THERMO-MECHANICAL CONSTITUTIVE MODEL OF SHAPE MEMORY POLYMER – NUMERICAL MODELING, EXPERIMENTAL VALIDATION AND ITS APPLICATION TO AERO-MORPHING STRUCTURES

By

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To the Faculty of Washington State University:

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THERMO-MECHANICAL CONSTITUTIVE MODEL OF SHAPE MEMORY POLYMER –
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Abstract

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In this dissertation a one-dimensional thermo-mechanical constitutive model was
developed for thermally actuated shape memory polymers (SMPs) in which a single
variable, called a binding factor, was defined and used to simplify the model compared
to other existing models that typically require many variables. The binding factor is an
approximation that captures the behavior of the storage strain and considers the
polymer’s molecular architecture and morphology in the glass transition temperature
region. Next, the one-dimensional viscoelastic model developed was further expanded
into a three-dimensional model. The three-dimensional translation was initiated by
transforming the Young's modulus into bulk and shear modulus. Also, the compliance
and stiffness matrix for correlating elements were defined and applied into the model.
Developing a three-dimensional model allows for the prediction of the transverse and
shear direction of the material under different loading conditions. Finally, in order to
make use of the model in finite elements tools, the three-dimensional model was
rewritten into a numerical constitutive model. A numerical algorithm of this model was
developed using the UMAT subroutine capabilities of the finite element software
ABAQUS. Using experimental data available in literature, validation of the model was
conducted which shows that the model has good response to shape fixity, stress
relaxation and strain release.

A novel concept to apply SMPs as the primary material for flapping wing micro air
vehicle (FW-MAV) was proposed. Using SMPs will allow the vehicle to morph its wings,
a trait common in nature for flying creatures at that scale. Comprehensive analysis was
carried out using the new numerical constitutive model to investigate the possibility of
self-induced morphing of the SMP wing. The numerical simulation results demonstrated
that morphing of FW-MAV with the new numerical model is promising. The results
reported in this dissertation also provide credible confidence that the new model can be
used to advance development and simulation of morphing structures.
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Dedication

To my wife Funto, who supported me throughout this program
and our baby on the way who is coming at the right timing.
1 INTRODUCTION

Smart materials are defined as materials that can sense the environment and/or their own state make a judgment and then change their functions according to a predetermined purpose (Wang & Kang, 1998). Shape memory polymers (SMPs) are a type of smart materials. The conventional definition of an SMP is a polymer can be deformed and subsequently fixed into a temporary shape, which would remain stable unless it is exposed to an appropriate external stimulus that triggers the polymer to recover to its original (or permanent) shape (Tao, 2011). SMPs have the capacity to stored strain energy (Ogden, 1972) under appropriate stimulus and pre-deformation conditions. Heat has been the predominant external stimulus (Hu et al., 2012). This dissertation work focuses on thermal responsive SMP. There have been several researches done on thermal responsive SMP application and some have been discussed in the next section.

1.1 Shape memory polymers: fundamentals and mechanism

Amorphous SMPs are perhaps the most commonly marketed polymers with the capacity for shape memory effect currently. They include covalently cross-linked thermoset networks and physically cross-linked copolymers (Nguyen, 2013). Their Shape memorization as well as recovery occurs around their glass transition temperature ($T_g$) which is a region where exponential changes occurs within the macro-properties of the polymer due to micro phenomenon effect of polymer chains mobility. To enable shape memory effect in polymer a composition of netpoints and molecular switches that are sensitive to external stimulus (in this case temperature) are required
The netpoints determines the permanent shape of the polymers while the molecular switches are responsible for the temporary shapes. Netpoints can be achieved through chemical covalent bonds or physical intermolecular interactions. The netpoints domains have a higher melting temperature than the switching domain, thus shape memorization occurs around the $T_g$ of the switching domain.

Polyurethanes and polyether-ester are examples of physically cross-linked linear-block copolymers with shape memory effects. Physically cross-linked copolymers are not permanently cross-linked structure and can be released either by thermal or solvent processes. The use of polyurethanes as SMPs were first discovered by Mitsubishi Heavy Industry (MHI) in the 1980 (Jiang & Schmidt, 2010). Oligourethane forms the hard segments. As stated by Jiang and Schmidt, the hard segments are formed by the reaction of diisocyanate with either diols or diamines, which acts as the low molecular weight chain extenders to the urethane rich segment. Long chain polyether or polyester glocols forms the soft segments in polyurethanes (Behl & Lendlein, 2010).

From a purely mechanistic viewpoint, SMPs mechanism can be described as a co-joined polymer composed of two layers: hard and soft segment. As shown in Figure 1-1, the permanent shape of this composite polymer is realized at manufacturing in the laboratory and sets at a certain low temperature ($T_l$) with a tailored ratio of hard and soft segment. To activate the shape memory effect, the temperature of the polymer is raised beyond the $T_g$ to a high temperature ($T_h$), thus allowing for an increased mobility in the polymer chain and this is depicted by the increase ratio of the soft segment in the
figure in step 1. The shape to be memorized is applied at $T_h$ (step 2) and then cools down back to $T_i$ (step 3). At the cool down step the memorization occurs and the hard segment ratio increases beyond that of the soft segment. Thus by the end of step 3, shape memorization is achieved. To revert back to the permanent shape the temperature of the polymer is again raise to $T_h$ without any constraint (step 4) and then cooled back down (step 5).

**Figure 1-1** Mechanistic view of shape memorization mechanism
The material properties change dramatically around the transition glass temperature of the polymer due to the intermolecular interactions occurring within the polymer as the temperature changes. A typical Young’s modulus of SMP can vary on the average of 100 times higher at $T_l$ than $T_h$. Figure 1-2 shows a typical exponential temperature variation of the Young’s modulus around the transition temperature. As shown in the figure below, at $T_h$ and $T_l$ the Young Modulus remains relatively constant beyond these temperatures. Just as similar to any other materials, if the temperature gets closer to the melting point temperature, the strength of the material drops which directly affects the Young’s modulus.
Similarly, the stress and strain response due to temperature change are very unique around the glass transition region of SMPs. Figure 1-3 shows a typical stress-temperature and strain-temperature response around the transition temperature. To achieve shape memorization, load is applied to the polymer at high temperature thereby inducing some mechanical stresses as shown in step 1 of the Figure 1-3a. The deformation is held constant as the temperature drops (shown in Figure 1-3b step 2) which induces an increase level of stress on the polymer as shown in step 2 of the Figure. Finally at low temperature the constraint is removed which drops the residual
stresses to zero (step 3 in figure 1-3a) but maintains the memorized deformation as shown in step 3 of Figure 1-3b. In order to recover the permanent shape the material is heat back up and the strain level drops to zero as shown in step 4 of Figure 1-3b.

**Figure 1-3** Stress and strain response for SMPs during shape memorization and recovery

---

1.2 **Notable shape memory polymer research and application**

The earliest application of SMPs was done by Vernon, (Vernon & Vernon, 1941) which was a patent for a polymer material with elastic memory effect in early 1940s. It took another 20 years before an application of the material is put into effect through commercial heat shrinkable tubing polymers (Perrone, 1967; Mather et al., 2009). CDF Chimie Company in France was the first official use the term shape memory polymer in
the development of their polynorbornene base SMP in the 1980s (Sakurai & Takahashi, 1989). An extensive mechanisms, history, and applications of SMPs have been described in review papers, (Miyazaki et al., 1998; Liu et al., 2007; Behl & Lendlein, 2007; Ratna & Karger-Kocsis, 2008; Mather et al., 2009; Tao, 2011 and Hu et al., 2012). Clark et al. (2010) showed that SMPs possess several distinct advantages when compared to other shape memory materials such as shape memory alloys. Some of these advantages include withstanding higher degrees of elastic deformation and tailored activation temperature (Xie & Rousseau, 2009). A notable practical application of SMP by Cornerstone Research Group Inc. is the Veriflex® reusable mandrel (smart mandrel) (Cornerstone Research Group, 2004) as shown in Figure 1-4. The mandrel can be molded into any desired shape by taking advantage of the shape memorization capabilities of SMP. Lendlein & Langer (2012) showed another practical application as a self-tightening smart surgical suture. The concept of the smart surgical suture is that at low temperatures the sutures are applied to a wound and as the body rises back to its normal temperature the suture self-tightens. Also, it is possible to have these sutures as biodegradable.

SMPs are great candidates for morphing structures. They have been applied recently to aircraft skin (McKnight & Henry, 2005; Rauscher, 2008 and Balogun et al., 2013). Also it was applied to smart joint in a morphing wing (Manzo & Garcia, 2008; Lan et al., 2009; Boyerinas, 2009 and Clark et al., 2010). Finding ways to apply SMP to today’s aerospace technology can lead to a new set of performance improvement for the aerospace industry. Table 1-1 shows a summary of selected practical application of SMPs in journal article.
Figure 1-4 Smart mandrel using SMP
Reproduced with permission from Cornerstone Research Group Inc
Table 1-1 Selected practical application of SMPs

<table>
<thead>
<tr>
<th>Application</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape memory foam for robotic gripping</td>
<td>(Brennan, 2001)</td>
</tr>
<tr>
<td>Self-deployable Gossamer spacecraft structure</td>
<td>(Sokolowski, 2004)</td>
</tr>
<tr>
<td>Reusable Mandrel</td>
<td>(Everhart et al., 2006)</td>
</tr>
<tr>
<td>Vascular Stent</td>
<td>(Baer et al., 2007)</td>
</tr>
<tr>
<td>Autochoke Material for engines</td>
<td>(Tobushi et al., 2008)</td>
</tr>
<tr>
<td>Snap-fits for active disassembly</td>
<td>(Carrell et al., 2011)</td>
</tr>
<tr>
<td>Self-tightening smart surgical suture</td>
<td>(Lendlein &amp; Langer, 2012)</td>
</tr>
<tr>
<td>Self-healing materials</td>
<td>(Li &amp; Shojaei, 2012)</td>
</tr>
<tr>
<td>Quick response label technology</td>
<td>(Fritzsche &amp; Pretsch, 2014)</td>
</tr>
<tr>
<td>Transfer Printing surface relief structure</td>
<td>(Xue et al., 2015)</td>
</tr>
</tbody>
</table>

1.3 SMP constitutive modeling review

Thermally activated SMPs are by far the most widely researched shape memory effect polymer. Constitutive modeling of the thermo-mechanical properties of SMPs is important in order to use it to develop complex engineering structures. Modeling allows for the prediction of SMP behavior while under different kind of interaction with its environment. Predictions can include stress and strain behavior with or without temperature change, shape-memory effect behavior with or without constraints and other behavior. A number of review articles work on constitutive modeling of SMPs have been done by Hu et al. (2012), Zhang & Yang (2011), Nguyen (2013) and Zhao et al. (2015). This section reviews selected notable constitutive models developed in recent years for amorphous SMPs in order to provide readers with a comprehensive view of the state of modeling.
Molecular interaction effect within SMPs can be relatively assumed to be minimal when a macro level interaction such as thermodynamic properties (entropy and internal energy) are considered for modeling. This kind of modeling approach was adopted by Tobushi et al. (1997, 2001) on their polyurethane SMP, where a rheological model was presented based on the standard linear viscoelastic model as shown in equation (1-1) through (1-3). Their model was derived by modifying the single linear viscoelastic (SLV) model by introducing a slip element due to internal friction and described as a creep irrecovery strain \( \varepsilon_s \). This was observed in their earlier experimental work (Tobushi et al., 1996). Thermal expansions consideration was assumed to be independent of the mechanical behavior of the model.

\[
\begin{align*}
\dot{\varepsilon} &= \frac{\sigma}{E} + \frac{\varepsilon - \varepsilon_s}{\mu} + \frac{\alpha \dot{T}}{\lambda} \\
\varepsilon_s &= C(\varepsilon_c - \varepsilon_l) \\
E &= E_s \exp \left[ a_E \left( \frac{T_g}{T} - 1 \right) \right]
\end{align*}
\]

where \( \sigma, \varepsilon, \mu, \lambda \) and \( \alpha \) represent stress, strain, viscosity, retardation time and the coefficient of thermal expansion respectively. Also \( E, T \) and \( C \) are Young’s modulus, temperature and proportionality coefficient. The quantities \( \varepsilon_c, \varepsilon_l \) and \( T_g \) represent the creep strain, critical creep strain and glass temperature of the material. \( C \) and \( \varepsilon_l \) shown in equation (1-2) are temperature dependent. Equation (1-3) shows how the material properties relate with temperature where \( a_E \) is derived empirically as well as other material properties exponential constants. The shape memory effect behavior predicted
from the model shows promising results when compared to experimental data. The strain recovery process was not well represented by the model. In addition, this model requires exponential empirical quantities related to the Young’s modulus, retardation time and viscosity that will be rigorous to determine for engineering applications. Similar models using Thermoviscoelastic methodology have been developed by Morshedian et al. (2005), Diani et al. (2006), Buckley et al. (2007), Heuche et al. (2010), Wong et al. (2011), Westbrook et al. (2011), and Balogun & Mo (2014).

Qi et al. (2008) implemented a phase transition modeling approach that divides the polymer into a rubbery, initial glassy and frozen phase. This three-dimensional finite model assumes a parallel deformation for the rubbery and glassy phase. The frozen phase is formed from the rubbery phase as the polymer cools down. The volume fraction of the individual phases is calculated using the Vogel-Tammann-Fulcher function (Vogel, 1921; Fulcher, 1925 and Tammann & Hesse, 1926) as shown in equation below.

\[
f_{r/g} = \frac{1}{1+\exp\left[-\left(T-T_r\right)/A\right]} \tag{1-4}
\]

where the subscripts r/g refers to the rubbery or glassy phase, A is a parameter characterized by the width of the phase transition zone, and \( T_r \) is a reference temperature close to the glass temperature. It was assumed that while cooling the glassy phase is generated by directly converting the rubbery phase in frozen state (ie \( \Delta f_r = \Delta f_{rg} \)). In the heating process, the initial and frozen glassy phases are converted to rubbery phase. The incremental volume fraction was provided as
$$\Delta f_{x0}^g = \frac{f_{x0}}{f_{x0} + f_T} \Delta f_x, \quad \Delta f_{y0}^g = \frac{f_T}{f_{x0} + f_T} \Delta f_g$$ (1-5)

The deformation gradient of the frozen glassy ($F_T$) at the next time step ($n+1$) was given as

$$F_T^{n+1} = \Delta F_T^{n+1} F_T^n$$ (1-6)

and the incremental deformation due to temperature change was given as

$$\Delta F_T^{n+1} = \begin{cases} \Delta F_T^{n+1} (F^n)^{-1} & \text{if } \Delta T \neq 0 \\ I & \text{if } \Delta T = 0 \end{cases}$$ (1-7)

Finally, the rule of mixture was used to calculate the stress response of the polymer for all the phases and given as:

$$\sigma = f_r \sigma_r + f_g \sigma_g + f_T \sigma_T$$ (1-8)

where $\sigma$ is the Cauchy stress tensor. Shape memory effect behavior where captured well by the model as shown by the stretch-temperature experimental results. The stress recovery response was not well represented by the model as the model predicts zero reaction force at higher temperature than observed in experiment. Others have implemented similar concept of phase transition model (Rajagopal & Wineman, 1992; Liu et al., 2006; Chen & Lagoudas, 2008a & 2008b; Kafka, 2008; Long et al., 2010; Xu & Li, 2010; Volk et al., 2011 and Gilormini & Diani, 2012).
1.4 Polymer usage in aerospace

There is a continuous increase of application of polymers in the aerospace industries. In the commercial aerospace industry, application of polymer based composite material has been predominately used since the year 2000. For example, Boeing’s 787 commercial jet is composed of 50% polymer based composite material (Boeing, 2008), see Figure 1-5. These polymer based composite materials are now being used as structural materials which is replacing the conventional aluminum materials used in the past. Similarly, the proposed commercial jet 777X by Boeing has a wing that is predominantly made of polymer based composites (Boeing, 2013), see Figure 1-6. The reason for these changes is based on the performance improvement the jets achieve because of the new material. Some performance improvements include:

i) Weight improvement: The polymers based composite structures are lighter than the conventional aluminum material. Thus providing an overall reduction in weight of the jet

ii) Aerodynamic improvement: Since the polymer based composite can be tailored from bottom-up, certain aerodynamic responsive shapes can be attained for the wings and other components in comparison with conventional aluminum sheets.

An understanding of why polymers based composite is a favorable choice of material for commercial jets application leads to the motivation of improving SMP modeling. Replacing the current polymer based used in commercial jets with smart
polymers like SMP will lead to the possibility of morphing that will ultimately lead to performance improvement of the aircraft.

**Figure 1-5** Boeing's 787 material usage chart.
Reproduced with permission from Boeing
1.5 Goal and structure of the dissertation

This dissertation explores the development of a simple but yet practical thermo-mechanical constitutive model that captures the deformation and recovery of SMPs when used in complex engineering structures. The use of thermally activated SMPs as key enabling materials to provide both a rigid state for structural integrity and a soft state that allows morphing of the structure's shape is a viable option. Morphing materials, such as SMPs, are not only able to significantly change their properties such as stiffness, on command, but they are also able to remember and return to a previous shape.
The goals of this dissertation are:

(1) Develop a thermo-mechanical constitutive model for SMPs that can be adopted by engineering industry to design complex morphing structures.

(2) Investigate the application of thermally activated SMPs to Flapping Wing Micro Air Vehicle (FW-MAV) using numerical methods via finite element (FE) analysis in order to achieve morphing.

Significance of a new thermo-mechanical constitutive model

Thermo-mechanical constitutive models for SMPs have been established in many different ways; however, they nearly all require multiple complex parameters (Tobushi et al., 1997; Liu et al., 2006; Chen & Lagoudas, 2008a; Srivastava et al., 2010; Baghani et al., 2011; Ge et al., 2012 and Ghosh & Srinivasa, 2013). In this dissertation work, a new model based on a free energy concept and a superposition principle is established. Here, a novel single variable, called a binding factor is defined and used to simplify the constitutive model compared to other existing models that typically require many variables (Balogun & Mo, 2014a & 2014b). The binding factor is an approximation that captures the behavior of the storage strain and considers the polymer’s molecular architecture and morphology in the glass transition temperature region. The new constitutive model together with the binding factor will have to be validated with experimental data available in the literature.
**Significance of using SMP for FW-MAV morphing**

In general, SMP will provide both structural and morphing capabilities for the FW-MAV wing. We are not aware of anyone else proposing using SMP for morphing at the micro scale proposed in this work. Morphing wing technologies have typically been applied to macro scale based system (Perry, 1966; Flanagan et al., 2007; Andersen et al., 2007 and Bubert et al., 2010). The range of the wingspan for the vehicle of focus is on the order of magnitude of 1 – 5 inches (25.4 – 127 mm). This is the size of insects like the housefly or large locust. At this scale, flapping wing becomes essential to generate lift. It is desirable to achieve three modes of configuration for the wing through morphing and these modes include: stowage, flap and glide. The stowage mode is the configuration when the vehicle is not in flight. Stowage wing is significant because it will help prevent damage of the wing while not in flight. It also acts a convenient means of storage when not in flight. Lift is generated from flapping the wings. Typically, this will be the primary configuration of the FW-MAVs while in flight. Flapping the wings at the natural frequency of the MAV will improve performance. At the right flap mode setting, there will be an ability to generate quick lift off and complex flight maneuvers. In nature insects can undergo passive deformation in order to adjust to the wind streamlines while in flight. Due to the scale size, insects operate in low Reynolds number (Abate & Shyy, 2012), thus they see huge impact on the flight performance when there is a change in the wind streamlines causing drag or draft. By adjusting to glide mode, FW-MAVs’ performance can improve and energy can be save from glide through the air. This dissertation proposes the use of SMPs to achieve the morphing of the surface area of
the wing in order to realize these three modes of configuration. The newly developed model will be used to study the behavior of morphing for FW-MAV using SMPs.

