \alpha'_m)b + (1 + \alpha'_a)d}{b^2(1 + \alpha_m)/(1 + \alpha_a) + bd} \right] L. \quad (3.30)$$

Since $g_P = 0$ is the baseline state relative price growth rate, the new relative price growth rate is

$$g'_P = - \left[ \frac{(1 + \alpha'_m)b + (1 + \alpha'_a)d}{b^2(1 + \alpha'_m)/(1 + \alpha'_a) + bd} - \frac{(1 + \alpha'_m)b + (1 + \alpha'_a)d}{b^2(1 + \alpha_m)/(1 + \alpha_a) + bd} \right] L. \quad (3.30)$$

Since $\alpha'_j > \alpha_j$, the above expression is positive. Take the first order derivative with respect to $\alpha'_m$ we get $\partial g'_P / \partial \alpha'_m > 0$.

### 3.7.3 Appendix C: Proof of Proposition 16

In the new state after import share changes, we have the following new growth rates similar to Proposition 14,

$$g'_a = \frac{1}{2b/(1 + \alpha'_a) + 2d/(1 + \alpha'_m)} L - \frac{1}{2} \rho - \frac{b/(1 + \alpha'_a) + 2d/(1 + \alpha'_m)}{2b/(1 + \alpha'_a) + 2d/(1 + \alpha'_m)} g_P; \quad (3.31)$$

$$g'_m = \frac{1}{2b/(1 + \alpha'_a) + 2d/(1 + \alpha'_m)} L - \frac{1}{2} \rho + \frac{b/(1 + \alpha'_a)}{2b/(1 + \alpha'_a) + 2d/(1 + \alpha'_m)} g_P. \quad (3.32)$$
The difference of labor productivity growth rate between agriculture and manufacturing is no longer zero as in the baseline state because \( g'_m - g'_a = g'_P \neq 0 \). Based on Proposition 12, we have \( g'_m - g'_a > g'_P \), and the first order derivative \( \partial(g'_m - g'_a) / \partial \alpha'_m \) is positive.
Bibliography


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