A Vegetative Pattern Formation Aridity Classification Scheme along a Rainfall Gradient: An Example of Desertification Control
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Research Area
The vegetative pattern formation in arid and semi-arid environments modeled by an interaction-diffusion model system is investigated by nonlinear stability analyses applied to the model system.

Interaction-Diffusion Model System
Our system is an extension of a pair of partial differential equations found in Klaasen (1999).

Let $(X, Y)$ be defined on an infinite two-dimensional domain. Define $W = \text{surface water}$, $N = \text{plant biomass}$, $\tau = \text{time}$.

\[ \frac{\partial N}{\partial \tau} = F(W, N) + D_1 \nabla^2 N \]

where

\[ \frac{\partial W}{\partial \tau} = G(W, N) + D_2 \nabla^2 W \]

\[ \nabla^2 = \frac{d^2}{dX^2} + \frac{d^2}{dY^2} \]

$D_1$ and $D_2$ are the diffusion coefficients for plants and water, respectively.

For simplicity in our techniques, we define

\[ F(W, N) = RJW^2 - MN \]

\[ G(W, N) = A - LW \]

$R$ is the functional response of plants to water and $g(N) \text{ describes how plants increase water infiltration}$

$J$ is the yield of plant biomass per unit water consumed

$M$ is the density-independent mortality and maintenance rate through which plant biomass is lost

$A$ is water supplied uniformly at rate $A$ and is lost due to evaporation at rate $LW$

Bare Ground Equilibrium Point
We find the equilibrium points of the system by considering $F(W, N) = 0$ and $G(W, N) = 0$, which yields two possible stable points:

\[ N = 0, \quad W = A \]

corresponding to a bare ground or no vegetation situation that always exists and is always stable.

Homogeneous Vegetation Equilibrium Point
\[ N = N_e \]
\[ W = W_e \]

corresponding to a situation of homogeneous vegetation that exists when

\[ \frac{(A_1)^2}{2A} \geq \frac{L}{R} \]

and the stability of which is the primary focus of this research.

Nondimensionalized System
Our new system:

\[ \frac{du}{d\tau} = aw - \alpha w + \mu \nabla^2 w \]

\[ \frac{dv}{d\tau} = v^2 - \beta (1 - v^2) - w + \nabla^2 w \]

where

\[ \psi^2 = \frac{d^2}{d\tau^2} - \frac{d^2}{d\tau^2} \]

Note: The equilibrium point for the nondimensionalized system is $(1,1)$.

Hexagonal Planform Expansion

\[ n(x,y,t) = 1 \sim A_1(t) \cos \left( \phi_1 + \phi_1 \right) \]

where

\[ a \approx \frac{1}{D_1} \sim \alpha A_1 - 4a_0 A_1^2 \cos \left( \phi_1 + \phi_1 \right) \]

\[ A_2(t) \cos \left( \phi_2 + \phi_1 \right) + A_3(t) \cos \left( \phi_3 + \phi_1 \right) \]

One-Dimensional Pattern Formation Results

\[ A_2 = A_3 = \phi_1 = \phi_2 = \phi_3 = 0 \]

Landau Constant

\[ \lambda_e = \lambda_e \left( \frac{D_1}{L} \right)^{1/2} \]

Two-Dimensional Pattern Formation Results

Behavior of the Landau Constants

Aridity Classification Scheme
\[ (a < 0.055 \text{ and } \mu > 0.001) \]

Dry-subhumid \((a > 0.1442)\) Homogeneous

Semi-arid \((a < 0.1442)\) Gaps and Stripes

Arid \((a < 0.0900)\) Stripes

Hyperarid \((a < 0.0900)\) Bare Ground

Other Examples of Vegetative Patterns

References