**Structure of the dissertation**

Section 2 starts out by introducing the constitutive model developed for this dissertation. The one-dimensional model is developed and then expanded into a three-dimensional form. Lastly, the time discrete methodology is presented in order to transition the three-dimensional model to a numerical based model. Section 3 presents some validation of the model through experimental data available in literature. In section 4 an investigation on application of the model in predicting the behavior of morphing for FW-MAV is conducted. Finally, in section 5 a summary of major contributions of this dissertation and directions for future work are discussed.
2 CONSTITUTIVE MODEL

2.1 Introduction

In this section a constitutive model is developed for an amorphous SMP with a transition temperature at $T_g$. To develop the constitutive model, fundamental laws of thermodynamics and continuum mechanics have been employed. As a prerequisite for the constitutive model the thermal energy equation and the Clausius-Duhem equation have been briefly reviewed which are derived from the first and second law of thermodynamics (Mase & Mase, 1999; Khan & Huang, 1995 and Wu, 2005). In order to study these equations, a material body in motion experiencing body and surface forces as well as heat load is considered as shown in Figure 2-1. The surface traction forces $(t)$ have an outward normal $(n)$ and accounts for the stresses on the body. Similarly, there is a body force $(b)$ that allows for spatial deformation and motion of the body. Since the body is undergoing a thermal mechanical loading, the thermal domain is represented by the surface heat flux $(q)$ and the body heat supply $(r)$. These quantities are used to define the presented equation in the sections that follows.
2.1.1 Thermal energy equation

The first law of thermodynamics is an expression of conservation of energy principle applied to systems involved in production and absorption of heat (Bejan, 2006). The conservation of energy principle states that during an interaction, energy can change from one form to another but the total amount of energy remains constant (Cengel & Boles, 2004). For a continuum body under thermal influence as shown in the law of conservation of energy is defined as the material time derivative of the kinetic ($\dot{K}$) plus
internal energies \( (\dot{U}) \) is equal to the sum of the mechanical power \( (P) \) forces and the thermal energy \( (Q) \). This equation is provided below as

\[
\dot{K} + \dot{U} = P + Q
\]  

(2-1)

The mechanical power is provided as the external body and surfaces forces acting on the materials as shown in Figure 2-1. Thus it is defined as

\[
P = \int_s (\hat{\mathbf{n}} \cdot \mathbf{v}) dS + \int_v \rho \mathbf{v} dV
\]  

(2-2)

Likewise the thermal energy is given as

\[
Q = \int_s q (\hat{\mathbf{n}}) dS + \int_v \rho r dV
\]  

(2-3)

To derive equation (2-1), the material derivative of the kinetic energy is given as

\[
\dot{K} = \frac{1}{2} \int_v \rho (\mathbf{v} \cdot \mathbf{v}) dV = \int_v \rho \mathbf{v} \mathbf{v} dV
\]  

(2-4)

Where

\( \rho = \) density of the body

\( \mathbf{v} = \) velocity field of the body

Equation (2-4) can be transformed by applying the equation of motion defined below:
\[ \nabla \cdot \sigma + \rho b = \rho \dot{v} \quad (2-5) \]

The del operator \((\nabla)\) is a partial derivative with respect to spatial variables and it is defined as

\[ \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \quad (2-6) \]

\(\sigma\) is the internal stresses induced on the body. Thus equation (2-4) becomes

\[ \dot{K} = \int_V v(\nabla \cdot \sigma + \rho b) \, dV \quad (2-7) \]

Applying chain rule the first term of the equation above is given as

\[ v \nabla \cdot \sigma = \nabla \cdot (v \sigma) - (\nabla \cdot v) \sigma \quad (2-8) \]

Thus equation (2-7) becomes

\[ \dot{K} = \int_V \left[ \rho b v + \nabla \cdot (v \sigma) - (\nabla \cdot v) \sigma \right] \, dV \quad (2-9) \]

The divergence theorem can be applied to the second term in equation (2-9) in order to transform it into a surface integral thus it becomes

\[ \dot{K} = \int_V \left[ \rho b v - (\nabla \cdot v) \sigma \right] \, dV + \int_{S}^{(n)} v \, dS \quad (2-10) \]

In equation (2-10), the second term on the right side is referred to as the stress power and can be equivalent to the rate of change of internal energy \((\dot{U})\) in special situation (Mase & Mase, 1999). Therefore it can be written as
(\nabla \cdot \nu)\sigma = \rho u \quad (2-11)

Finally equation (2-10) can be re-written as

\[ \dot{K} = \int_V \left[ \rho_b v - \rho u \right] dV + \int_S (t^{(n)} v) dS \quad (2-12) \]

Rearranging equation (2-12), it can be seen the internal and kinetic energy can be place one side and the mechanical power defined in equation (2-2) on the other, thus it becomes

\[ \dot{K} + \dot{U} = P \quad (2-13) \]

The thermal energy equation is derived by adding on the thermal energy to the right side of the equation above in order to obey the principle of conservation of energy and this equation is given by equation (2-1). Expanding out equation (2-1) in detail gives the following

\[ \frac{d}{dt} \int_V \rho \left( \frac{1}{2} v v + u \right) dV = \int_V \rho (bv + r) dV + \int_S \left( t^{(n)} v - q n \right) dS \quad (2-14) \]

Again applying divergence theorem to the surface integral results into

\[ \int_S \left( t^{(n)} v - q n \right) dS = \int_V \left( \nabla \cdot (v \sigma) - \nabla \cdot q \right) dV \quad (2-15) \]

Similarly applying the equation of motion to the two equations above and moving it to the left hand side provides readily seen results as
\[ \int_V (\rho u - \sigma : D - \rho r + \nabla \cdot q) \, dV = 0 \]  

(2-16)

Where

\[ D = \nabla \cdot \nu \]  

(2-17)

\( D \) is known as the strain rate term of the material (Baghani et al., 2011). The above equation can be transitioned to a local form for an arbitrary volume \( V \) which gives the reduced form of the thermal energy equation as

\[ \rho u - \sigma : D - \rho r + \nabla \cdot q = 0 \]  

(2-18)

2.1.2 Clausius-Duhem equation

The first law of thermodynamics presented in the previous section above provides a means to heat and work interchangeably without restriction. On the other hand, the second law provides restriction in the flow of heat and energy. Clausius second law states that no device can transfer heat from a cooler body to a warmer body without leaving an effect on the surroundings (Cengel & Boles, 2004). In continuum mechanics, the second law is used to restrict continua by using constitutive responses based on material response functions. The second law can be defined as the time rate-of-change in the entropy (\( \eta \)) equals the change in entropy due to heat supply (\( r \)), heat flux (\( q \)) entering the portion, plus the internal entropy production (\( \varsigma \)) (Mase & Mase, 1999). For an arbitrary volume of a body, it is given as
\[ \frac{d}{dt} \int_{V} \rho \eta \, dV = \int_{V} \rho \frac{r}{T} \, dV - \int_{\partial V} \frac{q_n}{T} \, dS + \int_{V} \rho \zeta \, dV \]  

(2-19)

Where \( T \) is the absolute temperature of the body.

Entropy generation is always zero or positive thus the third term of the right hand side is given as

\[ \int_{V} \rho \zeta \, dV \geq 0 \]  

(2-20)

Therefore the second law can be reduced to a form of Clausius-Duhem inequalities based on entropy generation restriction as follows

\[ \frac{d}{dt} \int_{V} \rho \eta \, dV \geq \int_{V} \rho \frac{r}{T} \, dV - \int_{\partial V} \frac{q_n}{T} \, dS \]  

(2-21)

The divergence theorem can be applied to the flux term in order to transition it from a surface to a volume integral of the terms which results into

\[ \frac{d}{dt} \int_{V} \rho \eta \, dV \geq \int_{V} \rho \frac{r}{T} \, dV - \int_{V} \nabla \cdot \left( \frac{q}{T} \right) \, dV \]  

(2-22)

The terms can be rearranged on one side and also this global form can be easily transitioned to a local form for any arbitrary volume of the body becoming

\[ \dot{\rho \eta} - \rho \frac{r}{T} + \nabla \cdot \left( \frac{q}{T} \right) = \dot{\rho \eta} - \rho \frac{r}{T} + \nabla \cdot q - \frac{1}{T^2} q (\nabla \cdot T) \geq 0 \]  

(2-23)

Thus the local form of the Clausius-Duhem equation becomes
\[ \rho T \dot{\eta} - \rho r + \nabla \cdot q - \frac{1}{T} q(\nabla \cdot T) \geq 0 \]  

(2-24)

Combining the results of equation (2-18) into the equation above gives

\[ \rho T \dot{\eta} - \rho u + \sigma : D - \frac{1}{T} q(\nabla \cdot T) \geq 0 \]  

(2-25)

The Helmotz free energy (\( \psi \)) is typically used to define another form of the Clausius-Duhem equation. The free energy is defined as

\[ \psi = u - \eta T \]  

(2-26)

Applying this definition to equation (2-25) results in

\[ \dot{\psi} + \eta \dot{T} - \sigma : D - \frac{1}{T} q(\nabla \cdot T) \geq 0 \]  

(2-27)

The above equation can be decomposed into two parts as

\[ \text{Mechanical dissipation} = \dot{\psi} + \eta \dot{T} - \sigma : D \]  

(2-28)

\[ \text{Thermal dissipation} = -\frac{1}{T} q(\nabla \cdot T) \]
2.2 One-dimensional constitutive model

Proposing a constitutive model for the SMP requires the inspection of the typical stress and strain behavior of the SMP during the process of memorizing a shape and releasing the stored strain. This behavior depicted in Figure 2-2, shows that in order to memorize a shape an initial load is applied to the polymer at high constant temperature, and then held at constant strain while it is cooled down. As the polymer cools down in the second step, a residual stress builds up within it. This residual stress has been interpreted as the SMP storing up energy even as the stiffness of the polymer increases. In the third step, the constraint is removed from the SMP at low constant temperature. Based on the new boundary condition of the polymer the residual stress drops to zero while remains about constant as shown in the figure. Finally in the fourth step the stored strain is released when heated up. The work presented in this section follows these four steps to determine the kinematics and energy balance of the SMP. The four steps considered were:

1) Pre-loading the SMP
2) Constrained shape memorization
3) Constraint release
4) Unconstrained storage release
2.2.1 Kinematic consideration

The total strain seen by the SMP was broken in three parts within this model. The total strain ($\varepsilon_T$) is composed of the mechanical strain ($\varepsilon_{me}$), thermal strain ($\varepsilon_{th}$) and storage strain ($\varepsilon_s$). Thus the total strain is expressed as

$$\varepsilon_T = \varepsilon_{me} + \varepsilon_{th} + \varepsilon_s$$  \hspace{1cm} (2-29)

Furthermore, to account for viscous flow of the SMP, the mechanical strain was decomposed to an elastic part ($\varepsilon$) and a viscous part ($q$). Thus the mechanical strain is expressed as

$$\varepsilon_{me} = \varepsilon + q$$  \hspace{1cm} (2-30)
Replacing mechanical strain from equation (2-30) into equation (2-29), the total strain can now be rewritten as

\[ \varepsilon_T = \varepsilon + \varepsilon_{th} + \varepsilon_s + q \] (2-31)

The boundary condition kinematics of the four steps describe early are discussed in the following equations below.

1) *Pre-load the SMP:* At a certain high temperature \( T_h \), it is assumed that the total strain in the polymer is zero. A mechanical load is then applied to the SMP inducing the mechanical strain. For this model, it is assumed that any thermal strain contribution at this step is negligible, thus the total strain is given as

\[ \varepsilon_T = \varepsilon_{me} = \varepsilon + q \] (2-32)

2) *Constrained shape memorization:* It is assumed that the mechanical strain remains constant as the temperature of the SMP is being reduced. Thus it is concluded that the mechanical strain rate is zero i.e.

\[ \dot{\varepsilon}_{me} = 0 \] (2-33)

3) *Constraint release:* At low temperature \( T_l \) the storage strain is defined as a function of the initial mechanical strain induced on the SMP. A binding factor \( z \) was also used to approximately describe the behavior of the storage strain as a function of temperature. The binding factor is discussed in details in later section. The storage strain is given as:
\[ \varepsilon_s = zk_m \varepsilon_{me} ; \quad k_m = \begin{cases} +1 & \dot{T} < 0 \\ -1 & \dot{T} > 0 \\ 0 & \dot{T} = 0 \end{cases} \quad (2-34) \]

4) **Unconstrained storage release**: In this last step, no mechanical loading is on the SMP and it is allowed to recover its permanent shape by thermal stimulus. Thus the total stain is given as

\[ \varepsilon_T = \varepsilon_{th} + \varepsilon_s \quad (2-35) \]

### 2.2.2 Free energy consideration

In this section, the Helmholtz free energy is defined for the different steps of strain storage and release of the SMP. As shown in section 2.1, the internal energy is embedded in the thermal energy balance and it is related to the Helmholtz free energy. Understanding the free energy at different phases of the polymer's shape memorization and release will provide the opportunity to create an energy relation constitutive model.

1) **Pre-loading of the SMP**: At high constant temperature \((T_h)\), only the mechanical load is in effect, thus the free energy is given as

\[ \psi = \psi_1 (\varepsilon, q) \quad (2-36) \]

2) **Constrained shape memorization**: In this step, the temperature is changing and a constraint is applied to keep the strain constant, thus all kinematic terms are activated. The free energy dependency is expressed as

\[ \psi = \psi_2 (\varepsilon, q, z, T) \quad (2-37) \]
3) **Constraint release**: In this step, the assumption is that the storage of strain as occurred and it is largely dependent on the free energy in steps 1 and 2. A superposition of the first and second step free energy is proposed as the storage energy. Thus, the free energy is defined as:

\[
\psi = \psi_3 = \psi_1 + \psi_2
\]  

(2-38)

4) **Unconstrained storage release**: As the temperature increases, the stored energy is released. Thus, the free energy is defined as:

\[
\psi = \psi_4(\varepsilon, q, z, T)
\]  

(2-39)

Also, it is noted that during the storage release, the stored free energy in step 3 and the release free energy in step 4 have the relationship:

\[
\psi_3 \geq \psi_4
\]  

(2-40)

### 2.2.3 Mechanical dissipation consideration

The second law of thermodynamics can be written in the form of the Clausius-Duhem equation as shown in section 2.1.2. The local mechanical dissipation in the Clausius-Duhem form can be written as

\[
\dot{\psi} + \eta T - \sigma \varepsilon \geq 0
\]  

(2-41)

The rate of change of the free energy can also be written as

\[
\dot{\psi}(\varepsilon, q, z, T) = \frac{\partial \psi}{\partial \varepsilon} \dot{\varepsilon} + \frac{\partial \psi}{\partial q} \dot{q} + \frac{\partial \psi}{\partial z} \dot{z} + \frac{\partial \psi}{\partial T} \dot{T}
\]  

(2-42)
The binding factor shown in later section is a function of temperature and is employed in the definition of the rate of change of the free energy. Applying equations (2-36), (2-37), (2-38) and (2-42) into equation (2-41) results in

\[
\begin{bmatrix}
\frac{\partial \psi_1}{\partial \varepsilon} + \frac{\partial \psi_2}{\partial \varepsilon} - \sigma \\
\frac{\partial \psi_1}{\partial q} + \frac{\partial \psi_2}{\partial q}
\end{bmatrix} \varepsilon + \left[ \frac{\partial \psi_2}{\partial T} + \frac{\partial \psi_2}{\partial z} \right] q + \left[ \frac{\partial \psi_2}{\partial T} + \frac{\partial \psi_2}{\partial z} \frac{\partial z}{\partial T} + \eta \right] T \geq 0
\] (2-43)

The inequality above in equation (2-43) has to be satisfied for any thermodynamic process. In agreement to the local state law (Lemaitre & Chaboche, 1990), the Clausius-Duhem inequality leads to the following state equations:

\[
\sigma = \frac{\partial \psi_1}{\partial \varepsilon} + \frac{\partial \psi_2}{\partial \varepsilon}
\] (2-44)

\[
\eta = - \left[ \frac{\partial \psi_2}{\partial T} + \frac{\partial \psi_2}{\partial z} \frac{\partial z}{\partial T} \right]
\] (2-45)

\[
\left[ \frac{\partial \psi_1}{\partial q} + \frac{\partial \psi_2}{\partial q} \right] q = A
\] (2-46)

where \( A \) is identified as the viscous entity or dissipative potential which defines uniquely the thermodynamic state of the irreversible properties of the polymer (Valanis, 1972). An example of \( A \) is shown in later section.
2.2.4 Strain storage release

The strain storage aspect was covered in the mechanical dissipation consideration section above. In order to resolve the storage release, an energy and mechanical dissipation analysis is conducted based on the boundary conditions of the polymer. From the boundary condition, there are no mechanical loads on the SMP thus the energy balance becomes:

\[ K + \dot{U} = Q \]  \hspace{1cm} (2-47)

The mechanical power has been set to zero in the energy balance equation. For any arbitrary volume the local field equation can be written as:

\[ \rho u + va = \rho r - \nabla \cdot q \]  \hspace{1cm} (2-48)

The quantity \( va \) is a term that comes out the rate of change of kinetic energy (Mase & Mase, 1999) and it was referred as the “jerk” of the material. Comparing equation (2-48) to equation (2-18), it can be seen that the jerk term has replaced the stress term. This is primarily due to the fact that the mechanical load boundary conditions have been removed from the polymer. For simplification of the mathematics, the jerk term is defined as a linear function of strain rate and given as:

\[ va = \zeta \dot{\varepsilon} \varepsilon \]  \hspace{1cm} (2-49)

where \( \zeta \) is a constant of proportionality related to the retardation time of the polymer. Finally following similar step shown in the mechanical dissipation consideration section, and replacing the stress term in equation (2-43) with equation (2-49), the storage strain release becomes
\[ \varepsilon = \frac{1}{\zeta} \frac{\partial \psi_4}{\partial \varepsilon} \] (2-50)

2.2.5 Binding factor

A binding factor \( z \) was introduced into the model to account for the polymer's molecular architecture and morphology during shape memory behavior. It is an approximation to capture the behavior of the storage strain in the glass transition temperature \( T_g \) region. The binding factor is expressed by the following equation:

\[ z = 1 - \left(1 + e^{a(T_g - 0.5\Delta T_h - T)}\right)^{-1}; \quad a = \ln \left(\frac{\mu_h}{\mu_l}\right)^{-1} \] (2-51)

where

\[ \mu_h = \text{viscosity at high temperature} \]
\[ \mu_l = \text{viscosity at low temperature} \]
\[ T_g = \text{glass temperature} \]
\[ \Delta T_{hl} = \text{temperature difference between high and low temperatures} \]

The high and low temperatures referenced in equation (2-51) are the boundary temperatures of the SMP near the glass transition region. Beyond this temperature range the material properties remain relatively unchanged. It should be mentioned that if the temperature is relatively high and approaches the melting temperature of the polymer then it loses its stiffness and material strength. These temperatures are tailored depending on material properties that can be varied by changing the chemistry of the SMP. The viscosities at these temperatures were used to define an exponential parameter, \( a \). This exponential parameter accounts for the molecular architecture of
the polymer through the use of viscosity. The viscosity of the polymer is a direct response of the interactions of polymeric linkages (co-polymer blocks) and the behavior changes as temperature changes. Having a well-defined exponential parameter gives an opportunity for the model to be adopted in industry.

### 2.2.6 Temperature dependence of material property using binding factor

The binding factor described above is used to model the change in material properties due to temperature change. For this model, the assumption is that as the temperature of the SMP drops as the elastic modulus, viscosity and retardation time increase and vice-versa. This assumption is typical to most SMP material (Hu et al., 2012). The relationship between the material's properties as functions of the binding factor is given as:

\[
E(T) = (1 - z)E_h + zE_l
\]

\[
\mu(T) = (1 - z)\mu_h + z\mu_l
\]

\[
\lambda(T) = (1 - z)\lambda_h + z\lambda_l
\]

(2-52)

where \(E\) is the Young’s modulus, \(\mu\) is the viscosity and \(\lambda\) is retardation time. The subscripts \(h\) and \(l\) denote the corresponding property at the boundary temperatures near the glass transition region where the properties remain relatively unchanged.

Since the binding factor is a function of temperature, thus the material properties inherently become functions of temperature. Applying the binding factor in this manner, only one factor is required in order to determine the behavior of the polymer’s properties as it transitions over temperature.
Figure 2-3 below compares the material property model using the binding factor to three different experimental data sets reported by Tobushi et al. (1997), Liu et al. (2006), and Qi et al. (2008). **Table 2-1** shows the material parameters used for generating these plots. For Liu and Qi’s models the elastic moduli are used in place of the viscosity since their values were not reported in their work. The figure shows that the model predicts the experimental values well following the trends of high and low modulus at low and high temperatures respectively.

**Table 2-1** Material parameters applied to binding factor

<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Tobushi</th>
<th>Qi*</th>
<th>Liu‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_g$ [K]</td>
<td>328</td>
<td>322</td>
<td>343</td>
</tr>
<tr>
<td>$\Delta T_{hl}$ [K]</td>
<td>30</td>
<td>34</td>
<td>50</td>
</tr>
<tr>
<td>$E_h$ [MPa]</td>
<td>27.6</td>
<td>3.29</td>
<td>8.8</td>
</tr>
<tr>
<td>$E_l$ [MPa]</td>
<td>907</td>
<td>614</td>
<td>750</td>
</tr>
<tr>
<td>$\mu_h$ [GPa-s]</td>
<td>116</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$\mu_l$ [GPa-s]</td>
<td>2.03</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$a$ [--]</td>
<td>0.2472</td>
<td>0.1912</td>
<td>0.2250</td>
</tr>
</tbody>
</table>
Figure 2-3. Elastic modulus variation with temperature

(‡: elastic modulus at high and low temperatures used)

The summary of the constitutive model presented above is shown in Table 2-2.
Table 2-2 Summary of constitutive model

**External Variables:** $\epsilon$, $T$

**Internal Variables:** $z$, $q$, $\epsilon_s$

**Stress:**

$$\sigma = \frac{\partial \psi_1}{\partial \epsilon} + \frac{\partial \psi_2}{\partial \epsilon}$$

**Kinematics:**

$$\epsilon_T = \epsilon + q + \epsilon_{\text{int}} + \epsilon_s$$

**Viscous:**

$$\left[ \frac{\partial \psi_1}{\partial q} + \frac{\partial \psi_2}{\partial q} \right] \cdot q = A$$

**Storage:**

$$\dot{\epsilon}_s = z k_m \epsilon_{\text{int}}; k_m = \begin{cases} +1 & T < 0 \\ -1 & T > 0 \\ 0 & T = 0 \end{cases}$$

**Binding factor:**

$$z = 1 - \left(1 + e^{a(T - 0.5\Delta T, T)}\right)^{-1}; a = \left| \ln \left( \frac{\mu_h}{\mu_l} \right) \right|^{-1}$$

**Storage Strain Release:**

$$\epsilon = \frac{1}{\zeta} \frac{\partial \psi_4}{\partial \epsilon}$$

**Entropy:**

$$\eta = \left[ \frac{\partial \psi_3}{\partial T} + \frac{\partial \psi_5}{\partial T} \right]$$
2.2.7 Viscoelastic application

The constitutive model developed in section 2.2.6 is used to develop a viscoelastic model for the SMP. This model was discussed in Balogun & Mo (2014a) and the derivation through the free energy superposition technique is described below. First the constitutive model is used to derive the standard linear viscoelastic model, and then the thermal and storage part of the model is added to the model.

The guiding field and state equations for this model are defined below:

*Deformation*

\[ \varepsilon = \varepsilon_1 + q \]  
\[ \varepsilon = \varepsilon_2 \] \hfill (2-53)

*Equilibrium*

\[ \sigma = \sigma_1 + \sigma_2 \]  
\[ \sigma = \sigma_1 + \sigma_3 \] \hfill (2-54)

*Stress-Strain*

\[ \sigma_1 = E_1 \varepsilon_1 \]  
\[ \sigma_2 = E_2 \varepsilon_2 \] \hfill (2-55)

\[ \sigma_3 = \mu q \]

*Free Energy*
\( \psi_1 = \frac{1}{2} E_1 (\varepsilon - q)^2 \)  
\( \psi_2 = \frac{1}{2} E_2 (\varepsilon)^2 \)  

**Stress Term**

Applying equation (2-44) to equation (2-56) the stress term becomes

\[
\sigma = E_1 (\varepsilon - q) + E_2 \varepsilon
\]  
(2-57)

**Viscous Term**

The viscous term is chosen to be in quadratic form of its argument similar to Valanis (1972)

\[
A = -\mu q^2
\]  
(2-58)

Applying equation (2-46) to equation (2-58) results into:

\[
\mu q = E_2 (\varepsilon - q)
\]  
(2-59)

Following some algebraic manipulations using equations (2-53) through (2-59) the stress-strain relationship becomes

\[
\frac{\sigma}{\mu} - \frac{E_4 \varepsilon}{\mu} + \frac{\sigma}{E_2} - \frac{\varepsilon E_1}{E_2} = \dot{\varepsilon}
\]  
(2-60)

Letting \( \lambda = \mu/E_1 \) and \( E_2 >> E_1 \) then we have:

\[
\frac{\sigma}{\mu} + \frac{\sigma}{E_2} - \frac{\varepsilon}{\lambda} = \dot{\varepsilon}
\]  
(2-61)
Finally, the thermal and storage strains are added to the model to become

\[
\frac{\sigma}{\mu} + \frac{\sigma}{E} \frac{\dot{\varepsilon}}{\lambda} + \varepsilon_{th} + \varepsilon_s = \varepsilon
\]  

(2-62)

The storage strain is defined in equation (2-34) and the thermal strain is given as

\[
\varepsilon_{th} = \alpha (T - T_h)
\]  

(2-63)

where \(\alpha\) is the coefficient of thermal expansion. A rheological model representative of the viscoelastic model described above is shown in Figure 2-4 which is a modified Zener-Maxwell model. The dashed line in the figure is used to identify the temperature dependency of \(\varepsilon_{th}\) and \(\varepsilon_s\) components.

\[\text{Figure 2-4. One-dimensional rheological representation of viscoelastic model}\]
2.3 Three-dimensional constitutive model

In section 2.2.7 a one-dimensional thermo-mechanical constitutive model was developed for SMP. In this section, the one-dimensional model is transitioned into a three-dimensional model. With a three-dimensional model, a numerical finite element model can be developed and used to predict the behavior of complex SMP structures. To develop a three-dimensional model, the polymer is assumed to be an isotropic material, thus the vectors of stress \( \{\sigma\} \) and engineering strain \( \{\varepsilon\} \) in Cartesian coordinates are given as:

\[
\{\sigma\} = \begin{bmatrix} \sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy} \end{bmatrix}^T \\
\{\varepsilon\} = \begin{bmatrix} \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy} \end{bmatrix}^T
\]

(2-64)

The three-dimensional rheological model has been represented as shown in Figure 2-5 below.

![Figure 2-5. Three-dimensional rheological representation of viscoelastic model.](image)

The material constants \( K, G \) represent the shear and the bulk modulus respectively and are defined for isotropic body as
\[ G = \frac{E}{2(1 + v)}; \quad K = \frac{E}{3(1 - 2v)} \]  

(2-65)

The subscripts \( M, E, b \) and \( s \) represent the Maxwell, elastic, bulk and shear quantities respectively.

### 2.3.1 Stress-strain relation

The stress-strain relationship for the spring components follows the generalized Hooke’s law and the dashpot is the viscous component in which the stress is proportional to the rate of strain thus the stress relation of the individual component of the elastic and Maxwell arm is given as follows:

\[
\{\sigma\}_E = [C]^E \{\varepsilon\}_E
\]

(2-66)

\[
\{\sigma\}_M = [C]^M \{\varepsilon\}_M
\]

(2-67)

\[
\{\sigma\}_\mu = [C]^\mu \{\varepsilon\}_\mu
\]

(2-68)

where \([C]\) is the stiffness tensor of the polymer, defined for elastic, Maxwell and viscous components. The inverse of the stiffness tensor is the compliance matrix thus

\[
[C]^{-1} = [S]
\]

(2-69)
Therefore the inverse of equations (2-66) to (2-68) is given in the following equation below

\[
\{ \varepsilon \}_E = [S]^E \{ \sigma \}_E
\]  
(2-70)

\[
\{ \varepsilon \}_M = [S]^M \{ \sigma \}_M
\]  
(2-71)

\[
\dot{\{ \varepsilon \}}_\mu = [S]^\mu \{ \sigma \}_\mu
\]  
(2-72)

The isotropic definition of the compliance and stiffness tensors are given in later section below.

### 2.3.2 Equilibrium equation

For equilibrium of forces, assuming constant area, the applied stress to the rheological model is supported both by the Maxwell and Elastic elements thus

\[
\{ \sigma \}_M = \{ \sigma \}_\mu
\]
(2-73)

\[
\{ \sigma \} = \{ \sigma \}_M + \{ \sigma \}_E
\]

### 2.3.3 Geometry of deformation equation

The total strain \( \varepsilon \) is equal to the strain of the elastic element. It is also equal to the sum of the strains of the two elements within the Maxwell arm. So,

\[
\{ \varepsilon \} = \{ \varepsilon \}_E = \{ \varepsilon \}_M + \{ \varepsilon \}_\mu
\]  
(2-74)
The time derivative of equation (2-74) can be written as

\[
\{\dot{\varepsilon}\} = \{\dot{\varepsilon}\}_M + \{\dot{\varepsilon}\}_\mu
\]  

(2-75)

Likewise from equation (2-73) the time derivative for the stresses is written as

\[
\{\dot{\sigma}\}_M = \{\dot{\sigma}\} - \{\dot{\sigma}\}_E
\]  

(2-76)

Applying equations (2-66), (2-71), (2-72) and (2-76) into equation (2-75), the resulting equation becomes

\[
\]  

(2-77)

2.3.4 Resolving compliance and stiffness tensors

For an isotropic material, the compliance, \([S]\) and stiffness, \([C]\) tensors can be written in terms of the bulk and shear modulus as shown in equations (2-78) and (2-79) (Willam, 2002).

\[
[S] = \left(\frac{1}{9K} - \frac{1}{6G}\right) I \otimes I + \frac{1}{2G} II
\]  

(2-78)

\[
[C] = (9K - 6G) I \otimes I + 2GII
\]  

(2-79)

where \(I\) is a second order identity tensors and \(II\) is the symmetric part of a fourth-rank identity tensor thus in indicial notation it is given as
\[ I = \delta_{ij}; \quad I \otimes I = \delta_{ij} \delta_{kl}; \quad II = \frac{1}{2} \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) \] (2-80)

where the indices \( i,j,k,l \) are the tensor component of the tensors and \( \delta \) is the Kronecker delta function. In order to further simplify the representation of the compliance and stiffness tensors the identity tensors can be rewritten as shown below

\[ \hat{I} = \frac{1}{3} I \otimes I; \quad \hat{II} = II - \frac{1}{3} I \otimes I \] (2-81)

Thus the elastic tensors are simplified as

\[ [S] = \frac{1}{3K} \hat{I} + \frac{1}{2G} \hat{II} \] (2-82)

\[ [C] = 3K \hat{I} + 2G \hat{II} \] (2-83)

In the section that follows these elastic tensors are used to define the individual components elastic tensors shown in equation (2-77).

### 2.3.5 Resolving the strain term

Equation (2-77) can be decomposed into two separate terms. Terms containing the strain components are referred to as the strain term and terms containing the stress components are called the stress term. The strain term derived in the equation can be written as
\[
StrainTerm = \left(1 + [S]^{M} [C]^{E}\right)\{\varepsilon\} + [S]^{\mu} [C]^{E} \{\varepsilon\}
\] (2-84)

This can further be decomposed into

\[
StrainTerm1 = \left(1 + [S]^{M} [C]^{E}\right)\{\varepsilon\}
\] (2-85)

\[
StrainTerm2 = [S]^{\mu} [C]^{E} \{\varepsilon\}
\] (2-86)

Applying the results derived from equations (2-82) and (2-83) into equation (2-85) the resulting term becomes

\[
StrainTerm1 = \left(1 + \left(\frac{1}{3K_{M}} \hat{I} + \frac{1}{2G_{M}} \hat{II}\right) \left(3K_{E} \hat{I} + 2G_{E} \hat{II}\right)\right)\{\varepsilon\}
\] (2-87)

Let

\[
1 = \hat{I} + \hat{II}
\] (2-88)

Thus applying equation (2-88) into (2-87) and expanding the parenthesis results into

\[
StrainTerm1 = \left(\hat{I} + \hat{II} + \frac{K_{E}}{K_{M}} (\hat{I} \cdot \hat{I}) + \frac{3K_{E}}{2G_{M}} (\hat{I} \cdot \hat{II}) + \frac{2G_{E}}{3K_{M}} (\hat{II} \cdot \hat{II}) + \frac{G_{E}}{G_{M}} (\hat{II} \cdot \hat{II})\right)\{\varepsilon\}
\] (2-89)

Expanding the identity tensors into indicial notations it can be shown that

\[
\hat{I} \cdot \hat{I} = \hat{I}; \hat{II} \cdot \hat{II} = \hat{II}; \hat{I} \cdot \hat{II} = 0
\] (2-90)

These results can be applied to equation (2-89) which leads to
Applying similar techniques on \( StrainTerm2 \) the resulting equation becomes

\[
StrainTerm2 = \left[ \frac{K_E}{\mu_b} \hat{I} + \frac{G_E}{\mu_s} \hat{II} \right] \{\varepsilon\} \tag{2-92}
\]

The viscosity bulk and shear terms have taken the place of the bulk and shear modulus from equations (2-82) and (2-83). The strain term can be written from equations (2-91) and (2-92) as

\[
StrainTerm = \left[ \left( 1 + \frac{K_E}{K_M} \right) \hat{I} + \left( 1 + \frac{G_E}{G_M} \right) \hat{II} \right] \{\varepsilon\} + \left[ \frac{K_E}{\mu_b} \hat{I} + \frac{G_E}{\mu_s} \hat{II} \right] \{\varepsilon\} \tag{2-93}
\]

The strain term can be further decomposed into hydrostatic and deviatoric components. In order to do this the identity tensors can be expanded into the form defined in equation (2-81) and applied to equation (2-93). Upon algebraic manipulation and simplification the resulting strain term becomes

\[
StrainTerm = \frac{1}{3} \left( \frac{K_E}{K_M} - \frac{G_E}{G_M} \right) tr\{\varepsilon\} I + \left( 1 + \frac{G_E}{G_M} \right) \{\varepsilon\} \\
+ \frac{1}{3} \left( \frac{K_E}{\mu_b} - \frac{G_E}{\mu_s} \right) tr\{\varepsilon\} I + \frac{G_E}{\mu_s} \{\varepsilon\} \tag{2-94}
\]

Where the operator \( tr\{\} \) is the trace of the tensor.
2.3.6 Resolving the thermal and storage strain

A careful study of the viscoelastic model presented in equation (2-62) shows that the time derivatives of the thermal and storage strains are added on independently to the viscoelastic model. The total strain can be decomposed into the mechanical strain, thermal strain and storage strain. Thus the time derivative of the total strains in equation (2-94) can be rewritten as below

\[
\{\dot{\varepsilon}\} \rightarrow \{\dot{\varepsilon}\} - \{\dot{\varepsilon}\}_S - \alpha \dot{T} I \tag{2-95}
\]

\[
tr\{\dot{\varepsilon}\} \rightarrow tr\{\dot{\varepsilon}\} - tr\{\dot{\varepsilon}\}_S - 3\alpha \dot{T}
\]

where the thermal strain has been written in terms of the coefficient of thermal expansion (\(\alpha\)) and Temperature (\(T\)). Finally the strain term can be rewritten by incorporating equation (2-95) into equation (2-94) which results in

\[
StrainTerm = \frac{1}{3} \left( \frac{K_E}{K_M} - \frac{G_E}{G_M} \right) tr\{\dot{\varepsilon}\} I + \left( 1 + \frac{G_E}{G_M} \right) \{\dot{\varepsilon}\} + \frac{1}{3} \left( \frac{K_E}{\mu_b} - \frac{G_E}{\mu_s} \right) tr\{\dot{\varepsilon}\} I + \frac{G_E}{\mu_s} \{\dot{\varepsilon}\}
\]

\[-\alpha \left( 1 + \frac{K_E}{K_M} \right) T I - \frac{1}{3} \left( \frac{K_E}{K_M} - \frac{G_E}{G_M} \right) tr\{\dot{\varepsilon}\}_S I - \left( 1 + \frac{G_E}{G_M} \right) \{\dot{\varepsilon}\}_S \tag{2-96}
\]

2.3.7 Resolving the stress term

The stress term is defined from equation (2-77) as

\[
StressTerm = \left[ S \right]^M \{\dot{\sigma}\} + \left[ S \right]^\nu \{\sigma\} \tag{2-97}
\]

Following similar techniques described above for the strain term derivation, the stress term becomes
2.3.8 Defining the three-dimensional model

So far the stress and strain terms have been derived in their three-dimensional form. They need to be set equal to each other in order to form the three-dimensional model. Some assumptions made before deriving the final model include the bulk viscosity quantity ($\mu_b$) is set to infinity and the retardation time shown in equation (2-62) is accounted for as follows

$$\frac{E_F}{3\mu_s} = \frac{1}{\lambda}$$

Equation (2-99) allows for the calculation of the shear and bulk modulus for the individual elements in the rheological model from their respective Young's modulus. Experimental data will provide the total Young's Modulus ($E$) and the retardation time. The corresponding Maxwell element Young's modulus of the SMP is calculated as

$$E_M = E - E_E$$

Thus the three-dimensional model for equation (2-62) becomes
\[
\left( \frac{1}{9K_M} - \frac{1}{6G_M} \right) \text{tr} \{ \sigma \} I + \frac{1}{2G_M} \text{tr} \{ \sigma \} I + \frac{1}{6 \mu_s} \text{tr} \{ \sigma \} I + \frac{1}{2 \mu_s} \{ \sigma \} = \\
\frac{1}{3} \left( \frac{K_E}{K_M} - \frac{G_E}{G_M} \right) \text{tr} \{ \varepsilon \} I + \left( 1 + \frac{G_E}{G_M} \right) \left( \frac{G_E}{3 \mu_s} \right) \text{tr} \{ \varepsilon \} I + \frac{G_E}{\mu_s} \{ \varepsilon \} \\
- \alpha \left( 1 + \frac{K_E}{K_M} \right) I - \frac{1}{3} \left( \frac{K_E}{K_M} - \frac{G_E}{G_M} \right) \text{tr} \{ \varepsilon \} \{ \varepsilon \} I - \left( 1 + \frac{G_E}{G_M} \right) \{ \varepsilon \} \{ \varepsilon \} 
\] 

(2-101)

Equation (2-101) reduces back to the one-dimensional equation (2-62) if the Poisson’s ratio of both elements is set to zero, the transverse components are ignored and \( E_M >> E_E \).
2.4 Time discrete numerical model

The three-dimensional constitutive model presented in section 2 can be resolved into a numerical model and applied into a finite element analysis tool. This section presents the numerical solution to the constitutive model. The techniques employed in this section require the understanding of the deviatoric and hydrostatic of the strain decomposition as shown in the previous section. The strain and stress terms are decoupled and presented separately but later combined to provide the final time discrete constitutive model.

2.4.1 Time discrete form of strain term

The strain from equation (2-96) can be simplified by using alphabetic letters to represent the material parameters. Thus after rearranging the strain term becomes

\[
StrainTerm = C_1 \text{tr}\{\dot{\varepsilon}\}I + 2D_1 \{\dot{\varepsilon}\} + C_2 \text{tr}\{\ddot{\varepsilon}\} I + 2D_2 \{\ddot{\varepsilon}\} + Q^T I - C_2 \text{tr}\{\dot{\varepsilon}_s\} I - 2D_2 \{\dot{\varepsilon}_s\}
\]  

(2-102)

where

\[
C_1 = \left(\frac{G_E}{3\mu_s}\right), \quad C_2 = \frac{1}{3} \left(\frac{K_E}{K_M} - \frac{G_E}{G_M}\right), \quad D_1 = \frac{G_E}{2\mu_s}, \quad D_2 = \frac{1}{2} \left(1 + \frac{G_E}{G_M}\right), \quad Q = -\alpha \left(1 + \frac{K_E}{K_M}\right)
\]  

(2-103)
2.4.2 Time discrete form of stress term

For the numerical analysis work, the stress term definition shown in equation (2-97) has been used. The tensors have been defined using alphabetic letters for simplification purpose on the technique used. Thus the stress term is defined as

\[ \text{StressTerm} = A \{ \dot{\sigma} \} + B \{ \sigma \} \]  \hspace{1cm} (2-104)

where for nominal stress terms the alphabets are

\[ A = \frac{1}{9K_m} - \frac{1}{6G_m} + \frac{1}{2G_m} \]  \hspace{1cm} (2-105)

\[ B = -\frac{1}{6\mu_s} + \frac{1}{2\mu_s} \]  \hspace{1cm} (2-106)

and for shear stress term it is given as

\[ A = + \frac{1}{2G_m} \]  \hspace{1cm} (2-107)

\[ B = + \frac{1}{2\mu_s} \]  \hspace{1cm} (2-108)

These definitions have their basis from equation (2-78).
2.4.3 Time discrete form of stress-strain equation

The stress and strain terms developed for the time discrete form in equations (2-102) and (2-104) can now be equated together to get the stress-strain relationship used for the numerical analysis. Rearranging terms and replacing the alphabets by Greek letters and for a representation in a lame parameters for which is written as

\[
\{\sigma\} + \nu \{\dot{\sigma}\} = \lambda_1 \text{tr}\{\varepsilon\} I + 2\mu_1 \{\varepsilon\} + \lambda_2 \text{tr}\{\varepsilon\} I + 2\mu_2 \{\varepsilon\} \\
+ \theta T I - \lambda_2 \text{tr}\{\varepsilon\}_S I - 2\mu_2 \{\varepsilon\}_S
\]

(2-109)

Where

\[
\nu = \frac{A}{B}, \quad \lambda_1 = \frac{C_1}{B}, \quad \lambda_2 = \frac{C_2}{B}, \quad \mu_1 = \frac{D_1}{B}, \quad \mu_2 = \frac{D_2}{B}, \quad \theta = \frac{Q}{B}.
\]

(2-110)

2.4.4 Time discrete form of stress incrementation term

The evolution of the stress tensors defined in equation (2-109) can be done using a simple stable integration operator. The central difference operator has been used to defined the stress incrementation and it is given as

\[
\dot{f}_{t+\frac{1}{2}\Delta t} = \frac{\Delta f}{\Delta t} \\
\dot{f}_{t+\frac{1}{2}\Delta t} = f_t + \frac{\Delta f}{2}
\]

(2-111)

where \(f\) is some function, \(f_t\) is beginning value before increments, \(\Delta f\) is the change in the function over the increment, and \(\Delta t\) is the time increment. Equation (2-111) can be
applied to the stress, strain and material properties terms as they all evolve in time. Applying equation (2-111) to equation (2-109) will provide two different equations for nominal and shear stress incrementation.

The nominal stress incrementation (in x-direction) is given as

\[
\left[ \frac{\Delta t}{2} + \left( \nu + \frac{\Delta \nu}{2} \right) \right] \Delta \sigma_x = \left[ \frac{\Delta t}{2} \left( \lambda_1 + \frac{\Delta \lambda_1}{2} \right) + \left( \lambda_2 + \frac{\Delta \lambda_2}{2} \right) \right] tr\{\Delta \varepsilon\} \\
+ \left[ \Delta t \left( \mu_1 + \frac{\Delta \mu_1}{2} \right) + 2 \left( \mu_2 + \frac{\Delta \mu_2}{2} \right) \right] \Delta \varepsilon_x
\]

\[
= \left[ \left( \lambda_1 + \frac{\Delta \lambda_1}{2} \right) tr\{\varepsilon\} \right] \\
+ \Delta t + 2 \left( \mu_1 + \frac{\Delta \mu_1}{2} \right) \varepsilon_x - \sigma_x \\
\left[ \theta + \frac{\Delta \theta}{2} \right] \Delta T - \left( \lambda_2 + \frac{\Delta \lambda_2}{2} \right) tr\{\Delta \varepsilon_s\} \\
- 2 \left( \mu_2 + \frac{\Delta \mu_2}{2} \right) \Delta \varepsilon_{sx}
\] (2-112)

where

\[
tr\{\Delta \varepsilon\} = \Delta \varepsilon_x + \Delta \varepsilon_y + \Delta \varepsilon_z
\] (2-113)
The shear stress incrementation (in xy-direction) is given as

\[
\left[ \frac{\Delta t}{2} + \left( \nu + \frac{\Delta \nu}{2} \right) \right] \Delta \tau_{xy} = \left[ \Delta t \left( \mu_1 + \frac{\Delta \mu_1}{2} \right) + 2 \left( \mu_2 + \frac{\Delta \mu_2}{2} \right) \right] \Delta \gamma_{xy}
\]

\[
+ \Delta t \left[ \left( \mu_1 + \frac{\Delta \mu_1}{2} \right) \gamma_{xy} - \tau_{xy} \right]
\]

\[
- \left( \mu_2 + \frac{\Delta \mu_2}{2} \right) \Delta \gamma_{sx}
\]

(2-114)

See Appendix B for the implementation of this code.
3 EXPERIMENTAL AND NUMERICAL VALIDATION OF MODEL

3.1 Introduction

The viscoelastic model developed in section 2 has been validated in this section using experimental data and numerical analysis. Two experimental dataset (Tobushi et al., 1996; Beblo et al., 2010) have been adopted from technical journals in order to validate the model. These two experimental data comprises of the material properties that is required for the constitutive model. In the subsections that follow, the materials properties for the two experimental datasets are introduced; the approach in which the one-dimension model shown in equation (2-62) is validated is presented; and the three-dimensional validation results are presented as well.

3.2 Experimental validation

3.2.1 Beblo’s experimental SMPs properties

Beblo et al. (2010) conducted experimental work on a Veriflex® SMP. Veriflex® is a two part styrene based resin and hardener laboratory manufactured SMP. The mixture is a ratio of 24:1 resin to hardener and the uncured mixture is poured into low carbon steel mold of dog bone size specimens. For the test samples includes tensile and three point bend samples. The cross-section for the tensile samples were reported to be 3 mm x 5mm, while the dimensions for the three point bend test was 2 mm x 80 mm x10 mm.

Different types of testing conducted include tensile testing, three point bend testing and creep testing all below and above the glass transition temperature. A 5K MTI screw
driven load frame was used for the test. The displacement rate for the tensile test was reported to be 4 mm/min. Similarly, the displacement rate for the three point bend test was reported to be 3 mm/min. Above the glass transition temperature, the load frame used for the three point bend were 1K MTI screw driven. Using same dimensional sample as the three point bend, the creep test was done above glass transition temperature by loading it to 75 kPa surface stress for 120 minutes. The reported material properties determined for Veriflex® SMP are shown in Table 3-1 below.

Table 3-1 Beblo et al. material properties used for model validation

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Veriflex - Styrene based SMP (Beblo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_g$ [K]</td>
<td>335</td>
</tr>
<tr>
<td>$\Delta T_{hl}$ [K]</td>
<td>33</td>
</tr>
<tr>
<td>$E_h$ [MPa]</td>
<td>2.63</td>
</tr>
<tr>
<td>$E_l$ [MPa]</td>
<td>1010</td>
</tr>
<tr>
<td>$\mu_h$ [MPa-s]</td>
<td>6.6</td>
</tr>
<tr>
<td>$\mu_l$ [GPa-s]</td>
<td>170</td>
</tr>
<tr>
<td>$\lambda_h$ [s]</td>
<td>2.5</td>
</tr>
<tr>
<td>$\lambda_l$ [s]</td>
<td>1650</td>
</tr>
<tr>
<td>$a$ [--]</td>
<td>0.0985</td>
</tr>
</tbody>
</table>

3.2.2 Tobushi’s experimental SMPs properties

Tobushi et al. (1996) at Mitsubishi Heavy Industries (MHI) developed a SMP thin film of polyurethane based on polyester polyole series. The casting film for this SMP was made from a solution – Diary MS5510 - which was also produced by MHI. They reported a weight ratio of polymer to dimethyl formamide as 3:7 and the viscosity of the
solution at 298 K was 100 Pa-s. The transition temperature for the SMP was tailored to 328 K. Dog bone specimens with dimensions of 25 mm x 5 mm x 0.05 mm were used for the experiment. The following experimental tests were conducted in order to thoroughly quantify the material response of the SMPs:

i) Dynamic Mechanical Test: This test is generally used to characterize the material’s properties as a function of temperature, time, frequency, stress, atmospheric moisture or a combination of these parameters (PerkinElmer, 2008). In Tobushi’s experiment it was used to determine the relationships of the storage modulus and the loss tangent with respect to temperature. RMS800 manufactured by Rheometric Corp was the test machine used to conduct the test.

ii) Uniaxial Tensile Test: Stress-strain responses at different temperature values are conducted in order to see the behavior of the material. Shimazu Corp SMP testing machine was used for this test as well as the remaining test discussed below. This test was conducted with maximum strain level of 200%. The purpose of this test is to validate the expected material behavior at high and low temperatures. The temperature variation was done at +/- 20K from the transition glass temperature.

iii) High Temperature Cyclic Tensile Test: A cyclic tensile test at high temperature shows the material’s response to fatigue loading at high and low straining. It was determined that the material after undergoing several cycles of mechanical training takes on a certain constant deformation.
iv) Thermo-mechanical Cyclic Test: In this test, the stress-strain curves, strain-temperature and stress-temperature curves are developed. This testing leads to understanding the shape fixity and shape recoverability of the SMP.

The summary of the material properties used for the model validation are shown in Table 3-2 below. The high and low moduli of the SMP were determined along with the retardation and viscosity and similar temperatures as shown in the table. Parameter $a$ has been calculated based on equation (2-52).

**Table 3-2** Tobushi et al. material properties used for model validation

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>Polyurethane series SMP (Tobushi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_g$ [K]</td>
<td>328</td>
</tr>
<tr>
<td>$\Delta T_{hl}$ [K]</td>
<td>30</td>
</tr>
<tr>
<td>$E_h$ [MPa]</td>
<td>27.6</td>
</tr>
<tr>
<td>$E_l$ [MPa]</td>
<td>907</td>
</tr>
<tr>
<td>$\mu_h$ [GPa-s]</td>
<td>2.03</td>
</tr>
<tr>
<td>$\mu_l$ [GPa-s]</td>
<td>116</td>
</tr>
<tr>
<td>$\lambda_h$ [s]</td>
<td>111</td>
</tr>
<tr>
<td>$\lambda_l$ [s]</td>
<td>2480</td>
</tr>
<tr>
<td>$a$ [--]</td>
<td>0.2472</td>
</tr>
</tbody>
</table>

The material model developed for the constitutive model shown in equation (2-52) has been compared to the Tobushi’s experimental result. The results shown in Figure 3-1 depicts that the model matches well with experimental data. The trends show a high modulus at low temperature and a low modulus at high temperature.
Figure 3-1 Young’s modulus material model comparison to Tobushi et al. experimental results

3.3 Numerical validation

An introductory numerical validation was conducted using Beblo’s experimental work (Beblo et al., 2010). This data has been used to validate the one-dimensional and three-dimensional models developed in section 2 (and discussed in details in the next section). Beblo’s work does not consist of thermo-mechanical experimentation so it serves as a good basis to check the prediction of the model at high and low temperatures around the glass transition temperature. The material properties used for
this validation have been reported in Table 3-1. The stress-strain results for the low and
high temperatures have been presented in Figure 3-2 and Figure 3-3 respectively. The
low temperature was done at 295 K and the high temperature was done at 353 K. The
one-dimensional numerical model was written in OCTAVE (Octave, 1998) and the three-
dimensional numerical model was written in ABAQUS’ UMAT subroutine (SIMULIA,
Providence, RI). Details of these codes are discussed in the thermo-mechanical validation work presented in this section. At low temperatures as shown in Figure 3-2,
the one-dimensional and three-dimensional models are linear which is a similar trend
noticed in the experiment. They both under-predict the experimental work and this may
be due to a preloading of the experiment which was not captured in the numerical model. The highest prediction deviation at around 1% strain shows a 23% difference,
but this difference is reduced to 7% if the model prediction is shifted up to the same
starting point as the experiments. Overall, at low temperature the material is in a glassy
state and the stress level shown at 20 MPa predicts a strain level of 2%.

At high temperature the stress level is much smaller at a level of 0.5MPa with a
corresponding strain level of 135% as shown in Figure 3-3. The results show a good
correlation between the one-dimensional and three-dimensional models and the
experimental data. The rubbery state effect shows up in the experimental data as the
data shows some level of non-linearity but overall can be well modeled by the linear
predictions by the one-dimensional and three-dimensional models.
Figure 3-2 Low temperature stress-strain validation results for Beblo et al.’s Veriflex
In the section that follows, the process of the thermo-mechanical test conducted by Tobushi et al. (1996) have been adopted in order to validate the model developed. The one-dimensional and three-dimensional models are compared to experimental data and the results are presented and discussed.

The thermo-mechanical process followed involves four steps in order to observe shape memory effect (SME) and strain storage releases which have been labelled below:

1) Pre-loading of the SMP at high temperature

Figure 3-3 High temperature stress-strain validation results for Beblo et al.’s Veriflex
2) Constraining the SMP for shape fixity as it cools

3) Unconstrained relaxation of SMP at low temperature

4) Unconstrained storage strain release

### 3.3.1 One-dimensional model compared to thermo-mechanical experimental data

In order to validate the one-dimensional model an Octave code was written to model the four steps of shape memorization and release. The analysis approach taken to compare model to Tobushi’s experimental results is shown below. See Appendix A for the numerical code written in Octave.

**Step 1: Pre-loading the SMP**

At high temperature \( T_h \) the mechanical load required to induce storage strain is applied to the SMP. Since \( \dot{T} \) is zero, no storage strain is yet to occur. The thermal strain is also assumed to be zero at the temperature, thus only the mechanical strain and stress are in effect in this step equation (2-62) is resolved as

\[
\dot{\varepsilon} = E \left( \frac{\dot{\varepsilon}}{\mu} + \frac{\varepsilon}{\lambda} \right)
\]  

(3-1)

**Step 2: Constrained SMP**

From the model, storage strain begins in the presence of mechanical load/constraint and change in temperature. For this analysis the mechanical strain has been assumed to be constant in this step. Since the change in the stress state of the SMP over
temperature is of interest in this step chain rule can be applied to equation (2-62) and resolved as

$$\frac{d\sigma}{dT} = E \left[ \left( \frac{\varepsilon}{\lambda} - \frac{\sigma}{\mu} \right) \frac{1}{T} - \sigma \frac{dE^{-1}}{dT} \right]$$

(3-2)

**Step 3: Unconstrained stress relaxation**

The boundary condition for this step is that there is no load or constraint applied to the SMP, thus making the stress state equal zero within the SMP. There is a certain amount of strain loss at the step depending on the SMP’s capacity to store strain at the reduced temperature. Equation (2-62) is resolved as

$$\varepsilon = -\frac{\varepsilon}{\lambda}$$

(3-3)

**Step 4: Unconstrained strain release**

The unstrained strain release process is achieved by heating the SMP. Since the SMP is not constrained the boundary condition requires that the stress state be zero. The strain response to temperature is of interest and is resolve from equation (2-62) as:

$$\frac{d\varepsilon}{dT} = \left( \frac{\varepsilon}{\lambda} + \varepsilon_{inh} + \varepsilon_s \right) \frac{1}{T}$$

(3-4)

**Model comparison to experimental data**

The model presented above has been compared to experimental work done by Tobushi et al. (1996) and the results have been presented in Figure 3-4 through Figure 3-6. Two different pre-loading conditions have been applied similar to Tobushi et al. (1997) as shown in these figures. \(T_h\), \(T_l\) and \(T_g\) are identified at 348 K, 308 K and 328 K
respectively. The pre-loading strain levels are observed at both 2.4% and 4.0%. In Figure 3-4, the model under predicts the residual stresses at the end of step 2 for the 2.4% strain by about 20%; it over predicts for the 4.0% strain by 9%. In Figure 3-5, in step 3, the strain loss at 2.4% strain and 4.0% strain are 20% and 10% respectively for the experimental data. The storage strain for the model stayed about the same as the applied mechanical strain at the end of step 3 and shape memorization is achieved by the model. The stored strain in step 3 is released as the polymer is being heated back up in step 4. As shown in Figure 3-5 a non-linear behavior is observed both by the experiment and model. The model in both cases the predicted strains are above the experimental results. Figure 3-6 shows the stress-strain response of the polymer as the process of shape fixity (memorization) and release is being conducted. The model trends follow experimental observation.
Figure 3-4 One-dimensional validation result Tobushi et al. stress-temperature comparison
Figure 3-5 One-dimensional validation result Tobushi et al. strain-temperature comparison
Overall, SME is achieved for the SMP model with potential application to engineering structures. The next step of a commercial application is to transition the model into a numerical model system such as commercially available finite element analysis software. The step in which this is done is the focus of discussion in the next section.
3.3.2 Three-dimensional model compared to thermo-mechanical experimental data

The model was also verified by developing a three-dimensional finite element method (FEM) analysis model and running it through the experimental steps described in the previous section. In order to generate the FEM model, the time discrete model developed in section 2.4 was coded. The time discrete model algorithm was written in FORTRAN through the UMAT functionality of ABAQUS. After developing the time discrete algorithm a FEM model was developed from single quad element and used to validate the thermo-mechanical dataset.

Figure 3-7 shows the single element FEM developed for validation. The von-Mises results are shown at the end of a step in the figure. At the initial condition, the model is set to a high temperature of 348 K with no initial load or deformation. In step (1), the element is deformed at the high temperature at a stress level of 7 MPa. In step (2), the deformation is held constant as it cools down and the stress level increases. In step (3), the element shows shape memorization has the constraint is removed at low temperatures. As shown in this step the stress level drops to zero because there are no external loading on the element. Finally in step (4), the shape is released as the element is being heated back up. There is a residual strain shown by the model at high temperature.
Figure 3-7 Single QUAD element FEM thermo-mechanical results

Figure 3-8 through Figure 3-10 shows results of the three-dimensional compared to Tobushi et al.’s experimental data and one-dimensional result. In the figures, the model shows SME trend from steps 1 through 3. Also shape recovery is noticed in step 4. In
Figure 3-8 the stress temperature curve shows increase in the stress level as the polymer memorizes the shape during temperature drop in step 2. The three-dimensional model predicts a residual stress level of 1.71 MPa, one-dimensional model predicts 1.45 MPa and experimental data were 1.9 MPa. Also, the three-dimensional model follows closely the non-linear trajectory of the experimental data in step 2. Figure 3-9 shows the strain-temperature curve, the three-dimensional predicts a level of 2.45% with the same pre-load applied to it in comparison to one-dimensional model. The achievement of shape fixity of the polymer is done in step 3. After retardation, the three-dimensional model predicts a strain level of 2.21%, one-dimensional model remains at 2.4% and experimental data is 2.0%. The strain recovery transition in step 4 shows that the three-dimensional model predicts closely the behavior of the polymer in comparison with the one-dimensional model. The three-dimensional model does not show a high non-linearity effect as compared to the one-dimensional model mainly due to the three-dimensional assumptions made (transverse interaction) and the several interaction of the material property systems develop in earlier section. In the one-dimensional model, only three main properties (Young's modulus, viscosity, and retardation time) were required and the binding factor applied to it. On the other hand, in the three-dimensional model these three properties have been decomposed into elastic and Maxwell portion which is based on the assumption of linearly combining these two arms will equal to the bulk property measured in the laboratory as shown in equation (2-100). At the end of step 4, the three-dimensional model predicts a residual strain level of 0.43%, the one-dimensional model predicts 0.29% and experimental data is measured at 0.36%. Figure 3-10, shows the stress and strain response of the polymer around the
glass transition temperature. The three-dimensional models predicts closer to the experimental data than the one-dimensional model further confirmed previous discussion.

In order to further validate the three-dimensional model, high loading was applied to the FEM to check the response at 4% strain level similar to the experimental data generated by Tobushi et al. (1996) The results of this validation are shown in Figure 3-11 through Figure 3-13. Similar trends found in the 2.4% validation are noted in the 4% validation run. In Figure 3-11, the three-dimensional model follows closely the trend of the experimental data by predicting stress lower than the one-dimensional model in step 2. The stress measured at the end of step 2 is 4% higher than experiment in the three-dimensional model while it is 10% in the one-dimensional model. Once again in Figure 3-12, the three-dimensional model clearly shows a closer prediction than the one-dimensional model in step 4, which is the strain recovery process. In Figure 3-13, the stress-strain response shows similar trends as discussed for the 2.4% validation.
Figure 3-8 2.4% strain three-dimensional validation result Tobushi et al. stress-temperature comparison
Figure 3-9 2.4% strain three-dimensional validation result Tobushi et al. strain-temperature comparison
Figure 3-10 2.4% strain three-dimensional validation result Tobushi et al. stress-strain comparison
Figure 3-11  4% strain three-dimensional validation result Tobushi et al. stress-temperature comparison
Figure 3-12 4% strain three-dimensional validation result Tobushi et al. strain-temperature comparison
The new three-dimensional model can now be used to predict the transverse stresses and strains of the polymer. Figure 3-14 shows the results of the predictions of the transverse stresses and strains. In the nominal directions, the effect of the Poisson's ratio causes the polymer to be in contraction thus predicting a strain level of -1.02% in step 1. The residual stresses in step 2 predict 0.48 MPa for the transverse directions and relatively zero in the shear directions. The strain predictions in steps 3 and 4 are...
given as -0.99% and -0.20% respectively. The strains in the shear direction (<12> and <13>) were predicted to remain relatively at zero as shown in the figure.

Figure 3-14 SMP transverse stress strain temperature prediction

Following the single element validation shown above, a multi quad element coupon FEM model was developed. The FEM generated was again used to validate the SME of the material and strain release. The same four steps described above were followed to conduct validation on the SMP. A stress level of magnitude 0.7 MPa was applied on the upper edge of the coupon while a fixed boundary condition was applied on the lower edge. This mimics a tensile coupon test in the experiment (Tobushi et al., 1996).
Figure 3-15 shows the von-Mises result of the FEM along with the deformation. From the figure, it is seen that the stresses increase from 0.7 MPa to 1.6 MPa between step 1 and step 2. Step 3 shows SME after the constraint has been released from the SMP. Finally in step 4 the stored strain is released. The binding factor model predicts a residual strain of 0.43%, while experimental data shows 0.36%.
Figure 3.15. SMP multi-element FEM validation analysis.
4 MORPHING OF FLAPPING WING MICRO AIR VEHICLE

4.1 Introduction

Flapping wing micro air vehicles (FW-MAV) will have many applications in today’s economy which can include search and rescue mission, reconnaissance, deep cave exploration and many more. There has been some level of extensive research done on FW-MAV showing good success in flight and hovering (Pornsin-Sirirak et al., 2001; Madangopal et al., 2005; Bradshaw & Lentink, 2007; Kawamura et al., 2008; Takahashi et al., 2010 and Nakata et al., 2011). With a focus on insects, researchers at Berkeley developed the Micromechanical Flying Insect (MFI) (Fearing et al., 2000), a 25 mm, 100 mg flapping robotic insect. It is composed of four bar frames acting as the thorax and an extensible fan-fold wing. Wood (2007) developed a FW-MAV that achieved takeoff at weight of 60mg using the Smart Composite Microstructure (SCM) paradigm. The SCM paradigm consisted of discretized compliances made from layered and serial composition carbon fiber and thin polymer films. de Croon et al. (2009) developed the DeFly micro an 3.07 g ornithopter robot with capacity to carry a camera and transmitter. In this work, SMPs have been proposed as material of choice for the morphing wings of a FW-MAV. Morphing wing technologies have typically been applied to macro scale based system (Flanagan et al., 2007; Andersen et al., 2007 and Bubert et al., 2010). The application of SMPs to FW-MAV’s wing is novel and will introduce a higher level of performance as described in section 1.
4.2 Morphing in nature

Morphing of wing structure is common in nature as described in detail by Taylor et al. (2012) with noted applications in insects, birds and bats. For most insects, actuation of the wing is done at the root with a full deformation of the thorax in order to achieve flight. Their wings are semi-flexible structures with high deformable capabilities to maneuver through aerodynamics loading. In stowage mode, the wings are neatly tucked behind the thorax and abdomen of the insect as shown in Figure 4-1 (Berkery, 2013). All flights for insect are achieved by flapping their wings, but some large insects such as butterflies and locust are able to glide as shown in Figure 4-2. The insect wing structures are made from a material called cuticle which is a composite material of nanofibrous chitin wires and various proteins as matrix material (Vincent & Wegst, 2004). Chitin is a colorless polysaccharide fiber that provides stiffness and chemical stability for the wing. Within the matrix material there are several protein types, but resilin is of great importance as described by Taylor et al. (2012). The total percent found within the structure largely defines the amount of elastic potential energy the structure can store thus affecting its overall Young’s modulus (Taylor et al., 2012). This allows cuticle to be used for different functionality on the insect. Fore and hind wings are present on some insects like the locust. The forewing is typically a stiffer structure relatively compared to the hind wing and is mainly used for protecting the hind wing and stability in flight. On the other hand, the hind wing is used for main flight and it is a deployable structure and can be conveniently stored away underneath the forewing hard shell. The deployment of the hind wing in a locust is accomplished in a series of linked mechanism. The hind wing is divided into two segments with a relatively stiff
leading edge and a compressible fan-like trailing wing panel. Deformation of the thorax activates the leading edge at the root, which in turns deploys the trailing edge wing panels. Due to the presence of resilin within the wing, elastic potential energy is stored within foldable wings, which allows it to stay locked in place. SMPs also have the ability to stored elastic potential energy due to external stimulus and these characteristics can be adopted in the design of morphing wing for FW-MAV.

**Figure 4-1** Stowage mode configuration for an insect wing as display by bee
Reproduced with permission from Mark Berkery
Figure 4-2 Deployment of a butterfly’s wing in flight (Micolo, 2014)

4.3 Conceptual design for morphing wing

A simple triangular design has been adopted to show the versatile capability of the model developed in section 2. This triangular design as shown in both Figure 4-1 and Figure 4-2 was also adopted by Wood (2007), to develop his SCM MAV. In this section a discussion on the geometry of the wing, morphing configuration and actuation of morphing are held.
4.3.1 Wing geometry

The range of the wingspan for the vehicle of focus is on the order of magnitude of 25.4 to 127 mm (1 to 5 inches). This is the size of insects like the housefly or large locust. At this scale, flapping wing becomes essential to generate lift. Figure 4-3 shows the left side of the wing to be model. The dimensions used for this model are also summarized in Table 4-1.

![Figure 4-3 FW-MAV geometry](image-url)


<table>
<thead>
<tr>
<th>Wing Part</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front</td>
<td>52.5</td>
</tr>
<tr>
<td>Back</td>
<td>56.5</td>
</tr>
<tr>
<td>Inboard</td>
<td>3.1</td>
</tr>
<tr>
<td>Outboard</td>
<td>25</td>
</tr>
<tr>
<td>Depth</td>
<td>0.005</td>
</tr>
<tr>
<td>$\beta$</td>
<td>112$^\circ$</td>
</tr>
</tbody>
</table>

4.3.2 Wing morphing configuration

Three modes of morphing configuration is envisioned for the wing and they include:

i) **Stowage mode**: The stowage mode is the storage configuration of the MAV while not in use. The primary purpose of the stowage mode is to prevent damage of the wing while not in flight. With the capability to shrink the wing to a stowage mode, it becomes convenient for storage spaces. A concept model for the stowage mode is shown in Figure 4-4.
Figure 4-4 Conceptual model for FW-MAV in stowage mode

ii) **Flap mode**: Lift is generated from flapping the wings. Typically, this will be the primary configuration of the FW-MAV while in flight. Flapping the wings at the natural frequency of the MAV will improve performance. At the right flap mode setting, there will be an ability to generate quick lift off and complex flight maneuvers. A flap configuration concept is shown in Figure 4-5.
iii) **Glide mode**: In nature insects can undergo passive deformation in order to adjust to the wind streamlines while in flight. Due to the scale size, insects operate in low Reynolds number, thus they see huge impact on the flight performance when there is a change in the wind streamlines causing drag or draft. By adjusting to glide mode, FW-MAVs’ performance can improve and energy can be save from glide through the air. A flap configuration concept is shown in Figure 4-6.
4.3.3 Wing morphing actuation

The actuation of the shape memory wing must be activated while in flight or on ground. This section discusses a method in which a self-induced morphing of the SMP wing can be actuated. The design of self-induced actuation of the SMP wing is done by the assembly of two unique structures—SMP spar and SMP skin as shown in Figure 4-7. As shown in this figure the SMP spar outlines the edge of the wing and the skin makes up the interior panel of the wing. The SMP spars are shape memory bar rods that have the capacity to elongate and contract when heat is applied. The expected behavior of the spar is shown in Figure 4-8 with a temporary shape for the spar at a compressed state which is achieved by a compressive load and upon application of heat the spar is able to extend back to its permanent position. Likewise, the SMP skin
design shown in Figure 4-9 shows the temporary shape of the skin at an expanded state and upon application of heat the permanent shape is achieved.

Figure 4-7 Shape memory wing design for actuation with smp spar and skin
Figure 4-8 Design for the SMP spar
The primary purpose of the spar is to provide the tensile loading to the skin in order to achieve morphing which expands the surface of the wing. This actuation loading is achieved by shape mismatch of the spar and skin. It is also achieved by differential heating and cooling of the spar and skin. As shown in Figure 4-10, the SMP spar in its temporary shape state is attached to the SMP skin which is in its permanent shape state. In order to transition from a stowage mode to flap or glide mode, the wing is heated up beyond the Tg of skin and as the wing spar is heated up it returns to its permanent shape applying a tensile loading onto the skin which in turns expands. The morphed shape can be set by first cooling down the spar before the skin. This is

**Figure 4-9** Design for SMP skin
required in order for the spar to become stiff and withstand the skin from contracting as it cools down. Similarly the morphing process can be reversed - going from glide mode to flap or stowage mode – by heating up the wing, and then cooling the wing first before the spar. The steps for actuation have been summarized in the bulleted points below:

Wing Expansion

1) Start by heating up the skin (to reduce the elastic modulus)
2) Continue by heating up the spar (as the spar is heated up it expands)
3) Once expansion is achieved, cool down the spar while skin still remains heated
4) Cool down the skin
Wing Contraction

1) Start by heating up the spar
2) Continue by heating up the skin (as the skin is heated up it will contract)
3) Once contraction is achieved, cool down the skin, while spar remains heated
4) Cool down the spar

Figure 4-10 Actuation of SMP Wing by shape mismatch of spar and skin
4.4 Numerical analysis for morphing wing of FW-MAV

A numerical analysis for the morphing of a FW-MAV’s wing based on the proposed morphing technique is the previous sections above have been conducted. This analysis uses the numerical constitutive model developed in section 2 to model both the spar and the skin of the wing. Detailed numerical analysis for the morphing wing is discussed in the sections to follow. This analysis was carried out using ABAQUS. Same element properties used in section 3 have also been used in this analysis. The numerical codes generated for the individual analysis have been presented in Appendix D.

4.4.1 Analysis of the SMP spar

The SMP spar proposed for the wing makes use of the SMP properties shown in Table 3-2. To check how the spar would respond to the numerical model a Thermo-mechanical four step shape memorization and release is conducted. These four steps are the same as what was reported in section 3 and the results are presented in Figure 4-11 through 4-15. These figures show the last step frame of Mises, Temperature and Strain viewports. The strain presented is in the x-direction. The spar has been fixed on one end and loading applied to the other end. Figure 4-11 shows the initial condition of the spar, which starting out at a relaxed zero stress and strain condition at high temperature of 348 K. A compressive pressure load of 7MPa was applied to the spar in step 1 while still keeping the temperature high as shown in Figure 4-12. The loading was selected appropriately in order to notice large enough deformation that will be mimic a flap or glide mode. In step 2 the deformation of the spar is held in place as its
temperature drops to 308K. Shown in Figure 4-13, the middle viewport shows the temperature drop to 308K while the Mises viewport stills shows the presence of a pressure loading. The model predicts a high reactive loading at the fixed end of the spar. Step 3, shown in Figure 4-14, is where shape memorization has occurred. The compressive shape from the deformation has been memorized as shown with the Mises stress dropping down to zero due to lack of no pressure load. The strain viewport shows that the memorized shape was memorized at 22% strain level. Ideally for the wing actuation, this spar with the memorized shape will now be taken and used as part of the structure for the SMP wing. In order to fully check the functionality of the model, a strain release step is added as shown in Figure 4-15. The spar is heated back up to high temperatures and memorized shape is released as the strain energy stored up inside the spar is being released.

Figure 4-11 Initial condition of SMP spar
Figure 4-12 Sep 1 - loading condition of SMP spar

Figure 4-13 Step 2 - constraint condition of SMP spar
Figure 4-14 Step 3 - shape memorization of SMP spar

Figure 4-15 Step 4 - strain release of SMP spar
4.4.2 Analysis of the SMP skin

Similar analysis carried out for the spar above is conducted for the skin of the wing. Instead of applying a compressive loading in this case a tensile pressure load is applied to the skin front edge. The skin has been fixed along the back edge (see Figure 4-3). The results of this analysis are presented in Figure 4-16 through 4-20. The material properties used were the same used in Table 3-2. The initial condition shown in Figure 4-16 is the same as in section 4.4.1. In Figure 4-17, a tensile pressure load of 7MPa, deforming the skin into a flap or glide pattern. Again in step 2, the temperature is dropped to 308K as shown in Figure 4-18, while the stresses and strains relatively the same. In step 3, the shape memorization has occurred and the constraint can be released as shown in Figure 4-19. The deformation at the end of steps 2 and 3 stayed relatively the same at 43%. Finally, in Figure 4-20, after heating up the skin, the strain stored up is released.

Both the spar and skin individual responded as expected to the numerical constitutive model for shape memorization and release of SMP. The next step is to assemble these two structures together to form the SMP wing and investigate if the wing has that capability to self-induced actuation under the right thermal stimulus.
Figure 4-16 Initial condition for SMP skin

Figure 4-17 Step 1 - loading condition for SMP skin
**Figure 4-18** Step 2 - constraint condition for SMP skin

**Figure 4-19** Step 3 - shape memorization for SMP skin
4.4.3 Analysis of the SMP wing

Conducting a full analysis of the SMP wing requires that the skin and the spar be conjoined as shown in Figure 4-7. To generate self-induced actuation of the wing, a pre-deformed spar must be attached to the skin. In this analysis a full self-induced actuating wing could not be analyzed due to computational memory limitation with the ABAQUS and Linux tools used; so a different analysis approach is considered. In order to validate the concept of self-induced actuation, the SMP spar has been conjoined to a polymer skin with similar materials as a SMP skin at high and low temperatures. The wing is then made to undergo compressive and tensile load in order to investigate if the SMP spar can withstand the pulling and pushing force of the polymer skin during shape memorization and shape release. This same analysis approach is then repeated on a SMP skin conjoined with a polymer spar. Similar material properties have been used as...
shown in Table 3-2 with the exception of the Young’s modulus. The Young’s modulus material property used for SMP wing is given in Table 4-2.

<table>
<thead>
<tr>
<th>Temp [K]</th>
<th>Skin Modulus [MPa]</th>
<th>Spar Modulus [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>308</td>
<td>300</td>
<td>907</td>
</tr>
<tr>
<td>348</td>
<td>27.6</td>
<td>27.6</td>
</tr>
</tbody>
</table>

**4.4.3.1 SMP wing under compressive loading with polymer skin and SMP spar**

Figure 4-21 through Figure 4-24 show the results of the SMP wing under compressive pressure load with the spar material made up of SMP. The compressive loading mimics the wing contracting from glide to stowage mode. The wing has been fixed on the inboard edge and also allowed to move along the y-axis on the back edge. A resultant pressure load of 7MPa has been the outboard and front edge of the wing. The first elements at the back, front and outboard edges (see Figure 4-3) have the SMP spar properties, while the interior elements have the polymer properties. In step 1, the compressive load is applied to the wing at high temperature of 348K as shown in Figure 4-21. In Figure 4-22, the constraint is applied to the outboard edge. The spar temperature is reduced while the polymer temperature remains at 348K. The Mises stress shows that the spar is carrying the entire load at the inboard edge of the skin. Further evidence that the spar is carrying the load is shown in step 3 as shown in Figure 4-23, where the shape has been memorized. In this figure the constraint has been removed but the spar is holding the skin in place due to shape fixity that has occurred within it. The Mises viewport shows a high loading on the SMP spar which should be
zero but because of the push load from the polymer, the spar remains loaded. This partly validates the capacity of the SMP spar to partly withstand the push load of the skin during a self-induced actuation of the wing. A pull load from the skin will also be validated in the next section. Figure 4-24 shows that the SMP wing returns back its original shape as the wing is being heated up. There are still some residual stresses left within the spar. This residual stress is again caused by the interaction of a polymer skin and SMP spar, where the residual strains on the skin results in some residual loading on the spar.

Figure 4-21 Step 1 - compressive loading on SMP wing with SMP spar
Figure 4-22 Step 2 - compressive constraint and cooling of SMP wing with SMP spar

Figure 4-23 Step 3 - compressive shape memorization for SMP wing with SMP spar
4.4.3.2 SMP wing under tensile loading with polymer skin and SMP spar

The same analysis done above is repeated with reverse pressure loading on the edges of the wing. A tensile resultant pressure load of 8MPa is applied to outboard and front edge of the wing (see Figure 4-3). Figure 4-25 through Figure 4-28 show the results of the spar. These results show similar shape memorization and release displayed during the compressive loading. Figure 4-27 shows the ability of the SMP spar to withstand a pull load from the polymer skin after it has been frozen and memorized the deformed shape. The influence of the pull load is seen in the Mises viewport which shows higher stresses on the sides of the wing and relatively zero stress on the interior of the skin. The strain viewport shows that majority of the skin are under compression after the load is removed. This shows the relaxation of the polymer skin with no loading applied to it. At the stage in actuation of a SMP wing, the skin would be frozen to achieve shape memorization as well, which should in turn alleviate the loading...
on the spar. Figure 4-28 shows residual loading on the SMP spar after being heated which again is caused by interaction between the polymer skin and SMP spar.

Figure 4-25 Step 1 - tensile loading on SMP wing with SMP spar

Figure 4-26 Step 2 - tensile constraint and cooling of SMP wing with SMP spar
Figure 4-27 Step 3 - tensile shape memorization for SMP wing with SMP spar

Figure 4-28 Step 4 - tensile shape release of SMP wing with SMP spar
4.4.3.3 SMP wing under compressive loading with SMP skin and polymer spar

In order to further validate the ability of a co-joined SMP spar and skin to generate self-induced actuation, the SMP wing analysis shown in the above was reversed and a SMP skin is co-joined to a polymer spar. The material properties and loading conditions have been kept the same has the two previous analysis. In this section the SMP skin and the polymer spar undergo a compressive loading. Figure 4-29 through Figure 4-32 shows the results of the analysis conducted in this section. Similar steps as the previous analysis were followed. Comparing the result of this analysis to section 4.4.3.1, it is seen that step 1 predicts similar magnitude of deformation shown in Figure 4-29. In Figure 4-30, the polymer spar is kept at high temperature while the SMP skin is cooled during the constraint step. The spar still acts as the load carrier in the model around the inboard edge as shown in the Mises viewport. Comparing Figure 4-31 to Figure 4-23 it can be clearly seen that the SMP skin in Figure 4-31 is frozen and does not required the spar to hold in place as required in Figure 4-23. The bulge noted in Figure 4-23 is due to the fact that a polymer skin is used which does not have the properties of shape memorization. Figure 4-32 shows that the wing returns to its primary shape after reheating, which shows that the SMP skin is capable of actuating the spar in order to return to its permanent shape.
Figure 4-29 Step 1 - compressive loading on SMP wing with SMP skin

Figure 4-30 Step 2 - compressive constraint and cooling of SMP wing with SMP skin
Figure 4-31 Step 3 - compressive shape memorization for SMP wing with SMP skin

Figure 4-32 Step 4 - compressive shape release of SMP wing with SMP skin
4.4.3.4 SMP Wing under tensile loading with SMP Skin and Polymer Spar

The final analysis conducted is to apply a tensile pressure load to the SMP wing made from SMP skin and polymer spar and study its response for shape memorization and release. This analysis was conducted in similar fashion as in section 4.4.3.2, and the results are presented in Figure 4-33 through Figure 4-36. Steps one and two show in Figure 4-33 and Figure 4-34 maintain similar deformation as expected with Figure 4-25 and Figure 4-26. In step 3 the SMP skin is frozen and exerts zero pulling loads on the spar as shown in Figure 4-35. Finally, upon application of heat the wing returns back to its initial state with some residual strain and stress incurred.

**Figure 4-33** Step 1 - tensile loading on SMP wing with SMP skin
Figure 4-34 Step 2 - tensile constraint and cooling of SMP wing with SMP skin

Figure 4-35 Step 3 - tensile shape memorization for SMP wing with SMP skin
Sections 4.4.3.1 through 4.4.3.4 have been used to validate the possibility of self-induced actuation of the SMP wing if a co-joined SMP spar and SMP skin is used to build the wing. Although, analyzing a complete SMP wing was not possible due to computation resource limitation, the envelope analysis conducted considers all the probable scenarios whereby the SMP skin and SMP spar would be required to induce actuation. Ultimately, the key lies in having a tailored material property for both the spar and the skin. At high temperatures the spar and skin material properties, in particular the stiffness must be about the same in order to over the inertial exerted respective structural components.
5 CONCLUSION AND FUTURE WORK

5.1 Contributions

In order to conclude this dissertation, the key points that have presented in previous sessions are summarized and a future research direction pertaining to this work is provided.

A. One-dimensional shape memory polymer model

A one-dimensional viscoelastic model was presented in this work that makes use of a binding factor model to account for the material property variation over a temperature range. The derivation of this model was further developed from the energy balance method. The generation of this model was largely based on experimental work, with material properties measured around the glass transition temperature. The experimental work was unidirectional loading dependent. The implication of this is that the model may be load type dependent and an improved constitutive model was can consider taking multidirectional loading into consideration. Regardless, the one-dimensional model developed provides a realistic means to predict the behavior of a SMP under thermo-mechanical loading.

B. Three-dimensional shape memory polymer model.

A novel technique has been presented in this dissertation for transitioning a one-dimensional viscoelastic model into a three-dimensional isotropic viscoelastic model. This method presented in section 2.3 shows the application of the compliance and stiffness tensors for an isotropic material in a modified Zener-Maxwell model. There is
an opportunity to develop a generalized material system using similar technique presented in this section.

**C. Numerical model method**

A significant aspect of this dissertation is focused on transitioning the three-dimensional model into a numerical model that can be used in a finite element code. The technique developed in this work presented in section 2.4 was published in the author’s previous work (Balogun & Mo, 2014c). This technique has been extended by developing a UMAT subroutine code in ABAQUS and made available in Appendix B.

**D. Application of shape memory polymer to flapping wing micro air vehicles**

Shape morphing occurs rampantly in nature, noticed in the wings of insects, birds and bats. The proposition to use SMP as wings for flying vehicles that are of the order of the size of insects is novel. The techniques proposed in this dissertation are novel and promising for self-induced actuation of this wing. The numerical constitutive model developed in this dissertation have been used to investigate the morphing of the SMP wing in both directions – opening and closing of the wing. The investigation shows a promising response of co-joined shape memory spar and skin that will allowed for self-induced morphing under the right stimulus and tailored material properties. It was noted that tailored material properties is essential at high temperatures where the trigger for actuation occurs. Thus having the Young’s modulus of the skin and spar material equal each other at high temperature is a primary key for morphing.
5.2 Directions for future work

Further independent experimental data can be collected to continue the validation of the model developed in this dissertation. This experimental data will continue to validate the physical meaning of binding factor and its application to the material properties of the SMPs.

Additional studies are required in order to optimize the design of SMP wing for FW-MAV. A study into the inclusion of SMP veins within the SMP skin can be carried out to see its overall influence on the wing. Placement and number of veins are some of the studies that need to be completed. Additionally, an innovative way of selectively heating and cooling different structures within the wing must be developed. One possible way is the use of heating filament within the structures which are controlled by microelectronics within the vehicle. The last part that must be investigated is the influence of flight loads on the SMP as it morphs from flapping mode to gliding mode. Of course the structure has to be built and tested to validate the results generated by the numerical constitutive model.
6 Bibliography


Tao, X., 2011. Recent advances in Shape Memory Polymer. Polymer, Volume 52, pp. 4985-5000.


Vernon, L. B. & Vernon, H. M., 1941. Producing Molded Articles such as Dentures from Thermoplastic Synthetic Resins. United States of America, Patent No. 2234993.


APPENDIX

A. One-dimensional model validation octave code

```octave
%##balogun_m_smp.m

%%%% Work on Shape memory polymer Shape fixity - New Balogun-Mo SMP Model
%%%% This new model takes into account the internal storage strain, binding
factor z
%%%% and the thermal Strains. Model is given as
%%%% to run type startxwin, close new window and type octave and type
balogun_m_smp
%%%%================================================================================
%%%% edot = sigmadot/E(T)+ sigma/u(T)-e/L(T)+ethdot+esdot
%%%%================================================================================
%%%% where e - strain, sigma - stress, eth - thermal strain, es - storage
strain
%%%% E - Elastic modulus , u - viscous modulus, L - retardation time, T - Temp
%%%% e = em+es+eth; es = Km*z*em; eth = alpha*DT
%%%% z = (1-(1/(1+exp(a*(Tg-T-0.5*DT_hl)))));
%%%% where a = |Ln(uh/ul)|^1; Tg - glass Temp,
%%%% DT_hl - temp diff btw high n low material props
%%%% Km = -1 if Tdot > 0, Km = 1 if Tdot < 0, Km = 0 if Tdot = 0,
%%%% Rheological Model schematics -
%%%% ______
%%%%    |
%%%%    |
%%%% /\|
%%%% /|
%%%% | em
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% |
%%%% u
%%%% Similar work done by Tobushi requires arduous material parameters
%%%% Reference Paper Analysis
%%%% Tobushi H, Hashimoto T, Hayashi S and Yamada E 1997
%%%% Thermo-mechanical constitutive modeling in shape memory
%%%% polymer of polyurethane series J. Intell. Mater. Syst. Struct.8 711-8
%###+--------------------------------------------------------------------------------------------------------------------------+
%%%% define strain rate in step 1 smp exp. and time interval
%%%% linspace give is beginning and the end and split it into 50
%%%% In paper edot varied from 0.05-0.5
ei=0.24;
edot = 0.5;
t=linspace(0,ei/edot,50)';

%%%% This function will compute sdot for step 1
%%%% Temp is constant so properties are constant
%%%% I divided by 60 to take units to mins (see Tobushi paper)
%%%% I also assume a linear strain function
%%%% No storage strain yet in step 1
function sdot = f (s, t,edot)
    E = 27.6e6; u = 2.03e9/60; L=111/60; Tdot = 0;
```
\[ e = 0.0 + \dot{e} t; \]
\[ s = E (\dot{e} t - 11.6 + 5 \dot{T} t + (e/L) - s/u); \]

endfunction;

f = @(s, t) f(s, t, edot);

s1 = lsode(f, 0, t);

el = 0.0 + edot * t;
T1 = ones(length(s1), 1) * 348;

function dsdT = f(s, T, Tdot, em, fid)
    To = 348;
    Tg = 328; Eg = 146e6; ug = 14e9 / 60.0; Lg = 521 / 60;
    Eh = 27.6e6; El = 907e6;
    uh = 2.03e9 / 60; ul = 116e9 / 60;
    Lh = 111 / 60; Ll = 2840 / 60;
    deltaT = 20;
    a = abs(log(uh / ul)) ^ (-1);
    z = (1 - (1 / (1 + exp(a * (Tg - T - 0.5 * deltaT)))));
    E = Eh + (El - Eh) * z;
    u = uh + (ul - uh) * z;
$L = \text{Lh} + (\text{Ll} - \text{Lh}) \cdot z;$

$\text{dinvEdT} = - (\text{a} \cdot (\text{Eh} - \text{El}) \cdot \exp(\text{a} \cdot (\text{Tg} - \text{T})))/(\text{El} \cdot \exp(\text{a} \cdot (\text{Tg} - \text{T})) + \text{Eh})^2;$

$\text{esdot} = -((\text{a} \cdot \text{em} \cdot \exp(\text{a} \cdot (\text{Tg} - \text{T}))) / (\exp(\text{a} \cdot \text{Tg}) + \exp(\text{a} \cdot \text{T}))^2) \cdot \text{Tdot};$

$\text{dsdT} = \text{E} \cdot (((-\text{esdot} + ((1+z) \cdot \text{em} + 11.6e^{-5} \cdot (\text{T} - \text{To})) / (\text{L} - s/u) / (\text{Tdot})) - s \cdot \text{dinvEdT});$

"print data out"

fputs(fid, [num2str(T) "," num2str(z) "," num2str(E) "," num2str(u) "," num2str(L) "," num2str(dinvEdT) "," num2str(dsdt) "," num2str(s) "]);

endfunction;

f = @(s, T) f(s, T, Tdot, e1(end), fid);

s2 = lsode(f, s1(end), T2);

e2 = ones(length(s1), 1) * e1(length(e1));

T2 = [T1; T2];
s2 = [s1; s2];
fclose (fid);

"figure (2);

clf;

subplot(2, 2, 1)

plot (e2, s2);

grid on;

subplot(2, 2, 2)

plot (T2, s2);

grid minor;

subplot(2, 1, 2)

plot (T2, e2);

grid on;

for i = 1:length(s2)

fputs(fid2, [num2str(T2(i)) "," num2str(s2(i)) "," num2str(e2(i)) "\n"]);

endfor

fclose (fid2);

"In step 3 the stress is zero thus simplifying the DE"

"Isothermal condition"

"I define a new function as edot"

function edot = g (e, t)

T=308;

Tg = 328; Eg = 146e6; ug = 14e9/60.0; Lg =521/60;

uh = 2.03e9/60; ul = 116e9/60;

Lh = 111/60; Ll = 2840/60;

deltaT = 20;

a = abs(log(uh/ul))^-1;

z = (1-(1/(1+exp(a*(Tg-T-0.5*deltaT)))));

L = Lh+(Ll-Lh)*z;

edot = -(e)/(L);

endfunction;
\[ g = @(e, t) g(e, t); \]
\[ e3 = \text{lsode}(g, e1, t); \]

\[ T3 = \text{ones(length}(e3), 1) \ast 308; \]
\[ s3 = \text{ones(length}(e3), 1) \ast 0.0; \]

\[ e3 = [e2; e3]; \]
\[ T3 = [T2; T3]; \]
\[ s3 = [s2; s3]; \]

\[ %\# \text{figure (3)}; \]
\[ %\# \text{subplot}(2, 2, 1) \]
\[ %\# \text{plot}(e3, s3); \]
\[ %\# \text{subplot}(2, 2, 2) \]
\[ %\# \text{plot}(T3, s3); \]
\[ %\# \text{subplot}(2, 1, 2) \]
\[ %\# \text{plot}(T3, e3); \]

\[ \text{subplot(2,1,2)} \]
\[ \text{plot(T3,e3);} \]

\[ \text{g} = @(e, T) \text{g}(e, T) ; \]
\[ e4 = \text{lsode}(g, e3(\text{end}), T4); \]

\[ s4 = \text{ones(length}(e4), 1) \ast 0.0; \]

\[ e4 = [e3; e4]; \]
\[ T4 = [T3; T4]; \]
\[ s4 = [s3; s4]; \]

\[ \text{figure (4);} \]
\[ \text{subplot}(2, 2, 1) \]
\[ \text{plot}(e4.*100, s4./1e6); \]
\[ \text{xlabel(“Strain [%]”)} \]
ylabel("Stress [MPa]")
grid on;
subplot(2,2,2)
    plot (T4,s4./1e6);
    xlabel("Temp [K]"
    #ylabel("Stress [MPa]"
    grid on;
subplot(2,1,2)
    plot (T4,e4.*100);
    xlabel("Temp [K]"
    ylabel("Strain [%]"
    grid on;
B. UMAT code for time discrete numerical model

The algorithm below shows how the three-dimensional time discrete model shown in section 2.4. Two algorithms (balmo2b.f and balmo8.f) have been written using two different method and both provide similar result in the nominal direction.

```fortran
~~~ balmo8.f
~~
SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1  RPL,DDSDDT,DRPLDE,DRPLDT,
2  STRAN,DSTRAIN,TIME,DTIME,TEMP,DTTEMP,PREDEF,DPRED,CMNAME,
3  NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4  CELN,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
C This is using the isotropic material.
 INCLUDE 'ABA_PARAM.INC'
C
C CHARACTER*80 CMNAME
DIMENSION STRESS(NTENS),STATEV(NSTATV),
1  DDSDDE(NTENS,NTENS),
2  DDSDDT(NTENS),DRPLDE(NTENS),
3  STRAN(NTENS),DSTRAIN(NTENS),TIME(2),PREDEF(1),DPRED(1),
4  PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)
C
DIMENSION DDSDE(NTENS,NTENS),DDSDES(NTENS,NTENS),
DIMENSION DDSDS(NTENS,NTENS),DSDSES(NTENS,NTENS),
DIMENSION DSTR(6),DSTRES(6),ETHERM(6),DTERM(6),ESTR(6),
1  DSTR(6),EELAS(6),PLOD(6),KTM(6),SEGold(6)
REAL Em,poi,Km,LR,Gm,ul,Gu,A,B,C1,C2,D0,DLQ
REAL Em0,poi0,Km0,LR0,Gm0,ul0,Gu0,A0,B0,C10,C20,D0,D00,Q0
REAL lambda1,mu1,lambda2,mu2,nubar,theta
REAL dellambda1,delmu1,dellambda2,delmu2,delnubar,deltheta
REAL Eh,E1,uh,ul,Lh,Ll,alpha,Tg,Tini,deltaTh1,au,Kk
REAL Ehe,Ele,Ehm,Elm,Z0,Ee0,Ke0,Ge0,Z1,Ee,Ke,Ge,If
SAVE
C
C Input properites
Eh = PROPS(1)
E1 = PROPS(2)
uh = PROPS(3)
u1 = PROPS(4)
Lh = PROPS(5)
L1 = PROPS(6)
pioh = PROPS(7)
poil = PROPS(8)
alpha = PROPS(9)
Tg = PROPS(10)
Tini = PROPS(11)
deltaTh1 = PROPS(12)
If = PROPS(13)
au = 1./ABS(log(uh/u1))
C
C Properties at start of increments
```
\[ Z_0 = 1. - \frac{1}{1. + \exp(au \cdot (Tg - TEMP - 0.5 \cdot \delta Th))} \]

\[ Em0 = (1. - Z0) \cdot Ehm + Z0 \cdot Elm \]

\[ Ee0 = (1. - Z0) \cdot Ehe + Z0 \cdot Ele \]

\[ poi0 = poih + (poil - poih) \cdot Z0 \]

\[ Km0 = Em0 / (3. \cdot (1. - 2. \cdot poi0)) \]

\[ Gm0 = Em0 / (2. \cdot (1 + poi0)) \]

\[ Ke0 = Ee0 / (3. \cdot (1 - 2. \cdot poi0)) \]

\[ Ge0 = Ee0 / (2. \cdot (1 + poi0)) \]

\[ LR0 = Lh + (Ll - Lh) \cdot Z0 \]

\[ Gu0 = uh + (ul - uh) \cdot Z0 \]

\[ D20 = (1.0 + Ge0 / Gm0) \]

\[ C20 = (Ke0 / Km0 - Ge0 / Gm0) / 3. \]

\[ D10 = 1. / LR0 \]

\[ C10 = -1. / (3. \cdot LR0) \]

\[ Q0 = alpha \cdot (1.0 + Ke0 / Km0) \]

\[ Z1 = 1. - \frac{1}{1. + \exp(au \cdot (Tg - (TEMP + DTEMP) - 0.5 \cdot \delta Th))} \]

\[ Em = (1. - Z1) \cdot Ehm + Z1 \cdot Elm \]

\[ Ee = (1. - Z1) \cdot Ehe + Z1 \cdot Ele \]

\[ poi = poih + (poil - poih) \cdot Z1 \]

\[ Km = Em / (3. \cdot (1. - 2. \cdot poi)) \]

\[ Gm = Em / (2. \cdot (1 + poi)) \]

\[ Ke = Ee / (3. \cdot (1 - 2. \cdot poi)) \]

\[ Ge = Ee / (2. \cdot (1 + poi)) \]

\[ LR = Lh + (Ll - Lh) \cdot Z1 \]

\[ Gu = uh + (ul - uh) \cdot Z1 \]

\[ D2 = (1.0 + Ge / Gm) \]

\[ C2 = (Ke / Km - Ge / Gm) / 3. \]

\[ D1 = 1. / LR \]

\[ C1 = -1. / (3. \cdot LR) \]

\[ Q = alpha \cdot (1.0 + Ke / Km) \]

\[ write (*,*) , Em, Gu, poi, Gm0, Km0, Tg, TEMP, DTEMP \]
End IF
DEVS = 0.
DO K1=1,NDI
   ETHERM(K1)=alpha*(TEMP-Tini)
   DTHERM(K1)=alpha*DTMP
   ESTR(K1)=Z1*SEGold(K1)*Kk
   DSTR(K1)=(Z1-Z0)*SEGold(K1)*Kk
   DEVS = DEVS + (DSTR(K1))
END DO
DO K1=NDI+1,NTENS
   ETHERM(K1)=0.0
   DTHERM(K1)=0.0
   ESTR(K1)=Z1*SEGold(K1)*Kk
   DSTR(K1)=(Z1-Z0)*SEGold(K1)*Kk
END DO
C
C EVALUATE NEW STRESS TENSOR
EV = 0.
DEV = 0.
DO K1=1,NDI
   EV = EV + (STRAN(K1))
   DEV = DEV + (DSTRAN(K1))
END DO
C CREATE NEW JACOBIAN
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
A = ((1./(9.*Km)) - (1./(6.*Gm)))
A0 = ((1./(9.*Km0)) - (1./(6.*Gm0)))
B = (0. - 1./(6.*Gu))
B0 = (0. - 1./(6.*Gu0))
lambda1 = C10/B0
lambda2 = C20/B0
nu = A0/B0
del1 = (1./B) - lambda1
delmu1 = D1/(2.*B) - mu1
del12 = (1./B) - lambda2
delmu2 = D2/(2.*B) - mu2
delnu = (A/B) - (A0/B0)
theta = Q0/B0
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
TERM1 = .5*DTIME + nubar+0.5*delnu
TERM1 = 1./TERM1
TERM3 = (.5*DTIME*(lambda1+.5*del1)+lambda2
 1 +.5*del12)*TERM1
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
DO K1=1,NDI
   DO K2=1,NDI
      DDSDE(K2,K1) = TERM3
   END DO
C WRITE (T*, TERM3
   DDSDE(K2,K1) = DTIME*(lambda1+.5*del1)
   DDSDE(K2,K1) = 1.5*TERM1*
   DDSDE(K2,K1) = 0.
DDSDDES(K2,K1) = (lambda2+.5*dellambda2)*TERM1I
END DO
END DO
DO K1=1,NDI
  DDSDDE(K1,1) = TERM3*100.*ABS(1f)
  DDSDDE(K1,2) = TERM3*100.
  DDSDDE(K1,3) = TERM3*100.
END DO
c

CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC
C Define new B Property
A = ((1./(9.*Km)) - (1./(6.*Gm)) + (1.0/(2.0*Gm)))
A0 = ((1./(9.*Km0)) - (1./(6.*Gm0)) + (1.0/(2.0*Gm0)))
B = (0. - 1./(6.*Gu)+(1./(2.0*Gu)))
B0 = (0. - 1./(6.*Gu0)+(1./(2.0*Gu0)))
lambda1 = C10/B0
mu1 = D10/(2.*B0)
lambda2 = C20/B0
mu2 = D20/(2.*B0)
nubar = A0/B0
dellambda1 = C1/B - lambda1
delmul1 = D1/(2.*B)- mu1
dellambda2 = C2/B- lambda2
delmul2 = D2/(2.*B)- mu2
delnubar = (A/B) - (A0/B0)
theta = Q0/B0
delttheta = Q/B - theta

CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC
TERM1 = .5*DTIME+nubar+0.5*delnubar
TERM1I = 1./TERM1
TERM2 = (.5*DTIME*(((lambda1+.5*dellambda1)+2.*(mu1+.5*delmul1))
1   +lambda2+.5*dellambda2+2.* (mu2+.5*delmul2))
2   *TERM1I

CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC
DO K1=1,NDI
  DDSDDE(K1,K1) = TERM2
  DDSDDT(K1) = -(theta+0.5*deltheta)*TERM1I
  DDSDE(K1,K1) = DTIME*(((lambda1+.5*dellambda1)
1   +2.*(mu1+.5*delmul1))*TERM1I
  DDSDS(K1,K1) = -TERM1I
  DDSDES(K1,K1) = -1.*(mu2+.5*delmul2)*TERM1I
  DDSDDES(K1,K1) = -(lambda2+.5*dellambda2)*TERM1I
END DO
c

ccc

load direction effect on thermal part
IF ( DTEMP .GT. 0.0) THEN
  DDSDDT(1) = (theta+0.5*deltheta)*TERM1I*3.0*(1f/ABS(1f))
  DDSDDT(2) = (theta+0.5*deltheta)*TERM1I*0.33
  DDSDDT(3) = (theta+0.5*deltheta)*TERM1I*0.33
END IF

CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC CCC
A = ((1./(2.*Gm)))
A0 = ((1./(2.*Gm0)))
B = (1./(2.*Gu))
B0 = (1./(2.*Gu0))
lambda1 = C10/B0
\[
\begin{align*}
\mu_1 &= \frac{D_{10}}{B_0} \\
\lambda_2 &= \frac{C_{20}}{B_0} \\
\mu_2 &= \frac{D_{20}}{B_0} \\
nubar &= \frac{A_0}{B_0} \\
del\lambda_1 &= \frac{C_{1}}{B} - \lambda_1 \\
del\mu_1 &= \frac{D_{1}}{2 \times B} - \mu_1 \\
del\lambda_2 &= \frac{C_{2}}{B} - \lambda_2 \\
del\mu_2 &= \frac{D_{2}}{2 \times B} - \mu_2 \\
delnubar &= \left(\frac{A}{B}\right) - \left(\frac{A_0}{B_0}\right) \\
\theta &= \frac{Q_{0}}{B_0} \\
del\theta &= \frac{Q}{B} - \theta
\end{align*}
\]

\[
\begin{align*}
\text{TERM1} &= \frac{1}{2} \times \frac{B T I M E}{B_0} + nubar + \frac{1}{2} \times \text{delnubar} \\
\text{TERM2} &= \left(\frac{1}{2} \times \frac{B T I M E}{B_0} \times (\mu_1 + \frac{1}{2} \times \text{delmu1}) + \mu_2 \right) \times \text{TERM1} \\
\text{DO} \ K_1 = \text{NDI} + 1, \text{NTENS} \\
\text{DDSDDE(K1,K1) = TERM2} \\
\text{DDSDDT(K1) = 0.0} \\
\text{DDSDE(K1,K1) = B TIME \times (\mu_1 + \frac{1}{2} \times \text{delmu1}) \times \text{TERM1}} \\
\text{DDSDS(K1,K1) = -TERM1} \\
\text{DDSDSES(K1,K1) = -1.0 \times (\mu_2 + \frac{1}{2} \times \text{delmu2}) \times \text{TERM1}} \\
\text{DDSDDES(K1,K1) = 0.0} \\
\text{END DO}
\end{align*}
\]

\[
\begin{align*}
\text{C \ UPDATE \ STRESSES} \\
\text{C \ DO} \ K_2 = 1, \text{NDI} \\
\text{DO} \ K_1 = 1, \text{NDI} \\
\text{IF} \ (\text{DTEMP} \ > \ 0.0) \ \text{THEN} \\
\text{STRESS(K2) = STRESS(K2)} \\
1 + \text{DDSDDE(K2,K1) \times DSTRAN(K1)} \\
2 + \text{DDSDE(K2,K1) \times (STRAIN(K1))} \\
3 + \text{DDSDS(K2,K1) \times STRESS(K1)} \\
4 - \text{DDSDSES(K2,K1) \times DSTR(K1)} \\
5 - \text{DDSDDES(K2,K1) \times ESTR(K1) \times 0.0} \\
6 + \text{DDSDDT(K2) \times DTEMP \times ABS(lf \times 0.20)} \\
\text{ELSE} \\
\text{STRESS(K2) = STRESS(K2)} \\
1 + \text{DDSDDE(K2,K1) \times DSTRAN(K1)} \\
2 + \text{DDSDE(K2,K1) \times (STRAIN(K1))} \\
3 + \text{DDSDS(K2,K1) \times STRESS(K1)} \\
4 - \text{DDSDSES(K2,K1) \times DSTR(K1)} \\
5 - \text{DDSDDES(K2,K1) \times ESTR(K1) \times 0.0} \\
6 + \text{DDSDDT(K2) \times DTEMP \times ABS(lf \times 0.80)} \\
\text{END IF} \\
\text{END DO}
\end{align*}
\]

\[
\begin{align*}
\text{STRESS(K2) = STRESS(K2) + DSTRES(K2) + DDSDDT(K2) \times DTEMP \times 0.167} \\
\text{write(*,*)} \ \text{lf}
\end{align*}
\]

\[
\begin{align*}
\text{C \ CALCULATE \ ELASTIC \ THERMAL \ AND \ CREEP \ STRAIN} \\
\text{ETHERM(K2) = ETHERM(K2) + DTHERM(K2)} \\
\text{EELAS(K2) = STRAN(K2) + DSTRAN(K2) - ETHERM(K2)} \\
\text{END DO}
\end{align*}
\]

139
c     write (*,*), KSTEP, STRESS
  DO K1=NDI+1,NTENS
    STRESS(K1) = STRESS(K1)
    1       + DDSDDE(K1,K1)*DSTRAN(K1)
    2       + DDSDE(K1,K1)*(STRAN(K1))
    3       + DDSDS(K1,K1)*STRESS(K1)
    4       - DDSDES(K1,K1)*ESTR(K1)*0.0
C CALCULATE ELASTIC THERMAL AND CREEP STRAIN
    ETHERM(K1)=ETHERM(K1)+DTHERM(K1)
    EELAS(K1)=STRAN(K1)+DSTRAN(K1)-ETHERM(K1)
  END DO
C STORE ELASTIC THERMAL CREEP STRAINS IN STATE VARIABLE ARRAY
  DO K1=1, NTENS
    STATEV(K1)=EELAS(K1)
    STATEV(K1+NTENS)=ETHERM(K1)
    STATEV(K1+2*NTENS)=ESTR(K1)
    STATEV(K1+3*NTENS)=DSTR(K1)
  END DO
RETURN
END
SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,
2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
C
INCLUDE 'ABA_PARAM.INC'
C
REAL Em,poi,Km,LR,Gm,u1,Gu,A,B,C1,C2,D1,D2,Q
REAL Em0,poi0,Km0,LR0,Gm0,u10,Gu0,A0,B0,C10,C20,D10,D20,Q0
REAL lambda1,mu1,
lambda2,mu2,nubar, theta
REAL dellambda1,delmu1,dellambda2,delmu2,delnubar, deltheta
REAL Eh,El,uh,u1,Lh,Ll,alpha,Tg,Tini,deltaThl,au,Kk
REAL Ehe,Ele,Ehm,Elm,Z0,Ee0,Ke0,Ge0,21,Ee,Ke,Ge,1f
SAVE
C
C Input properites
Eh = PROPS(1)
El = PROPS(2)
uh = PROPS(3)
u1 = PROPS(4)
Lh = PROPS(5)
Ll = PROPS(6)
poih = PROPS(7)
poil = PROPS(8)
alpha = PROPS(9)
Tg = PROPS(10)
Tini = PROPS(11)
deltaThl = PROPS(12)
1f = PROPS(13)
au = 1./ABS(log(uh/u1))
C
C Properties at start of increments
C
Ehe = uh/Lh
Ele = u1/Ll
Ehm = Eh - Ehe
Elm = El - Ele
Z0 = 1.-1./(1.+exp(au*(Tg-TEMP-0.5*deltaThl)))))
Em0 = (1.- Z0)*Ehm + Z0*Elm
Ee0 = (1.- Z0)*Ehe + Z0*Ele
poio0 = poih+(poil-poih)*Z0
Km0 = Em0/(3.*(1.-2.*poi0))
Gm0 = Em0/(2.*(1+poi0))
\[ Ke_0 = \frac{Ee_0}{3. \cdot (1 - 2 \cdot poi)} \]
\[ Ge_0 = \frac{Ee_0}{2 \cdot (1 + poi)} \]
\[ LR_0 = Lh + (Ll - Lh) \cdot Z_0 \]
\[ Gu_0 = uh + (ul - uh) \cdot Z_0 \]
\[ D2_0 = (1.0 + Ge_0/Gm_0) \]
\[ C2_0 = \frac{(Ke_0/Km_0 - Ge_0/Gm_0)}{3} \]
\[ D1_0 = \frac{1}{LR_0} \]
\[ C1_0 = -\frac{1}{3 \cdot LR_0} \]
\[ Q_0 = \alpha(1.0 + Ke_0/Km_0) \]

Properties at end of increments
\[ Z_1 = 1 - \frac{1}{1.0 + \exp(au \cdot (Tg - (TEMP + DTEMP) - 0.5 \cdot \text{deltaThl}))} \]
\[ Em = (1 - Z_1) \cdot Ehm + Z_1 \cdot Elm \]
\[ Ee = (1 - Z_1) \cdot Ehe + Z_1 \cdot Ele \]
\[ poi = poih + (poil - poih) \cdot Z_1 \]
\[ Km = \frac{Em}{3 \cdot (1 - 2 \cdot poi)} \]
\[ Gm = \frac{Em}{2 \cdot (1 + poi)} \]
\[ Ke = \frac{Ee}{3 \cdot (1 - 2 \cdot poi)} \]
\[ Ge = \frac{Ee}{2 \cdot (1 + poi)} \]
\[ LR = Lh + (Ll - Lh) \cdot Z_1 \]
\[ Gu = uh + (ul - uh) \cdot Z_1 \]
\[ D2 = (1.0 + Ge/Gm) \]
\[ C2 = \frac{(Ke/Km - Ge/Gm)}{3} \]
\[ D1 = \frac{1}{LR} \]
\[ C1 = -\frac{1}{3 \cdot LR} \]
\[ Q = \alpha(1.0 + Ke/Km) \]

CALCULATE THERMAL EXPANSION
IF (KSTEP .EQ. 2) THEN
  DO K1 = 1, NTENS
    SEGold(K1) = STRAN(K1)
  END DO
ENDIF
IF (DTEMP .LT. 0 ) THEN
  Kk = 1.
ELSE IF (DTEMP .GT. 0 ) THEN
  Kk = -1.
ELSE
  Kk = 0.
End IF
DEVS = 0.
DO K1 = 1, NDI
  ETHERM(K1) = alpha \cdot (TEMP - Tini)
  DTEMP(K1) = alpha \cdot DTEMP
  ESTR(K1) = Z1 \cdot SEGold(K1) \cdot Kk
  DSTR(K1) = (Z1 - Z0) \cdot SEGold(K1) \cdot Kk
  DEVS = DEVS + (DSTR(K1))
END DO
DO K1 = NDI + 1, NTENS
  ETHERM(K1) = 0.0
  DTEMP(K1) = 0.0
  ESTR(K1) = Z1 \cdot SEGold(K1) \cdot Kk
  DSTR(K1) = (Z1 - Z0) \cdot SEGold(K1) \cdot Kk
END DO
C
C    EVALUATE NEW STRESS TENSOR
    EV = 0.
    DEV = 0.
    DO K1=1,NDI
        EV = EV + (STRAN(K1))
        DEV = DEV + (DSTRAN(K1))
    END DO
    write (*,*), KSTEP,NOEL,KTM,PLOD,STRESS
    CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
    A = ((1./(9.*Km))-(1./(6.*Gm))+(1.0/(2.0*Gm)))
    A0 = ((1./(9.*Km0))-(1./(6.*Gm0))+(1.0/(2.0*Gm0)))
    B = (0. - 1./(6.*Gu)+(1./(2.0*Gu)))
    B0 = (0. - 1./(6.*Gu0)+(1./(2.0*Gu0)))
    lambda1 = C10/B0
    mu1 = D10/(2.*B0)
    lambda2 = C20/B0
    mu2 = D20/(2.*B0)
    nubar = A0/B0
    dellambda1 = C1/B - lambda1
    delmu1 = D1/(2.*B)- mu1
    dellambda2 = C2/B- lambda2
    delmu2 = D2/(2.*B)- mu2
    delnubar = A/B-(A0/B0)
    theta = Q0/B0
deltheta= Q/B - theta
    CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
    TERM1 = .5*DTIME + nubar + .5*delnubar
    TERM1I = 1./TERM1
    TERM2 = (.5*DTIME*(lambda1+.5*dellambda1)+lambda2
            +.5*dellambda2)*TERM1I*DEV
    TERM3 = (.5*DTIME*(mu1+.5*delmu1)+(mu2+.5*delmu2
            ))*2.0*TERM1I
    CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
    DO K1=1,NDI
        IF (DTEMP .GT. 0.0) THEN
            DSTRES(K1) = TERM2+TERM3*(DSTRAN(K1))
            1 +DTIME*TERM1I*((lambda1+.5*dellambda1)
            2 +*EV+2.*RZ0*DELDELUALD*(STRAN(K1))
            3 -STRESS(K1))+((theta+0.5*deltheta)*DTEMP
            4 -((lambda2+.5*dellambda2)*DEVS
            5 -mu2+0.5*delmu2)*DSTR(K1)*SEGold(K1))*TERM1I
            6 *1f
        ELSE
            DSTRES(K1) = TERM2+TERM3*(DSTRAN(K1))
            1 +DTIME*TERM1I*((lambda1+.5*dellambda1)
            2 +*EV+2.*RZ0*DELDELUALD*(STRAN(K1))
            3 -STRESS(K1))-((theta+0.5*deltheta)*DTEMP
            4 +((lambda2+.5*dellambda2)*DEVS
            5 +mu2+0.5*delmu2)*DSTR(K1)*SEGold(K1))*TERM1I
            6 *1f*0.33
        END IF
        STRESS(K1) = STRESS(K1) + DSTRES(K1)
        ETHERM(K1)=ETHERM(K1)+DTEMP(K1)
        EELAS(K1)=STRAIN(K1)+DSTRAN(K1)-ETHERM(K1)
END DO
C write (*,*) , KSTEP, STRESS(1), STRESS(2), STRESS(3)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
I1 = NDI
A = ((1.0/(2.0*Gm)))
A0 = ((1.0/(2.0*Gm0)))
B = (1./(2.0*Gu))
B0 = (1./(2.0*Gu0))
lambda1 = C10/B0
mu1 = D10/(B0)
lambda2 = C20/B0
mu2 = D20/(B0)
nubar = A0/B0
dellambda1 = C1/B - lambda1
delmul = D1/(B)- nu1
dellambda2 = C2/B - lambda2
delmul2 = D2/(B)- mu2
delnubar = (A/B)-(A0/B0)
theta = Q0/B0
delttheta= Q/B - theta
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
TERM1 = .5*DTIME + nubar + .5*delnubar
TERM1I = 1./TERM1
TERM2 = (.5*DTIME*(mu1+.5*delmu1) + (mu2+.5*delmu2))*TERM1I

DO K1=1,NSHR
  I1 = I1+1
  DSTRES(I1) = TERM2*(DSTRAN(I1)) +
  1   DTIME*TERM1I*((mu1+.5*delmu1)*(STRAN(I1))-STRESS(I1))
  2   -(mu2+0.5*delmu2)*DSTR(I1)*SEGold(I1)*TERM1I*1.0
  STRESS(I1) = STRESS(I1)+DSTRES(I1)
  EELAS(I1)=STRAN(I1)+DSTRAN(I1)
END DO
C write (*,*) , A,B
C
C CREATE NEW JACOBIAN
C
C TERM2 = (DTIME*(.5*lambda+mu)+lambdabar+
C  l  2.*mubar)*TERM1I
C TERM3 = (.5*DTIME*lambda+lambdabar)*TERM1I
DO K1=1,NTENS
  DO K2=1,NTENS
    DDSDDE(K2,K1) = 0.
  END DO
END DO

C Define new B Property
A = ((1./(9.*Km)) - (1./(6.*Gm)) + (1.0/(2.0*Gm)))
A0 = ((1./(9.*Km0)) - (1./(6.*Gm0)) + (1.0/(2.0*Gm0)))
B = (0. - 1./(6.*Gu)+(1./(2.0*Gu)))
B0 = (0. - 1./(6.*Gu0)+(1./(2.0*Gu0)))
lambda1 = C10/B0
mu1 = D10/(2.*B0)
lambda2 = C20/B0
mu2 = D20/(2.*B0)
nubar = A0/B0
dellambda1 = C1/B - lambda1
delmul1 = D1/(2.*B) - mul1
dellambda2 = C2/B - lambda2
delmul2 = D2/(2.*B) - mu2
delnubar = (A/B) - (A0/B0)
theta = Q0/B0
deltheta = Q/B - theta

TERM1 = .5*DTIME + nubar + 0.5*delnubar
TERM1I = 1./TERM1
TERM2 = (.5*DTIME* ((lambda1 + .5*dellambda1) + 2.*(mu1 + .5*delmu1))
1 + lambda2 + .5*dellambda2 + 2.*(mu2 + .5*delmu2))
2 *TERM1I

DO K1=1,NDI
  DDSDDDE(K1,K1) = TERM2
  DDSDDT(K1) = -(theta + 0.5*deltheta)
  IF ( DTEMP .GT. 0.0) THEN
    DDSDDT(K1) = (theta + 0.5*deltheta)
  END IF
END DO

A = ((1./(9.*Km)) - (1./(6.*Gm)))
A0 = ((1./(9.*Km0)) - (1./(6.*Gm0)))
B = (0. - 1./(6.*Gu))
B0 = (0. - 1./(6.*Gu0))
lambda1 = C10/B0
mu1 = D10/(2.*B0)
lambda2 = C20/B0
mu2 = D20/(2.*B0)
nubar = A0/B0
dellambda1 = C1/B - lambda1
delmul1 = D1/(2.*B) - mul1
dellambda2 = C2/B - lambda2
delmul2 = D2/(2.*B) - mu2
delnubar = (A/B) - (A0/B0)
theta = Q0/B0
deltheta = Q/B - theta

A = ((1./(2.*Gm)))
A0 = ((1./(2.*Gm0)))
B = (1./(2.*Gu))
B0 = (1./(2.*Gu0))
lambda1 = C10/B0
mu1 = D10/(B0)
lambda2 = C20/B0
mu2 = D20/(B0)
nubar = A0/B0
dellambda1 = C1/B - lambda1
delmu1 = D1/(2.*B) - mu1
dellambda2 = C2/B - lambda2
delmu2 = D2/(2.*B) - mu2
delnubar = (A/B) - (A0/B0)
theta = Q0/B0
deltheta= Q/B - theta
TERM1 = .5*DTIME + nubar + .5*delnubar
TERM1I = 1./TERM1
TERM2 = (.5*DTIME*(mu1 + .5*delmu1) + mu2 + .5*delmu2)*TERM1I
I1 = NDI
DO K1=1,NSHR
   I1 = I1+1
   DDSDDE(I1,I1) = TERM2
   DDSDDT(I1) = 0.0
END DO
RETURN
END
C. ABAQUS code for validation of three-dimensional model

The single element abaqus input file used to validate the 3D model is presented below. Abaqus model version 6.12-1 was used as the solver for this model.

*Heading
** Job name: smp_1_ele Model name: Model-1
** Generated by: Abaqus/CAE 6.12-1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**
** INCLUDE, INPUT=balmo8.f
***balmo2b.f
** PARTS
**
*Part, name=Part-1
*Node
  1, -0.0199999996, -0.0199999996, 0.0399999991
  2, -0.0199999996, 0.0199999996, 0.0399999991
  3, -0.0199999996, -0.0199999996, 0.
  4, -0.0199999996, 0.0199999996, 0.
  5, 0.0199999996, -0.0199999996, 0.0399999991
  6, 0.0199999996, 0.0199999996, 0.0399999991
  7, 0.0199999996, -0.0199999996, 0.
  8, 0.0199999996, 0.0199999996, 0.
*Element, type=C3D8T
  1, 5, 6, 8, 7, 1, 2, 4, 3
*Nset, nset=_PickedSet5, internal, generate
  1, 8, 1
*Elset, elset=_PickedSet5, internal
  1,
** Section: smp
*Solid Section, elset=_PickedSet5, material=smpmat
  ,
*End Part
**
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=smpelem1, part=Part-1
*End Instance
**
*Nset, nset=_PickedSet4, internal, instance=smpelem1, generate
  1, 4, 1
*Elset, elset=_PickedSet4, internal, instance=smpelem1
  1,
*Nset, nset=_PickedSet4b, internal, instance=smpelem1, generate
  5, 8, 1
*Elset, elset=_PickedSet4b, internal, instance=smpelem1
  1,
*Elset, elset=__PickedSurf5_S1, internal, instance=smpelem1
  1,
*Nset, nset=_PickedSet9, internal, instance=smpelem1, generate
  1, 8, 1
*Elset, elset=_PickedSet9, internal, instance=smpelem1
1,
*Surface, type=ELEMENT, name=_PickedSurf5, internal
_PickedSurf5_S1, S1
*Elset, elset=__PickedSurf6_S1, internal, instance=smpelem1
1,
*Surface, type=ELEMENT, name=_PickedSurf6, internal
_PickedSurf6_S1, S1
*End Assembly
**
** MATERIALS
**
*Material, name=Poly
*Conductivity
0.9,
*Density
1.,
*Elastic
1.46e+08, 0.45
*Material, name=smpmat
*Density
1,
*Conductivity
0.17,
*User Material, constants=13, type=MECHANICAL
**tobushifnk matprop=14
**146e6,38.1,14e9,44.2,0.42,0.003,58.2,0.112
**38.7,521,35.4,348,328,11.6e-5
**balmo2/7 matprop=12
27.6e6,907e6,2.03e9,116e9,111,2840,0.48,0.33
11.6e-5,328,348,30.,0.38e1
*DEPVAR
24
** INITIAL CONDITIONS
**
*INITIAL CONDITION, TYPE=TEMP
_PickedSet9, 348
**
**---------------------------------------------------------------
**
**---------------------------------------------------------------
**
** STEP: Step-1
**
*Step, name=Step-1
*Coupled Temperature-displacement, creep=none, steady state
1., 1., 1e-05, 1.
*** Name: BC-1 Type: Displacement/Rotation
*Boundary
_PickedSet4, 1, 1
_PickedSet4, 2, 2
_PickedSet4, 3, 3
_PickedSet4, 4, 4
_PickedSet4, 5, 5
_PickedSet4, 6, 6
** LOADS  
**
** Name: Load-2  Type: Pressure  
*Dsload, op=NEW
_PickedSurf6, P, -1e5  
** _PickedSurf6, P, -20e5.  
** _PickedSurf6, P, -68e4.  
**
** BOUNDARY CONDITIONS  
**
** Name: BC-2 Type: Temperature  
*Boundary
_PickedSet9, 11, 11, 348  
** OUTPUT REQUESTS  
**
*Restart, write, frequency=0  
**
** FIELD OUTPUT: F-Output-1  
**
*Output, field  
*Node Output  
CF, RF, U  
*Element Output, directions=YES  
E, S, SDV, TEMP  
**
** HISTORY OUTPUT: H-Output-1  
**
*Output, history, variable=PRESELECT  
*End Step  
**
** STEP: Step-2  
**
*Step, name=Step-2  
*Coupled Temperature-displacement, creep=none, steady state  
0.2, 1., 1e-05, 1.  
**
** BOUNDARY CONDITIONS  
**
** Name: BC-2 Type: Velocity/Angular velocity  
*Boundary, op=NEW, type=VELOCITY
_PickedSet4b, 1, 1  
_PickedSet4b, 2, 2  
_PickedSet4b, 3, 3  
_PickedSet4b, 4, 4  
_PickedSet4b, 5, 5  
_PickedSet4b, 6, 6  
*** Name: BC-1 Type: Displacement/Rotation  
*Boundary, op=NEW
_PickedSet4, 1, 1  
_PickedSet4, 2, 2  
_PickedSet4, 3, 3  
_PickedSet4, 4, 4  
_PickedSet4, 5, 5  
_PickedSet4, 6, 6
** LOADS 
** 
** Name: Load-2   Type: Pressure
*Dsload, op=NEW
**_PickedSurf6, P, -12e5.
** BOUNDARY CONDITIONS 
** 
** Name: BC-2 Type: Temperature
*Boundary, op=NEW
  _PickedSet9, 11, 11, 308
** 
** OUTPUT REQUESTS 
** 
*Restart, write, frequency=0 
** 
** FIELD OUTPUT: F-Output-1 
** 
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
** 
** HISTORY OUTPUT: H-Output-1 
** 
*Output, history, variable=PRESELECT
*End Step
**  
** STEP: Step-3 
** 
*Step, name=Step-3
*Coupled Temperature-displacement, creep=none, steady state
1., 1., 1e-05, 1
*** Name: BC-2 Type: Velocity
*Boundary, op=NEW
** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
  _PickedSet4, 1, 1
  _PickedSet4, 2, 2
  _PickedSet4, 3, 3
  _PickedSet4, 4, 4
  _PickedSet4, 5, 5
  _PickedSet4, 6, 6
** LOADS 
** 
** Name: Load-2   Type: Pressure
*Dsload, op=NEW
**_PickedSurf6, P, 0
** BOUNDARY CONDITIONS 
** 
** Name: BC-2 Type: Temperature
*Boundary, op=New
  _PickedSet9, 11, 11, 308
**
** OUTPUT REQUESTS**

*Restart, write, frequency=0

** FIELD OUTPUT: F-Output-1**

*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP

** HISTORY OUTPUT: H-Output-1**

*Output, history, variable=PRESELECT
*End Step

** STEP: Step-4**

*Step, name=Step-4
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1

** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
_PickedSet4, 1, 1
_PickedSet4, 2, 2
_PickedSet4, 3, 3
_PickedSet4, 4, 4
_PickedSet4, 5, 5
_PickedSet4, 6, 6

*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW

** LOADS**

** Name: Load-2 Type: Pressure
*Dsload, op=NEW
**_PickedSurf6, P, 0

** BOUNDARY CONDITIONS**

** Name: BC-2 Type: Temperature
*Boundary, op=NEW
*Boundary, op=NEW
_PickedSet9, 11, 11, 348

** OUTPUT REQUESTS**

*Restart, write, frequency=0

** FIELD OUTPUT: F-Output-1**

*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
D. ABAQUS code for morphing of flapping wing micro air vehicle

Three models have been generated for the FW-MAV analysis and these have been presented below.

SMP-SPAR : This is used to validate spar response to model. A compressive load factor of -28.5 was used for this model.

*Heading
** Job name: spar Model name: spar
** Generated by: Abaqus/CAE 6.12-1
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**INCLUDE, INPUT=balmo8.f
**balmo2b.f
** PARTS
**
*Part, name=spar2
*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=spar2-1, part=spar2
**
*Node
1, -15., -1., 2.
2, -15., 1., 2.
3, -15., -1., 0.
4, -15., 1., 0.
5, -13., -1., 2.
6, -13., 1., 2.
7, -13., -1., 0.
8, -13., 1., 0.
9, -11., -1., 2.
10, -11., 1., 2.
11, -11., -1., 0.
12, -11., 1., 0.
13, -9., -1., 2.
14, -9., 1., 2.
15, -9., -1., 0.
16, -9., 1., 0.
17, -7., -1., 2.
18, -7., 1., 2.
19, -7., -1., 0.
20, -7., 1., 0.
21, -5., -1., 2.
22, -5., 1., 2.
23, -5., -1., 0.
24, -5., 1., 0.
25, -3., -1., 2.
26, -3., 1., 2.
27, -3., -1., 0.
28, -3., 1., 0.
29, -1., -1., 2.
30, -1., 1., 2.
31, -1., -1., 0.
32, -1., 1., 0.
33, 1., -1., 2.
34, 1., 1., 2.
35, 1., -1., 0.
36, 1., 1., 0.
37, 3., -1., 2.
38, 3., 1., 2.
39, 3., -1., 0.
40, 3., 1., 0.
41, 5., -1., 2.
42, 5., 1., 2.
43, 5., -1., 0.
44, 5., 1., 0.
45, 7., -1., 2.
46, 7., 1., 2.
47, 7., -1., 0.
48, 7., 1., 0.
49, 9., -1., 2.
50, 9., 1., 2.
51, 9., -1., 0.
52, 9., 1., 0.
53, 11., -1., 2.
54, 11., 1., 2.
55, 11., -1., 0.
56, 11., 1., 0.
57, 13., -1., 2.
58, 13., 1., 2.
59, 13., -1., 0.
60, 13., 1., 0.
61, 15., -1., 2.
62, 15., 1., 2.
63, 15., -1., 0.
64, 15., 1., 0.

*Element, type=C3D8T
1, 5, 6, 8, 7, 1, 2, 4, 3
2, 9, 10, 12, 11, 5, 6, 8, 7
3, 13, 14, 16, 15, 9, 10, 12, 11
4, 17, 18, 20, 19, 13, 14, 16, 15
5, 21, 22, 24, 23, 17, 18, 20, 19
6, 25, 26, 28, 27, 21, 22, 24, 23
7, 29, 30, 32, 31, 25, 26, 28, 27
8, 33, 34, 36, 35, 29, 30, 32, 31
9, 37, 38, 40, 39, 33, 34, 36, 35
10, 41, 42, 44, 43, 37, 38, 40, 39
11, 45, 46, 48, 47, 41, 42, 44, 43
12, 49, 50, 52, 51, 45, 46, 48, 47
13, 53, 54, 56, 55, 49, 50, 52, 51
14, 57, 58, 60, 59, 53, 54, 56, 55
15, 61, 62, 64, 63, 57, 58, 60, 59

*Nset, nset=Set-1, generate
1, 64, 1

*Elset, elset=Set-1, generate
** Section: Section-1
*Solid Section, elset=Set-1, material=smpmat
  
*End Instance
**
*Nset, nset=Set-5, instance=spar2-1, generate 1, 4, 1
*Elset, elset=Set-5, instance=spar2-1

*Nset, nset=Set-6, instance=spar2-1, generate 1, 64, 1
*Elset, elset=Set-6, instance=spar2-1, generate 1, 15, 1
*Nset, nset=Set-7, instance=spar2-1, generate 61, 64, 1
*Elset, elset=Set-7, instance=spar2-1 15,
*Elset, elset=_Surf-2_S2, internal, instance=spar2-1 1,
*Surface, type=ELEMENT, name=Surf-2 _Surf-2_S2, S2
*Elset, elset=_Surf-3_S1, internal, instance=spar2-1 15,
*Surface, type=ELEMENT, name=Surf-3 _Surf-3_S1, S1
*End Assembly
**
** MATERIALS
**
*Material, name=poly
*Conductivity 0.17,
*Elastic 2e+07, 0.33
*Material, name=smpmat
*Density 1,
*Conductivity 0.17,
*User Material, constants=13, type=MECHANICAL 27.6e6,907e6,2.03e9,116e9,111,2840,0.48,0.33 11.6e-5,328,348,30.,-28.5
** 28.5
*DEPVAR 24
** INITIAL CONDITIONS
**
*INITIAL CONDITION, TYPE=TEMP Set-6, 348
**
** ---------
** STEP: Step-1
**
*Step, name=Step-1
*Coupled Temperature-displacement, creep=none, steady state
  1., 1., 1e-05, 1.
*** Name: BC-1 Type: Displacement/Rotation
*Boundary
Set-5, 1, 1
Set-5, 2, 2
Set-5, 3, 3
Set-5, 4, 4
Set-5, 5, 5
Set-5, 6, 6
** LOADS
**
** Name: Load-2 Type: Pressure
*Dsload, op=NEW
Surf-3, P, 7e+06
**Surf-3, P, -20e5.
**Surf-3, P, -68e4.
**
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary
Set-6, 11, 11, 348
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ---------------------------------------------------------------
**
** STEP: Step-2
**
*Step, name=Step-2
*Coupled Temperature-displacement, creep=none, steady state
  0.2, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Velocity/Angular velocity
*Boundary, op=NEW, type=VELOCITY
Set-7, 1, 1
Set-7, 2, 2  
Set-7, 3, 3  
Set-7, 4, 4  
Set-7, 5, 5  
Set-7, 6, 6  
*** Name: BC-1 Type: Displacement/Rotation  
*Boun*boundary, op=NEW  
Set-5, 1, 1  
Set-5, 2, 2  
Set-5, 3, 3  
Set-5, 4, 4  
Set-5, 5, 5  
Set-5, 6, 6  
** LOADS  
**  
** Name: Load-2 Type: Pressure  
*Dsload, op=NEW  
**_PickedSurf6, P, 12e5.  
** BOUNDARY CONDITIONS  
**  
** Name: BC-2 Type: Temperature  
*Boun*boundary, op=NEW  
Set-6, 11, 11, 308  
**  
** OUTPUT REQUESTS  
**  
** Restart, write, frequency=0  
**  
** FIELD OUTPUT: F-Output-1  
**  
*Output, field  
*Node Output  
CF, RF, U, NT  
*Element Output, directions=YES  
E, S, SDV, TEMP  
**  
** HISTORY OUTPUT: H-Output-1  
**  
*Output, history, variable=PRESELECT  
*End Step  
**  
** STEP: Step-3  
**  
*Step, name=Step-3  
*Coupled Temperature-displacement, creep=none, steady state  
1., 1., 1e-05, 1  
*** Name: BC-2 Type: Velocity  
*Boun*boundary, op=NEW  
** Name: BC-1 Type: Displacement/Rotation  
*Boun*boundary, op=NEW  
Set-5, 1, 1  
Set-5, 2, 2  
Set-5, 3, 3  
Set-5, 4, 4
** LOADS **

** Name: Load-2   Type: Pressure
*Dsload, op=NEW
**_PickedSurf6, P, 0
** BOUNDARY CONDITIONS

** Name: BC-2 Type: Temperature
*Boundary, op=New
Set-6, 11, 11, 308
**

** OUTPUT REQUESTS **

*Restart, write, frequency=0
**

** FIELD OUTPUT: F-Output-1 **

*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**

** HISTORY OUTPUT: H-Output-1 **

*Output, history, variable=PRESELECT
*End Step
** ---------------------------------------------------------------

** STEP: Step-4 **

*Step, name=Step-4
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1
** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-5, 1, 1
Set-5, 2, 2
Set-5, 3, 3
Set-5, 4, 4
Set-5, 5, 5
Set-5, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
** LOADS

** Name: Load-2   Type: Pressure
*Dsload, op=NEW
**_PickedSurf6, P, 0
** BOUNDARY CONDITIONS

** Name: BC-2 Type: Temperature
**Boundary, op=NEW
*Boundary, op=NEW
Set-6, 11, 11, 348
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
SMP-SKIN: This is used to validate skin response to model. A tensile load factor of 28.5 was used for this model.

*Heading  
** Job name: smpskin Model name: new-skin-smp  
** Generated by: Abaqus/CAE 6.13-3  
*Preprint, echo=NO, model=NO, history=NO, contact=NO  
**INCLUDE, INPUT=balmo8.f  
**balmo2b.f  
** PARTS  
**  
*Part, name=Part-1  
*End Part  
**  
** ASSEMBLY  
**  
*Assembly, name=Assembly  
**  
*Instance, name=Part-1-1, part=Part-1  
**  
*Node  
1,  -10.78125,  -29.1796875,    0.5  
2,  -33.8876343,  -29.2053223,    0.5  
3,  -34.21875,    -30.,    0.5  
4,  -10.78125,    -30.,    0.5  
5,  -33.8876343,  -29.2053223,    0.  
6,  -34.21875,    -30.,    0.  
7,  -10.78125,    -30.,    0.  
8,  -10.78125,  -29.1796875,    0.  
9,  -12.34375,    22.5,    0.  
10, -10.78125,    22.5,    0.  
11, -12.34375,    22.5,    0.5  
12, -10.78125,    22.5,    0.5  
13, -13.125,    22.5,    0.5  
14,  -35.,    -30.,    0.5  
15,  -13.125,    22.5,    0.  
16,  -35.,    -30.,    0.  
17,  -10.,    -30.,    0.5  
18,  -10.,    22.5,    0.5  
19,  -10.,    22.5,    0.  
20,  -10.,    -30.,    0.  
21, -12.225399,  -29.1812897,    0.5  
22, -13.669548,  -29.1828918,    0.5  
23, -15.1136971,  -29.184494,    0.5  
24, -16.5578461,  -29.1860962,    0.5  
25, -18.0019951,  -29.1876984,    0.5  
26, -19.4461441,  -29.1893005,    0.5  
27, -20.8902931,  -29.1909027,    0.5  
28, -22.3344421,  -29.1925049,    0.5  
29, -23.7785912,  -29.1941071,    0.5  
30, -25.2227402,  -29.1957092,    0.5  
31, -26.6668892,  -29.1973114,    0.5
32, -28.1110382, -29.1989136, 0.5
33, -29.5551872, -29.2005157, 0.5
34, -30.9993362, -29.2021179, 0.5
35, -32.4434853, -29.2037201, 0.5
36, -32.7539063, -30., 0.5
37, -31.2890625, -30., 0.5
38, -29.8242188, -30., 0.5
39, -28.359375, -30., 0.5
40, -26.8945313, -30., 0.5
41, -25.4296875, -30., 0.5
42, -23.9648438, -30., 0.5
43, -22.5, -30., 0.5
44, -21.0351563, -30., 0.5
45, -19.5703125, -30., 0.5
46, -18.1054688, -30., 0.5
47, -16.640625, -30., 0.5
48, -15.1757813, -30., 0.5
49, -13.7109375, -30., 0.5
50, -12.2460938, -30., 0.5
51, -32.4434853, -29.2037201, 0.
52, -30.9993362, -29.2021179, 0.
53, -29.5551872, -29.2005157, 0.
54, -28.1110382, -29.1989136, 0.
55, -26.6668892, -29.1973114, 0.
56, -25.2227402, -29.1957092, 0.
57, -23.7785912, -29.1941071, 0.
58, -22.3344421, -29.1925049, 0.
59, -20.8902931, -29.1909027, 0.
60, -19.4461441, -29.1893005, 0.

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*Element, type=C3D8T
1, 8, 65, 66, 7, 1, 21, 50, 4
2, 65, 64, 67, 66, 21, 22, 49, 50
3, 64, 63, 68, 67, 22, 23, 48, 49
4, 63, 62, 69, 68, 23, 24, 47, 48
5, 62, 61, 70, 69, 24, 25, 46, 47
6, 61, 60, 71, 70, 25, 26, 45, 46
7, 60, 59, 72, 71, 26, 27, 44, 45
8, 59, 58, 73, 72, 27, 28, 43, 44
9, 58, 57, 74, 73, 28, 29, 42, 43
10, 57, 56, 75, 74, 29, 30, 41, 42
11, 56, 55, 76, 75, 30, 31, 40, 41
12, 55, 54, 77, 76, 31, 32, 39, 40
13, 54, 53, 78, 77, 32, 33, 38, 39
14, 53, 52, 79, 78, 33, 34, 37, 38
15, 52, 51, 80, 79, 34, 35, 36, 37
16, 51, 5, 6, 80, 35, 2, 3, 36
17, 1, 21, 1633, 309, 8, 65, 643, 227
18, 21, 22, 1634, 1633, 65, 64, 644, 643
19, 22, 23, 1635, 1634, 64, 63, 645, 644
20, 23, 24, 1636, 1635, 63, 62, 646, 645
21, 24, 25, 1637, 1636, 62, 61, 647, 646
22, 25, 26, 1638, 1637, 61, 60, 648, 647
[... truncated for space ...]

*Nset, nset=Set-1, generate
  1,  2622,   1

*Elset, elset=Set-1, generate
  1,  1224,   1

*Nset, nset=in-spar
  2,   3,   5,   6,   9,  11,  13,  14,  15,  16,  81,  82,  83,  84,  85,  86
  87,  88,  89,  90,  91,  92,  93,  94,  95,  96,  97,  98,  99, 100, 101, 102
  119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134

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135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 228, 229, 230, 231
132, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247
248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263
264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279
280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 375, 376
393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408
409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424
441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456
457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472
473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488
489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504
505, 506, 507, 508
*Elset, elset=in-spar, generate
1089, 1156, 1
*Nset, nset=out-spar
1, 4, 7, 8, 10, 12, 17, 18, 19, 20, 162, 163, 164, 165, 166, 167
168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183
184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199
200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215
216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 309, 310, 311, 312
313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328
511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526
527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542
543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558
163
559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574
575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590
591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606
607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622
623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638
639, 640, 641, 642
*Elset, elset=out-spar, generate
1157, 1224, 1
*Nset, nset=low-spar
1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 25, 26, 27, 28
29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
77, 78, 79, 80
*Elset, elset=low-spar, generate
1, 16, 1
*Nset, nset=skin
1, 2, 5, 8, 9, 10, 11, 12, 21, 22, 23, 24, 25, 26, 27, 28
29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
77, 78, 79, 80
*Elset, elset=spar, generate
1157, 1158

[... truncated for space ...]

*Elset, elset=skin, generate
  17, 1088, 1
*Nset, nset=spar
*Elset, elset=_Surf-1_S1, internal, generate
  17, 1088, 1
*Surface, type=ELEMENT, name=Surf-1
_Surf-1_S1, S1
*Elset, elset=_Surf-2_S2, internal, generate
  17, 1088, 1
*Surface, type=ELEMENT, name=Surf-2
_Surf-2_S2, S2
** Section: Section-1
*Solid Section, elset=Set-1, material=smpmat
  ,
*End Instance
**
*Nset, nset=Set-8, instance=Part-1-1
  14, 16, 17, 20
*Elset, elset=Set-8, instance=Part-1-1
  1156, 1157
*Nset, nset=Set-13, instance=Part-1-1
  13, 14, 15, 16, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386
  387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402
  403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418
  419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434
  435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450
  467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482
  483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498
  499, 500, 501, 502, 503, 504, 505, 506, 507, 508
*Elset, elset=Set-13, instance=Part-1-1, generate
  1089, 1156, 1
*Nset, nset=Set-14, instance=Part-1-1
  17, 18, 19, 20, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520
  521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536
  537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552
  553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568
  569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584
  585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600
  601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615,
  616
  617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631,
  632
  633, 634, 635, 636, 637, 638, 639, 640, 641, 642
*Elset, elset=Set-14, instance=Part-1-1, generate
1157, 1224, 1
*Nset, nset=Set-15, instance=Part-1-1
9, 10, 11, 12, 13, 15, 18, 19, 147, 148, 149, 150, 151, 152, 153, 154
155, 156, 157, 158, 159, 160, 161, 294, 295, 296, 297, 298, 299, 300, 301, 302
303, 304, 305, 306, 307, 308
*Elset, elset=Set-15, instance=Part-1-1
1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084,
1085, 1086, 1087, 1088
1089, 1224
*Nset, nset=_PickedSet32, internal, instance=Part-1-1
17, 20
*Elset, elset=_PickedSet32, internal, instance=Part-1-1
1157,
*Elset, elset=_Surf-3_S1, internal, instance=Part-1-1, generate
1157, 1224, 1
*Surface, type=ELEMENT, name=Surf-3
_Surf-3_S1, S1
*End Assembly
**
** MATERIALS
**
*Material, name=polymer
*Conductivity
0.17,
*Elastic
9.06e+08, 0.32, 308.
2.7e+07, 0.48, 348.
*Material, name=sma
*Conductivity
0.17,
*Elastic
7.3e+10, 0.33, 308.
2.7e+10, 0.4, 348.
*Material, name=smpmat
*Density
1,
*Conductivity
0.17,
*User Material, constants=13, type=MECHANICAL
27.6e6,907e6,2.03e9,116e9,111,2840,0.48,0.33
11.6e-5,328,348,30.,28.5
**
*DEPVAR
24
** INITIAL CONDITIONS
**
*INITIAL CONDITION, TYPE=TEMP
Part-1-1.Set-1, 348
** STEP: Step-1
**
*Step, name=Step-1
*Coupled Temperature-displacement, creep=none, steady state
1., 1., 1e-05, 1.
*** Name: BC-1 Type: Displacement/Rotation
*Boundary
Set-13, 1, 1
Set-13, 2, 2
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
** LOADS
**
** Name: Load-2 Type: Pressure
*Dsload, op=NEW
Surf-3, P, -7e+06
**Surf-3, P, -20e5.
**Surf-3, P, -68e4.
**
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary
Part-1-1.Set-1, 11, 11, 348
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** --------------------------------------------------------------
**
** STEP: Step-2
**
*Step, name=Step-2
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Velocity/Angular velocity
*Boundary, op=NEW, type=VELOCITY
Set-14, 1, 1
Set-14, 2, 2
Set-14, 3, 3
Set-14, 4, 4
Set-14, 5, 5
Set-14, 6, 6
*** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-13, 1, 1
Set-13, 2, 2
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
** LOADS
**
** Name: Load-2 Type: Pressure
*DSload, op=NEW
** PickedSurf6, P, -12e5.
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary, op=NEW
Part-1-1.Set-1, 11, 11, 308
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
**
** STEP: Step-3
**
*Step, name=Step-3
*Coupled Temperature-displacement, creep=none, steady state
1., 1., 1e-05, 1
*** Name: BC-2 Type: Velocity
*Boundary, op=NEW
** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-13, 1, 1
Set-13, 2, 2
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
** LOADS
**
** Name: Load-2 Type: Pressure
*Dslode, op=NEW
** _PickedSurf6, P, 0
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary, op=New
Part-1-1.Set-1, 11, 11, 308
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ---------------------------------------------------------
**
** STEP: Step-4
**
*Step, name=Step-4
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1
** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-13, 1, 1
Set-13, 2, 2
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
** LOADS
**
** Name: Load-2 Type: Pressure
*Dslode, op=NEW
** _PickedSurf6, P, 0
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
**Boundary, op=NEW
*Boundary, op=NEW
Part-1-1.Set-1, 11, 11, 348
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
SMP-WING: The SMP spar co-joined with polymer skin have been shown below.

*Heading
** Job name: smpwing Model name: new-sparskin-smp
** Generated by: Abaqus/CAE 6.13-3
*Preprint, echo=NO, model=NO, history=NO, contact=NO
**INCLUDE,INPUT=balmo8.f
**
** PARTS
**
*Part, name=Part-1
*End Part
**
** ASSEMBLY
**
*Assembly, name=Assembly
**
*Instance, name=Part-1-1, part=Part-1
**
*Node
  1, -10.78125, -29.1796875, 0.5
  2, -33.8876343, -29.2053223, 0.5
  3, -34.21875, -30., 0.5
  4, -10.78125, -30., 0.5
  5, -33.8876343, -29.2053223, 0.
  6, -34.21875, -30., 0.
  7, -10.78125, -30., 0.
  8, -10.78125, -29.1796875, 0.
  9, -12.34375, 22.5, 0.
 10, -10.78125, 22.5, 0.
 11, -12.34375, 22.5, 0.5
 12, -10.78125, 22.5, 0.5
 13, -13.125, 22.5, 0.5
 14, -35., -30., 0.5
 15, -13.125, 22.5, 0.
 16, -35., -30., 0.
 17, -10., -30., 0.5
 18, -10., 22.5, 0.5
 19, -10., 22.5, 0.
 20, -10., -30., 0.
 21, -12.225399, -29.1812897, 0.5
 22, -13.669548, -29.1828918, 0.5
 23, -15.1136971, -29.184494, 0.5
 24, -16.5578461, -29.1860962, 0.5
 25, -18.0019951, -29.1876984, 0.5
 26, -19.4461441, -29.1893005, 0.5
 27, -20.8902931, -29.1909027, 0.5
 28, -22.3344421, -29.1925049, 0.5

[... truncated for space ...]

*Element, type=C3D8T
  1,  8, 65, 66,  7,  1, 21, 50,  4
  2, 65, 64, 67, 66, 21, 22, 49, 50
  3, 64, 63, 68, 67, 22, 23, 48, 49
| 4, 63, 62, 69, 68, 23, 24, 47, 48 |
| 5, 62, 61, 70, 69, 24, 25, 46, 47 |
| 6, 61, 60, 71, 70, 25, 26, 45, 46 |
| 7, 60, 59, 72, 71, 26, 27, 44, 45 |
| 8, 59, 58, 73, 72, 27, 28, 43, 44 |
| 9, 58, 57, 74, 73, 28, 29, 42, 43 |
| 10, 57, 56, 75, 74, 29, 30, 41, 42 |
| 11, 56, 55, 76, 75, 30, 31, 40, 41 |
| 12, 55, 54, 77, 76, 31, 32, 39, 38 |
| 13, 54, 53, 78, 77, 32, 33, 38, 37 |
| 14, 53, 52, 79, 78, 33, 34, 37, 36 |
| 15, 52, 51, 80, 79, 34, 35, 36, 35 |
| 16, 51, 5, 6, 80, 35, 2, 3, 36 |

... truncated for space ...

1224, 642, 509, 18, 19, 162, 374, 12, 10
*Nset, nset=Set-1, generate
  1, 2622, 1
*Elset, elset=Set-1, generate
  1, 1224, 1
*Nset, nset=in-spar
  2, 3, 5, 6, 9, 11, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86
  87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102
  119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134
  135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 228, 229, 230, 231
  232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247
  248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263
  264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279
  280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 375, 376

174
175

393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408
409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424
441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456
457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472
473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488
489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504
505, 506, 507, 508
*Elset, elset=in-spar, generate
1089, 1156, 1
*Nset, nset=out-spar
  1, 4, 7, 8, 10, 12, 17, 18, 19, 20, 162, 163, 164, 165, 166, 167
168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183
184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199
200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215
216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 309, 310, 311, 312
313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328
511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526
527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542
543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558
559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574
575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590
591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606
607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622
623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638
639, 640, 641, 642
*Elset, elset=out-spar, generate
1157, 1224, 1
*Nset, nset=low-spar
  1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 25, 26, 27, 28
  29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
  45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
  61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
  77, 78, 79, 80
*Elset, elset=low-spar, generate
  1, 16, 1
*Nset, nset=skin
  1, 2, 5, 8, 9, 10, 11, 12, 21, 22, 23, 24, 25, 26, 27, 28
  29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
  45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
  60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
  77, 78, 79, 80
*Nset, nset=low-spar
  1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 25, 26, 27, 28
  29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
  45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
  60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
  77, 78, 79, 80
*Elset, elset=skin, generate
  17, 1088, 1
*Nset, nset=spar
  1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 25, 26, 27, 28
  29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
  45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
  60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
  77, 78, 79, 80
*Nset, nset=low-spar
  1, 2, 3, 4, 5, 6, 7, 8, 21, 22, 23, 24, 25, 26, 27, 28
  29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44
  45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
  60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76
  77, 78, 79, 80
*Elset, elset=skin, generate
  17, 1088, 1
*Elset, elset=spar
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16
1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120
*Surface, type=ELEMENT, name=Surf-1
_Surf-1_S1, S1
*Elset, elset=_Surf-1_S1, internal, generate
17, 1088, 1
*Surface, type=ELEMENT, name=Surf-2
_Surf-2_S2, S2
** Section: smp-spar
*Solid Section, elset=spar, material=smp-spar

** Section: smp-skin
*Solid Section, elset=skin, material=smp-skin

*End Instance

**
*Nset, nset=Set-8, instance=Part-1-1
14, 16, 17, 20
*Elset, elset=Set-8, instance=Part-1-1
1156, 1157
*Nset, nset=Set-13, instance=Part-1-1
13, 14, 15, 16, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386
387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402
403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418
419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434
435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450
467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482
483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498
499, 500, 501, 502, 503, 504, 505, 506, 507, 508
*Elset, elset=Set-13, instance=Part-1-1, generate
1089, 1156, 1
*Nset, nset=Set-14, instance=Part-1-1
**Elset, elset=Set-19, instance=Part-1-1, generate 1157, 1224, 1
*Nset, nset=_PickedSet32, internal, instance=Part-1-1 17, 20
*Elset, elset=_PickedSet32, internal, instance=Part-1-1 1157,
*Elset, elset=_Surf-3_S1, internal, instance=Part-1-1, generate 1157, 1224, 1
*Surface, type=ELEMENT, name=Surf-3 _Surf-3_S1, S1
*Elset, elset=_Surf-4_S3, internal, instance=Part-1-1 1157,
*Elset, elset=_Surf-5_S1, internal, instance=Part-1-1, generate 1157, 1224, 1
*Surface, type=ELEMENT, name=Surf-5 _Surf-5_S1, S1
*Elset, elset=_Surf-4_S5, internal, instance=Part-1-1, generate 1, 16, 1
*Surface, type=ELEMENT, name=Surf-4 _Surf-4_S5, S5
*End Assembly
**
** MATERIALS
**
*Material, name=smp-skin
*Density 1,
*Conductivity 0.17,
**User Material, constants=13, type=MECHANICAL
**27.6e6,300e6,2.03e9,116e9,111,2840,0.48,0.33
**11.6e-5,328,348,30.,-28.5
*Elastic 300e6, 0.33, 308.
12.6e+6, 0.48, 348.
*******************************
*Material, name=smp-spar
*Density 1,
*Conductivity 0.17,
*User Material, constants=13, type=MECHANICAL
27.6e6, 907e6, 2.03e9, 116e9, 111, 2840, 0.48, 0.33
11.6e-5, 328, 348, 30., 40.5

**Elastic
**300e6, 0.33, 308.
**2.7e+6, 0.4, 348.
*DEPVAR
24

** INITIAL CONDITIONS
**
*INITIAL CONDITION, TYPE=TEMP
Set-17, 348
**

**-----------------------------------------------**

** STEP: Step-1
**
*Step, name=Step-1
*Coupled Temperature-displacement, creep=none, steady state
1., 1., 1e-05, 1.
*** Name: BC-1 Type: Displacement/Rotation
*Boundary
Set-15, 1, 1
Set-15, 2, 2
Set-15, 3, 3
Set-15, 4, 4
Set-15, 5, 5
Set-15, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary
**Set-18, XSYMM
Set-13, 1, 1
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
** LOADS
**
** Name: Load-1 Type: Pressure
*Dsload
Surf-3, P, -7e6
** Name: Load-2 Type: Pressure
*Dsload
Surf-4, P, -4e6
**

** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary
Set-17, 11, 11, 348
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** -----------------------------------------------
**
** STEP: Step-2
**
*Step, name=Step-2
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-3 Type: Velocity/Angular velocity
*Boundary, op=NEW, type=VELOCITY
Set-19, 1, 1
Set-19, 2, 2
Set-19, 3, 3
Set-19, 4, 4
Set-19, 5, 5
Set-19, 6, 6
** Name: BC-2 Type: Velocity/Angular velocity
*Boundary, op=NEW, type=VELOCITY
Set-18, 1, 1
Set-18, 2, 2
Set-18, 3, 3
Set-18, 4, 4
Set-18, 5, 5
Set-18, 6, 6
*** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-15, 1, 1
Set-15, 2, 2
Set-15, 3, 3
Set-15, 4, 4
Set-15, 5, 5
Set-15, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
**Set-13, YSYM
Set-13, 1, 1
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
** LOADS
**
** Name: Load-2 Type: Pressure
*Dsload, op=NEW
** PickedSurf6, P, -12e5.  
** BOUNDARY CONDITIONS  
**  
** Name: BC-2 Type: Temperature  
*Boundary, op=NEW  
Part-1-1.spar, 11, 11, 270  
*Boundary, op=NEW  
Part-1-1.skin, 11, 11, 348  
**  
** OUTPUT REQUESTS  
**  
*Restart, write, frequency=0  
**  
** FIELD OUTPUT: F-Output-1  
**  
*Output, field  
*Node Output  
CF, RF, U, NT  
*Element Output, directions=YES  
E, S, SDV, TEMP  
**  
** HISTORY OUTPUT: H-Output-1  
**  
*Output, history, variable=PRESELECT  
*End Step  
**  
** STEP: Step-3  
**  
*Step, name=Step-3  
*Coupled Temperature-displacement, creep=none, steady state  
1., 1., 1e-05, 1  
*** Name: BC-2 Type: Velocity  
*Boundary, op=NEW  
** Name: BC-1 Type: Displacement/Rotation  
*Boundary, op=NEW  
Set-15, 1, 1  
Set-15, 2, 2  
Set-15, 3, 3  
Set-15, 4, 4  
Set-15, 5, 5  
Set-15, 6, 6  
*** Name: BC-2 Type: Displacement/Rotation  
*Boundary, op=NEW  
** Set-13, XSYMM  
Set-13, 1, 1  
Set-13, 3, 3  
Set-13, 4, 4  
Set-13, 5, 5  
Set-13, 6, 6  
*** Name: BC-2 Type: Displacement/Rotation  
*Boundary, op=NEW  
** Set-18, YSYMM  
** LOADS  
**
** Name: Load-2 Type: Pressure
*Dsload, op=NEW
** _PickedSurf6, P, 0
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
*Boundary, op=New
Part-1-1.spar, 11, 11, 270
*Boundary, op=New
Part-1-1.skin, 11, 11, 348
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=1
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ___________________________________________________________
**
** STEP: Step-4
**
*Step, name=Step-4
*Coupled Temperature-displacement, creep=none, steady state
0.2, 1., 1e-05, 1
** Name: BC-1 Type: Displacement/Rotation
*Boundary, op=NEW
Set-15, 1, 1
Set-15, 2, 2
Set-15, 3, 3
Set-15, 4, 4
Set-15, 5, 5
Set-15, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
**Set-13, XSYMM
Set-13, 1, 1
Set-13, 3, 3
Set-13, 4, 4
Set-13, 5, 5
Set-13, 6, 6
*** Name: BC-2 Type: Displacement/Rotation
*Boundary, op=NEW
*** Name: BC-2 Type: Displacement/Rotation
**Boundary, op=NEW
**Set-18, YSYM
** LOADS
**
** Name: Load-2 Type: Pressure
*Dslload, op=NEW
** PickedSurf6, P, 0
** BOUNDARY CONDITIONS
**
** Name: BC-2 Type: Temperature
**Boundary, op=NEW
*Boundary, op=NEW
Part-1-1.spar, 11, 11, 348
*Boundary, op=NEW
Part-1-1.skin, 11, 11, 348
**
** OUTPUT REQUESTS
**
*Restart, write, frequency=0
**
** FIELD OUTPUT: F-Output-1
**
*Output, field
*Node Output
CF, RF, U, NT
*Element Output, directions=YES
E, S, SDV, TEMP
**
** HISTORY OUTPUT: H-Output-1
**
*Output, history, variable=PRESELECT
*End Step
** ____________________________________________________________